References
- Rosenbluth '57
- Gasiorowicz '56
- Escande et al '2015

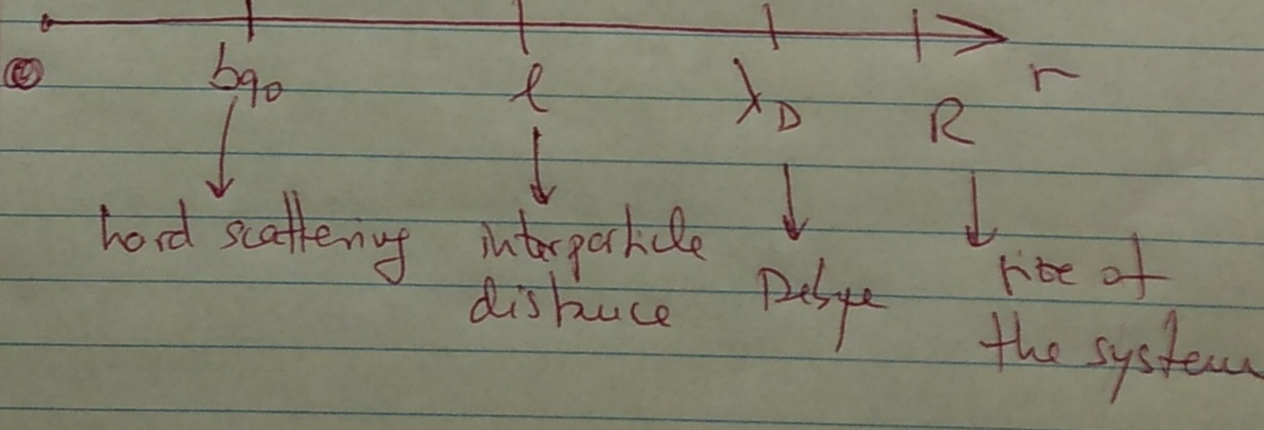
For simplicity we consider non collective situation.

<u>Question we address</u>: different derivations of the Chandrasekhar approximation

- "Landau-like": Fourier space, perturbative
- Rosenbluth: "two-body collisions"

What is the right picture? Successive two-body collisions? Successive soft scatterings? A mixture?

Scales in the problem:



For gravitational systems:

$$b_{90} \sim \frac{R}{N}, \qquad \ell \sim \frac{R}{N^{1/3}}$$

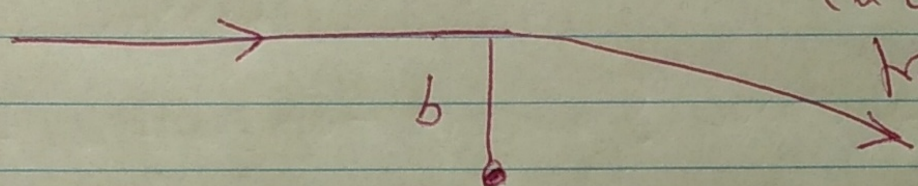
plasmas only

$\lambda_D \sim$ fraction of R

$$b_{90} \ll \ell \, (\ll \lambda_D) \ll R$$

① For impact factors $b \ll \ell$: single (hard) collision.

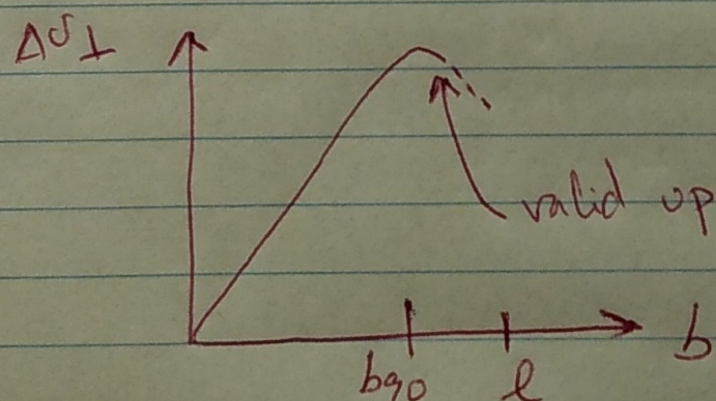
(in center of mass frame)



Two-body scattering, we can neglect all the other particles because $b \ll \ell$.

↳ we can use Rutherford formula

$$\Delta v_\perp \sim \frac{b/b_{90}}{1 + (b/b_{90})^2}$$



2 For impact factors $b \gg \ell$: many particle interaction, not allowed to use Rutherford scattering formula. But because $b \gg b_{90}$, soft collisions, and we can therefore use a perturbative approach:

$$\ddot{\vec{r}}_i = \dot{\vec{v}}_i = g \sum_{j \neq i} \frac{\vec{r}_i(t) - \vec{r}_j(t)}{\|\vec{r}_i(t) - \vec{r}_j(t)\|^3} \Rightarrow$$

(exact)

$$\Rightarrow \Delta\vec{v}_i = g \int_{-\infty}^{\infty} dt \sum_{j \neq i} \frac{\vec{r}_i(t) - \vec{r}_j(t)}{\|\vec{r}_i(t) - \vec{r}_j(t)\|^3}$$

(exact)

At lowest order: $\vec{r}_i(t) = \vec{r}_i(0) + \vec{v}_i(0)t$

We define $\vec{b}_{ij} = \vec{r}_i(0) - \vec{r}_j(0)$, $\Delta\vec{v}_{ij} = \vec{v}_i(0) - \vec{v}_j(0)$

t_{ij} = time closest approach.

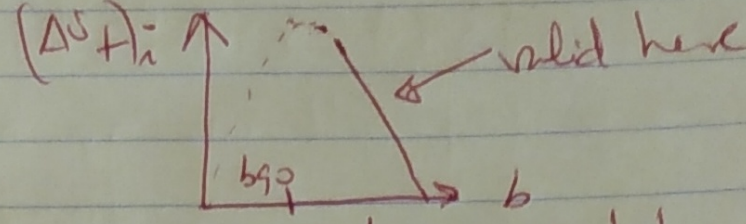
Then:

$$\Delta\vec{v}_i = g \int_{-\infty}^{\infty} dt \sum_{j \neq i} \frac{\vec{b}_{ij} + \Delta\vec{v}_{ij} t}{\|\vec{b}_{ij} + \Delta\vec{v}_{ij} t\|^3} =$$

$$= \underbrace{g \sum_{j \neq i} \int_{-\infty}^{\infty} dt \frac{\vec{b}_{ij} + \Delta\vec{v}_{ij}(t - t_{ij})}{\|\vec{b}_{ij} + \Delta\vec{v}_{ij}(t - t_{ij})}}_{\text{successive independent scatterings}}$$

In the center of mass frame, each ~~collision~~ scattering will give a contribution

$$(\Delta v_\perp)_i \sim \frac{b_{90}}{b}$$



which corresponds to the large b asymptotics of Rutherford scattering! We can therefore use here also Rutherford formula:

$$(\Delta v_\perp)_i \sim \frac{b/b_{90}}{1+\left(\frac{b}{b_{90}}\right)^2}$$

because we are in the regime $b \gg b_{90}$.

Indeed this calculation is valid for $b \gg b_{90}$, and not only $b \gg l$.

Summary