Kinetics (and fluid dynamics) of astrophysical plasmas

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Interconnections between the Physics of Plasmas and Self-gravitating systems
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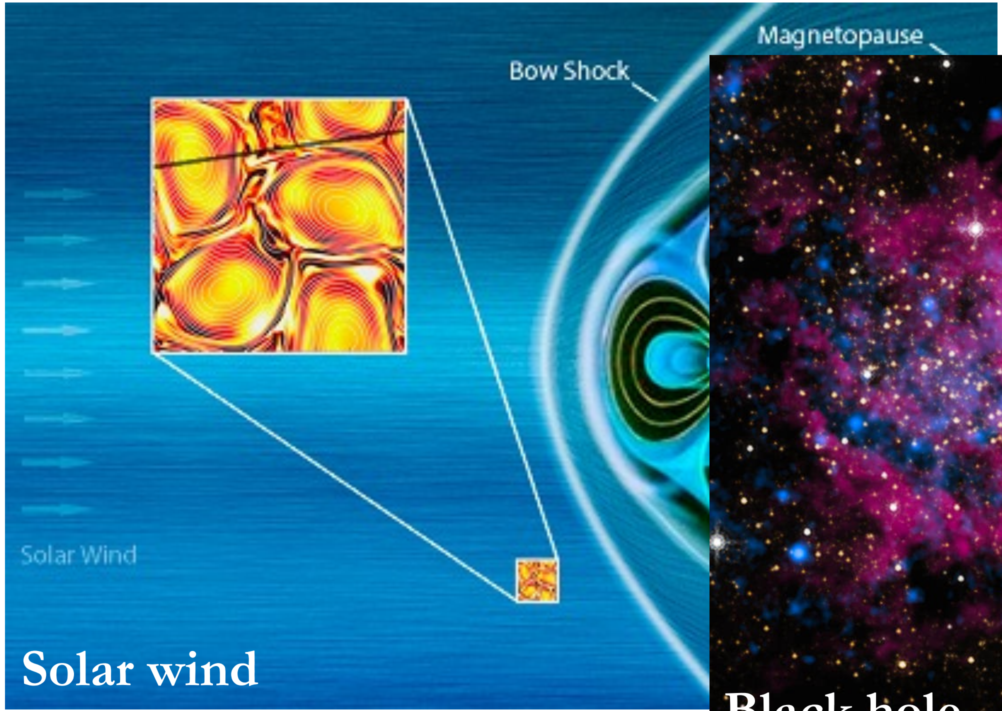
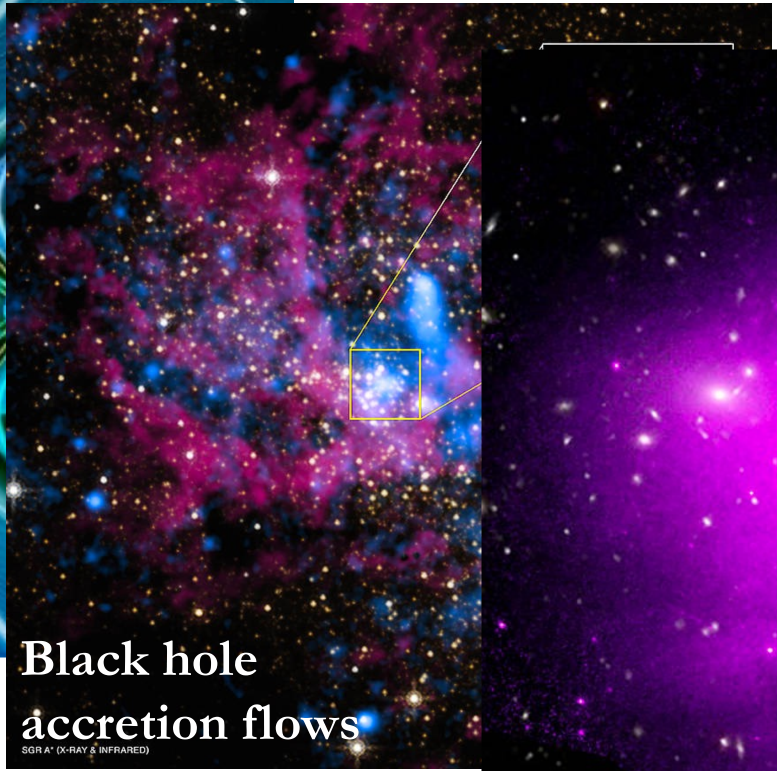
Motivation

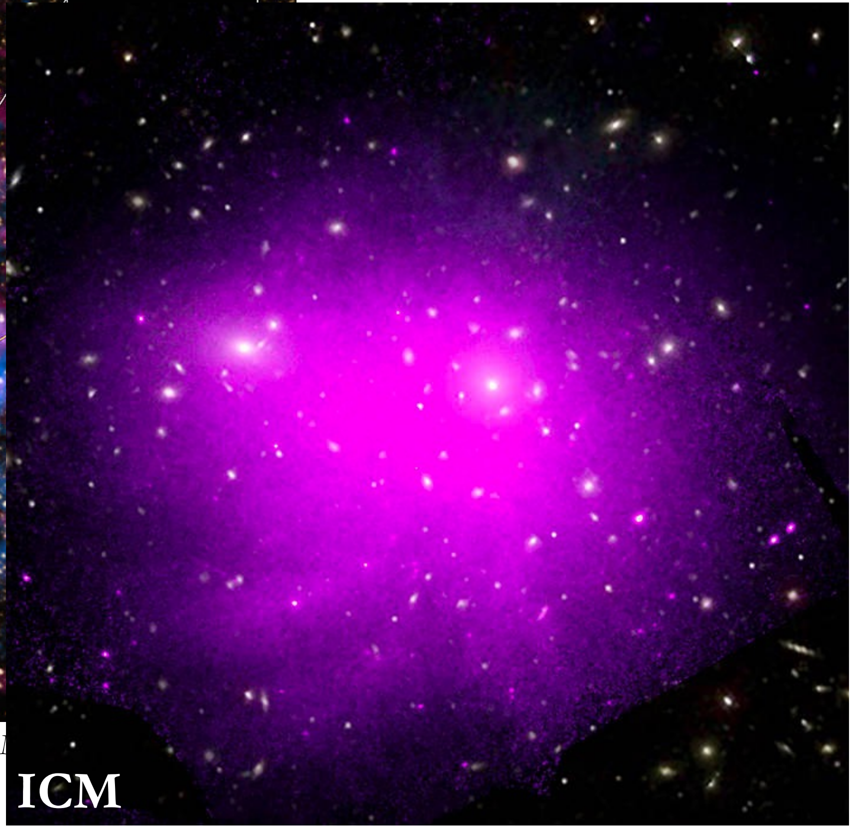
Plasma is ubiquitous in many different astrophysical contexts of interest



ESA/ATG Medialab; inset: J. Dorelli (NASA) – see Perro (2012)



X-ray: NASA/UMass/D.Wang et al., IR: I



X-ray: NASA/CXC/Univ. of Chicago, I. Zhuravleva et al, Optical: SDSS

Plasma physics → Magnetogenesis, cosmic rays, astronomical observations

What type of plasma?

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These astrophysical plasmas are (typically) classical, magnetised and weakly collisional/collisionless

Before we go any further, some terminology as I use it…

1) "Classical" = "typical particle separation $n^{-1/3} \ll$ Debye screening length $\lambda_{\rm D} = \sqrt{\dfrac{k_{\rm B}T_e}{4\pi n_e e^2(1+ZT_e/T_i)}}$

2) "Magnetised" = "macroscopic magnetic field", "electron Larmor radius $\rho_e \lesssim \lambda_{\rm mfp}$"

3) "Collisional" = "collisional mean free path $\lambda_{\rm mfp} \ll$ macroscopic scale length L"…

…*and, in classical plasma,* $\lambda_{\rm mfp}$ = "distance of 90° deflection due to small-angle Coulomb scattering events" $\sim \dfrac{T^2}{Z^2 e^4 n_i \log\Lambda}$

Coulomb logarithm

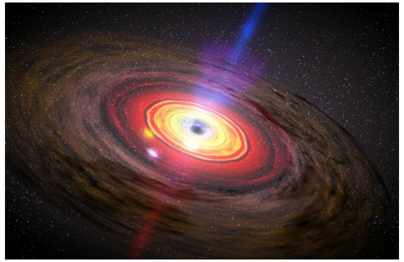
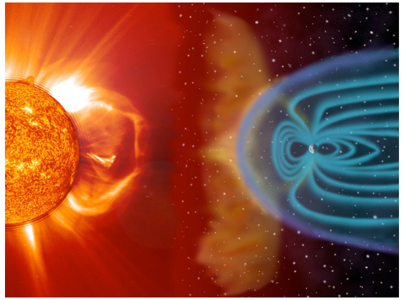
$$\log\Lambda = \log\left(\lambda_{\rm De}/\lambda_{\rm dB}\right) \gg 1$$

4) "Weakly collisional" = "Larmor radius $\rho_i \ll \lambda_{\rm mfp} \ll L$"

5) "Collisionless" = "$\rho_i \ll L \lesssim \lambda_{\rm mfp}$"

What type of plasma?

These astrophysical plasmas are (typically) classical, magnetised and weakly collisional/collisionless

		Parameters	*Scales*
ICM at R_{cool}		$n \sim 10^{-2}\,\mathrm{cm}^{-3}$ $T \sim 10\,\mathrm{keV}$ $B \sim 1\,\mu\mathrm{G}$	$L \sim 100\,\mathrm{kpc}$ --- $\lambda_{\mathrm{mfp}} \sim 1\,\mathrm{kpc}$ --- $\rho_i \sim 1\,\mathrm{npc}$
Sgr A* at R_{Bondi}		$n \sim 100\,\mathrm{cm}^{-3}$ $T \sim 2\,\mathrm{keV}$ $B \sim 1\,\mathrm{mG}$	$L \sim 0.1\,\mathrm{pc}$ --- $\lambda_{\mathrm{mfp}} \sim 0.01\,\mathrm{pc}$ --- $\rho_i \sim 1\,\mathrm{ppc}$
Solar wind at 1 au		$n \sim 10\,\mathrm{cm}^{-3}$ $T \sim 1\,\mathrm{eV}$ $B \sim 20\,\mu\mathrm{G}$	$L \sim 1\,\mathrm{au}$, $\lambda_{\mathrm{mfp}} \sim 1\,\mathrm{au}$ --- $\rho_i \sim 10^{-6}\,\mathrm{au}$

Weakly collisional ↓ *Collisionless*

How can we describe such plasmas?

- ***Kinetic description*** of plasma: distribution functions f_α for particle species α satisfy Maxwell-Vlasov-Landau equations:

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{v} \cdot \nabla f_\alpha + \frac{Z_\alpha e}{m_\alpha}\left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c}\right) \cdot \frac{\partial f_\alpha}{\partial \boldsymbol{v}} = \sum_\beta \mathcal{C}(f_\alpha, f_\beta) \ ,$$

combined with Faraday's and Ampère's law (for $k\lambda_\mathrm{D} \lesssim 1$): $\dfrac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E} \, , \quad \nabla \times \boldsymbol{B} = \dfrac{4\pi}{c}\boldsymbol{j} + \dfrac{1}{c}\dfrac{\partial \boldsymbol{E}}{\partial t} \, .$

- Integrating Maxwell-Vlasov-Landau equations over velocity space leads to ***fluid description***:

$$\begin{aligned}
\left.\frac{\mathrm{D}n_\alpha}{\mathrm{D}t}\right|_\alpha + n_\alpha \nabla \cdot \boldsymbol{V}_\alpha &= 0 \, , \\
m_\alpha n_\alpha \left.\frac{\mathrm{D}\boldsymbol{V}_\alpha}{\mathrm{D}t}\right|_\alpha &= -\nabla p_\alpha - \nabla \cdot \boldsymbol{\pi}_\alpha + Z_\alpha n_\alpha \left(\boldsymbol{E} + \boldsymbol{V}_\alpha \times \boldsymbol{B}\right) + \boldsymbol{R}_\alpha \, , \\
\frac{3}{2} n_\alpha \left.\frac{\mathrm{D}T_\alpha}{\mathrm{D}t}\right|_\alpha + p \nabla \cdot \boldsymbol{V}_\alpha &= -\nabla \cdot \boldsymbol{q}_\alpha - \boldsymbol{\pi}_\alpha : \nabla \boldsymbol{V}_\alpha + Q_\alpha \, ,
\end{aligned}$$

where $\boldsymbol{q}_\alpha$ is the heat flux, $\boldsymbol{\pi}_\alpha$ the viscosity tensor, and so on.

Fluid equations "simpler" – but require closure!

Classical transport: collisional plasmas are fluids

Key concept: frequent collisions drive plasma towards local thermodynamic equilibrium, allowing for closure to be obtained

- (Recall) conditions for plasma to be collisional:

$$\lambda_\alpha \ll L \,, \quad \nu_\alpha \gg \frac{1}{T} \,,$$

for ν_α collision frequency of species α, $\lambda_\alpha = v_{\mathrm{th}\alpha}/\nu_\alpha$ particle mean-free path, T and L macroscopic plasma scales

- Expand electron and ion distribution functions around Maxwellian distribution [***Chapman-Enskog (CE) expansion***]:

$$f_\alpha = f_\alpha^{(0)} + f_\alpha^{(1)} + \dots \,, \quad \text{for} \quad \frac{f_\alpha^{(1)}}{f_\alpha^{(0)}} \sim \epsilon_\alpha \sim \frac{\nabla \boldsymbol{V}_\alpha}{\nu_\alpha} \sim \frac{1}{T\nu_\alpha} \,, \quad \text{and} \quad f_\alpha^{(0)} = \frac{n_\alpha}{\pi^{3/2} v_{\mathrm{th}\alpha}^3} \exp\left(-\frac{v'^2}{v_{\mathrm{th}\alpha}}\right).$$

- Substitute into kinetic equation, solve first-order equation to determine $f_\alpha^{(1)}$

   → E.g., in strongly magnetised ($\rho_\alpha \ll \lambda_\alpha$), two-species plasma (first order in λ_α/L, zeroth in ρ_α/L):

$$f_e(\tilde{v}_{e\|}, \tilde{v}_{e\perp}) = \frac{n_e}{v_{\mathrm{th}e}^3 \pi^{3/2}} \exp\left(-\tilde{v}_e^2\right) \left\{1 + \left[\left(\eta_e^a A_e^a(\tilde{v}_e) + \eta_e^b A_e^b(\tilde{v}_e) + \eta_e^c A_e^c(\tilde{v}_e)\right) \tilde{v}_{e\|} + \epsilon_e C_e(\tilde{v}_e) \left(\tilde{v}_{e\|}^2 - \frac{\tilde{v}_{e\perp}^2}{2}\right)\right]\right\},$$

$$f_i(\tilde{v}_{e\|}, \tilde{v}_{e\perp}) = \frac{n_i}{v_{\mathrm{th}i}^3 \pi^{3/2}} \exp\left(-\tilde{v}_i^2\right) \left\{1 + \left[\eta_i A_i(\tilde{v}_i) \tilde{v}_{i\|} + \epsilon_i C_i(\tilde{v}_i) \left(\tilde{v}_{i\|}^2 - \frac{\tilde{v}_{i\perp}^2}{2}\right)\right]\right\},$$

where

$$\eta_e^a = \lambda_e \nabla_\| \ln T_e, \quad \eta_e^b = \lambda_e \frac{R_{e\|}}{p_e}, \quad \eta_e^c = \lambda_e \frac{m_e u_{ei\|}}{T_e \tau_e}, \quad \eta_i = \lambda_i \nabla_\| \ln T_i, \quad \epsilon_e = \lambda_e \frac{(\hat{\boldsymbol{z}} \cdot \mathbf{W}_e \cdot \hat{\boldsymbol{z}})}{v_{\mathrm{th}e}}, \quad \epsilon_i = \lambda_i \frac{(\hat{\boldsymbol{z}} \cdot \mathbf{W}_i \cdot \hat{\boldsymbol{z}})}{v_{\mathrm{th}i}},$$

Fluxes in collisional Braginskii MHD

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Non-Maxwellian components of distribution functions give rise to macroscopic fluxes, achieving desired closure

Friction force

$$\begin{cases} \boldsymbol{R}_e &= \boldsymbol{R}_u + \boldsymbol{R}_T\,, \\ \boldsymbol{R}_u &= -\dfrac{m_e n_e}{\tau_e}\left(0.51\boldsymbol{u}_\parallel + \boldsymbol{u}_\perp\right)\,, \\ \boldsymbol{R}_T &= -0.71 n_e \nabla_\parallel T_e + \dfrac{3n_e}{2\Omega_e\tau_e}\hat{\boldsymbol{b}}\times\nabla T_e\,, \end{cases}$$

Electron heat flux

$$\begin{cases} \boldsymbol{q}_e &= \boldsymbol{q}_u^e + \boldsymbol{q}_T^e\,, \\ \boldsymbol{q}_u^e &= 0.71 n_e T_e \boldsymbol{u}_\parallel - \dfrac{3n_e}{2\Omega_e\tau_e}\hat{\boldsymbol{b}}\times\boldsymbol{u}_\perp\,, \\ \boldsymbol{q}_T^e &= \underbrace{3.16\dfrac{n_e T_e \tau_e}{m_e}}_{\kappa_\parallel^e}\nabla_\parallel T_e + \underbrace{\dfrac{5n_e T_e}{2m_e\Omega_e}}_{-\kappa_\wedge^e}\hat{\boldsymbol{b}}\times\nabla T_e - \underbrace{\dfrac{4.66 n_e T_e}{m_e\Omega_e^2\tau_e}}_{\kappa_\perp^e}\nabla_\perp T_e\,, \end{cases}$$

Ion heat flux

$$\boldsymbol{q}_T^e = \underbrace{2.7\frac{n_i T_i \tau_{ii}}{m_i}}_{\kappa_\parallel^i}\nabla_\parallel T_i + \underbrace{\frac{5n_i T_i}{2m_i\Omega_i}}_{\kappa_\wedge^i}\hat{\boldsymbol{b}}\times\nabla T_i - \underbrace{\frac{\sqrt{2}n_i T_i}{m_i\Omega_i^2\tau_{ii}}}_{\kappa_\perp^i}\nabla_\perp T_i\,,$$

Fluxes in collisional Braginskii MHD cont.

Non-Maxwellian components of distribution functions give rise to macroscopic fluxes, achieving desired closure

Heat exchanged

$$Q_i = -Q_e - \boldsymbol{R}_e \cdot \boldsymbol{u} = \frac{3 n_e m_e}{m_i \tau_e} (T_e - T_i)$$

Viscosity tensor

$$\begin{cases} \pi_{xx} = -\frac{\eta_0}{2}(M_{xx} + M_{yy}) - \frac{\eta_1}{2}(M_{xx} - M_{yy}) - \eta_3 M_{xy} \,, \\ \pi_{yy} = -\frac{\eta_0}{2}(M_{xx} + M_{yy}) - \frac{\eta_1}{2}(M_{xx} - M_{yy}) + \eta_3 M_{xy} \\ \pi_{xy} = \pi_{yx} = -\eta_1 M_{xy} + \frac{\eta_3}{2}(M_{xx} - M_{yy}) \,, \\ \pi_{xz} = \pi_{zx} = -\eta_2 M_{xz} - \eta_4 M_{yz} \\ \pi_{yz} = \pi_{zy} = -\eta_2 M_{yz} + \eta_4 M_{xz} \\ \pi_{zz} = -\eta_0 M_{zz} \end{cases}$$

where η_0 to η_4 are viscosity coefficients (different for ions and electrons).

Q: how well does this model describe astrophysical plasmas?

Testing classical transport theory

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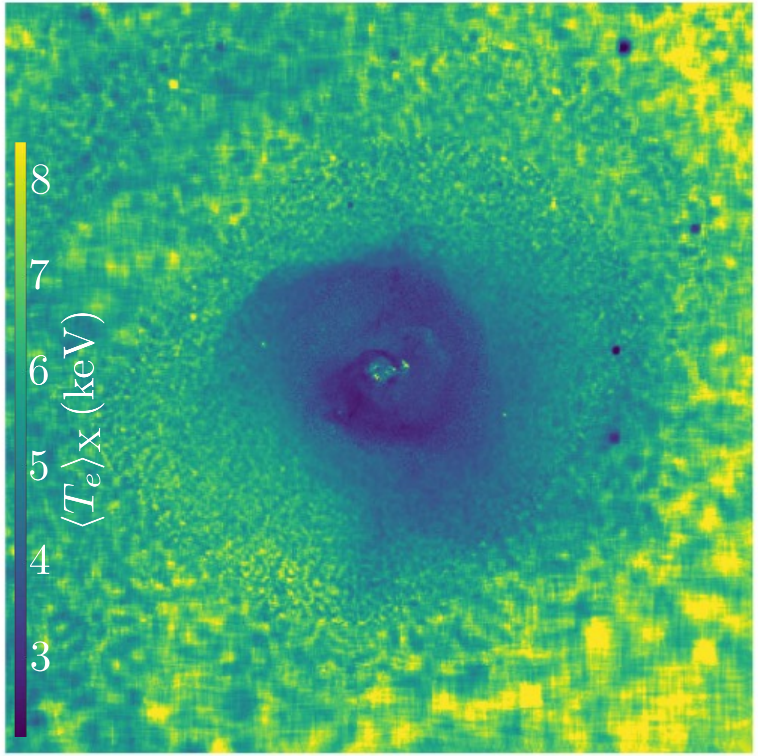
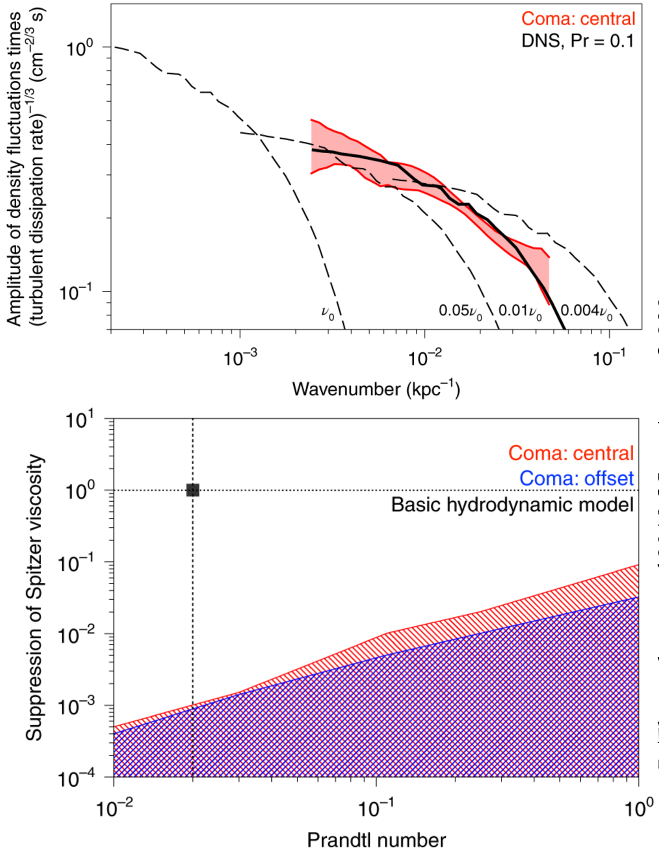
Recent observational evidence: MHD with classical transport often fails

ICM

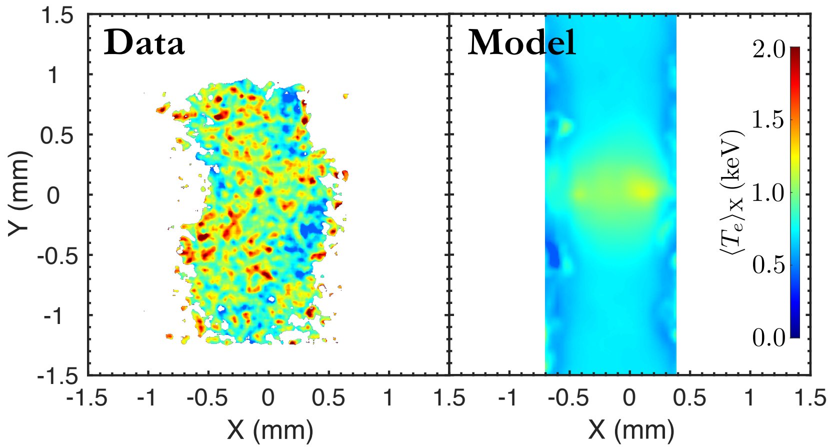


E. Churazov *et al.* 2016 *MNRAS* **463** 1057



I. Zhuravleva *et al* 2019 *Nat. Astron.* **3** 832

Laser-plasma experiments



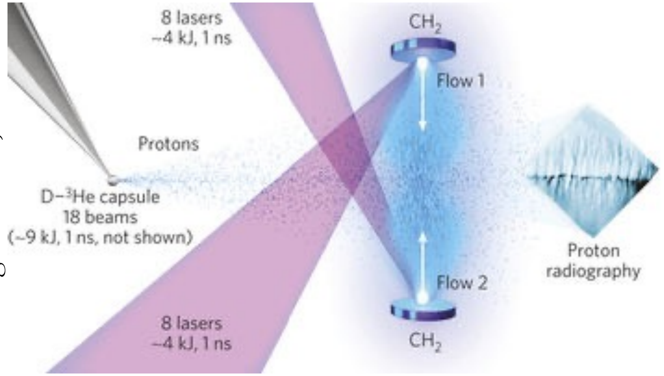
J. Meinecke *et al* 2022 *Sci. Adv.* **8** eabj6799

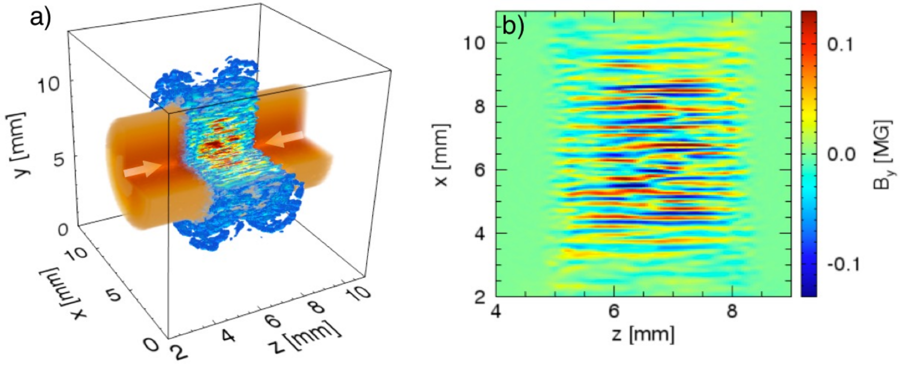
A plausible culprit: kinetic instabilities (microinstabilities)!

(Very brief) review of collisionless microinstabilities

- Microinstabilities studied in great depth by many researchers (too many to list all!)
- Generically arise in plasmas with anisotropic distribution functions.
  → Example (unmagnetised plasma): Weibel instability in counter-streaming plasmas (Weibel 1959, Fried 1959)

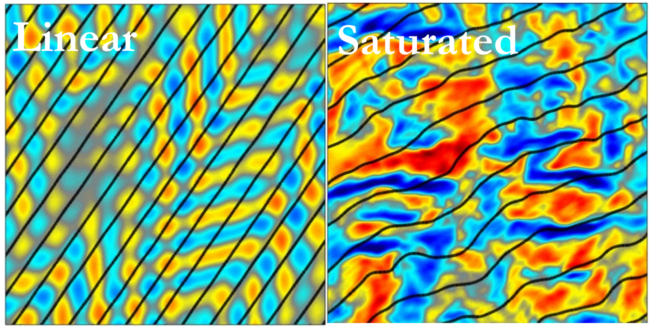
Huntingdon et. al., 2015



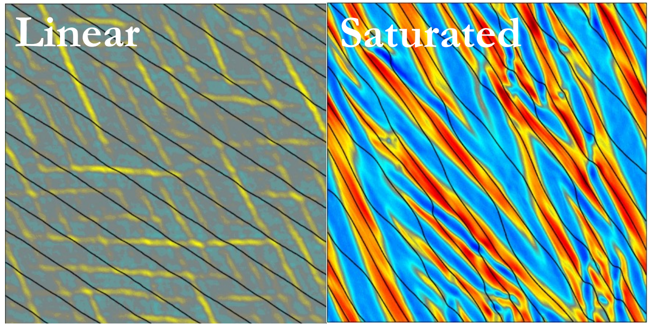


- Microinstabilities in magnetised plasma also comprehensively studied
  → Firehose and mirror instabilities (Rosenbluth 1958, Chandrasekhar et al. 1958, and so on…)
  → Electron instabilities (Kennel & Petschek, 1966, and so on…)

Firehose



Mirror



Kunz *et. al.* (2014)

Microinstabilities in weakly collisional plasma

<u>Q:</u> how can weakly collisional plasmas be subject to microinstabilities?

- Naïve view I: how can 'almost Maxwellian' CE distribution function be unstable?
- Naïve view II: doesn't collisional damping prevent microinstabilities in weakly collisional plasmas?

<u>A</u>: both views are overly simplistic:

- *Thresholdless instabilities* (Kahn 1964, Ramani & Laval 1978, Epperlein 1984) e.g. thermal anisotropy Weibel instability
- Characteristic scales of microinstabilities are *not* macroscopic scales

→ In (for example) a classical hydrogen plasma,

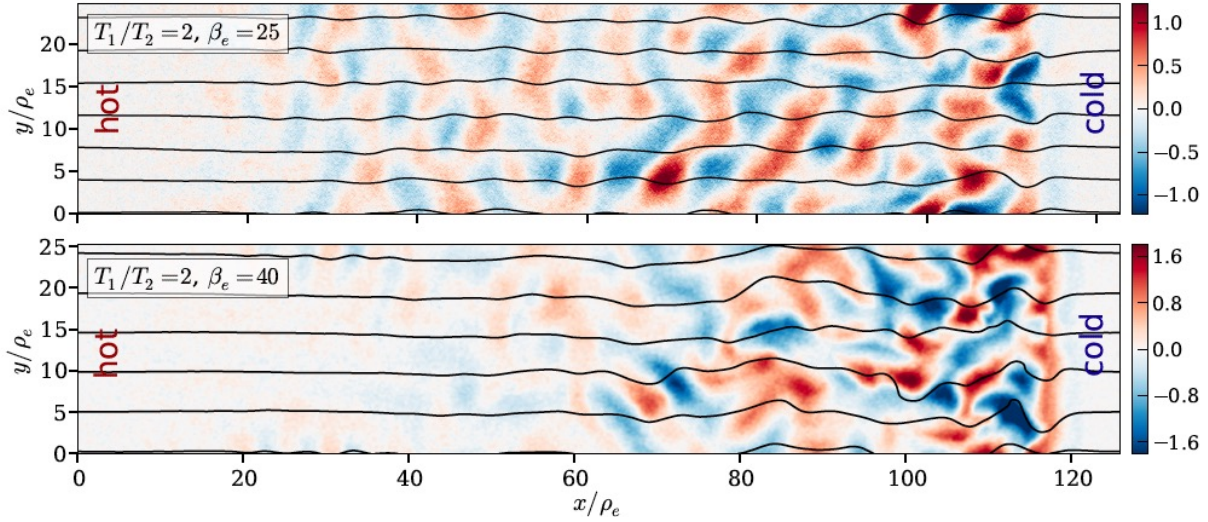
$$\frac{\lambda_{\rm mfp}}{\lambda_{\rm D}} \sim n_e \lambda_{\rm D}^3 \gg 1 \implies$$

Non-trivial range of collisionless scales in classical plasma

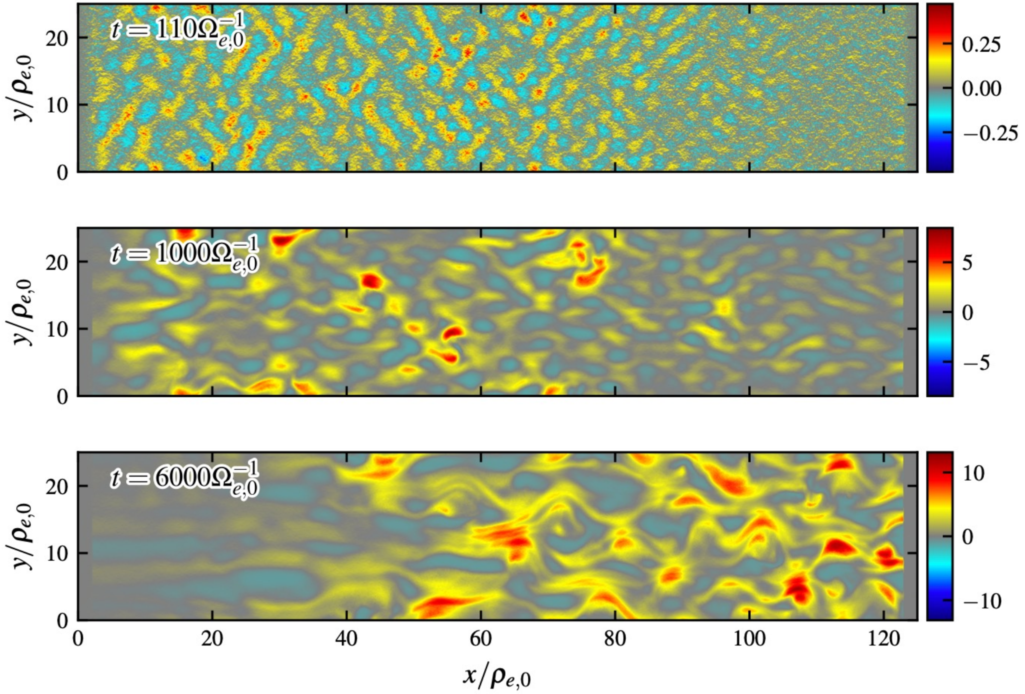
Studies on CE microinstabilities I

- Problem considered by various researchers, usually focused on only one or two instabilities.
  - Transverse (aka thermal-anisotropy Weibel) instability (Kahn 1964, Albright 1970)
  - Ramani-Laval instability (Ramani & Laval 1978)
  - Whistler heat-flux instability (Levinson 1992, Pistinner 1998)
  - Mirror/firehose instability (Schekochihin 2005)
- Recent sims of CE whistler-heat flux instability (Roberg-Clarke 2016, 2018, Komarov 2018, Yerger 2024)

Komarov *et al.* (2018)



Yerger *et al.* (2024)

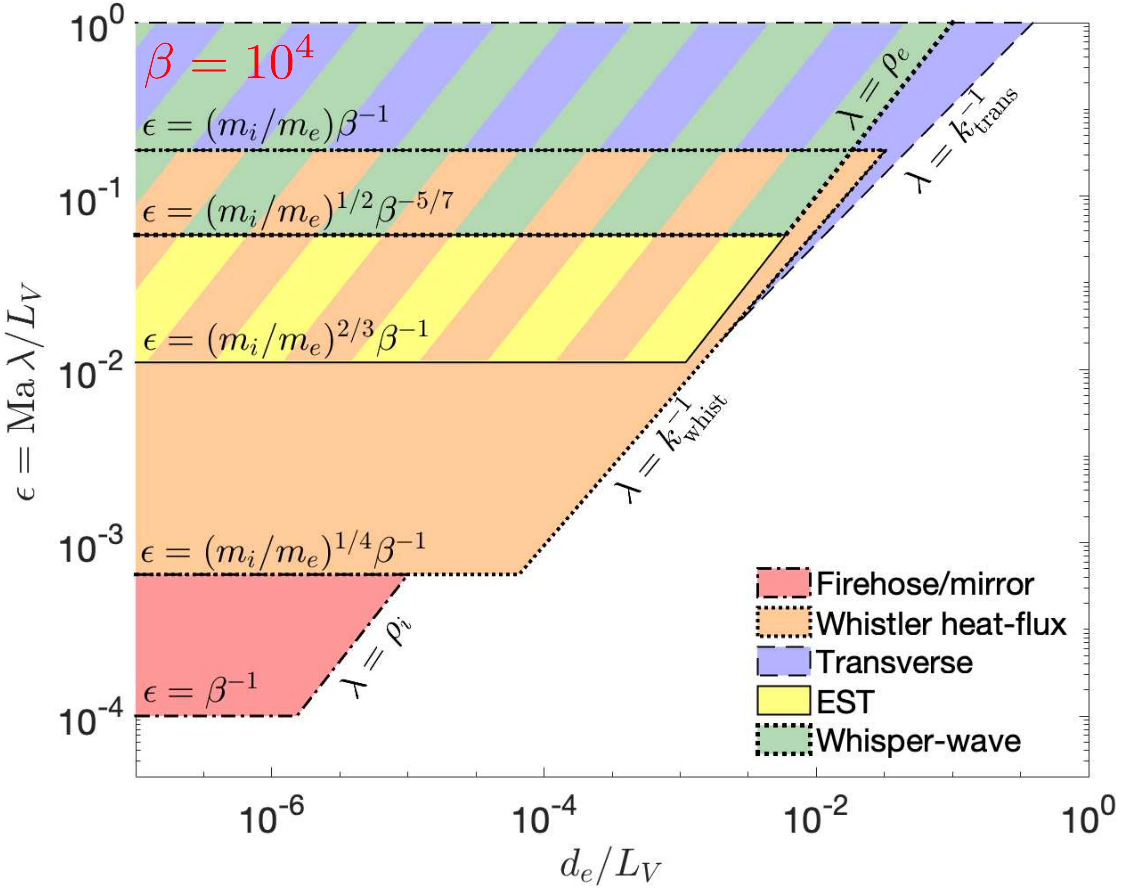


Studies on CE microinstabilities II

Recent study (Bott et al 2024) characterised 'zoo' of microinstabilities arising in weakly collisional plasmas:

Microinstability name	Wavenumber scale	Growth rate ($\times \Omega_e$)	β-threshold
Mirror instability	$k_\parallel \rho_i \lesssim k_\perp \rho_i \sim 1$	$\mu_e \epsilon$	$\epsilon\beta \sim 1$
Parallel firehose instability	$k_\parallel \rho_i \sim k_\perp \rho_i \sim \epsilon^{1/2}$, $k_\perp \ll \epsilon^{1/4} k_\parallel$	$\mu_e \epsilon$	$\epsilon\beta \sim 1$
Oblique firehose instability	$k_\parallel \rho_i \sim \epsilon^{1/4}$, $k_\perp \sim k_\parallel$	$\mu_e \epsilon^{3/4}$	$\epsilon\beta \sim 1$
Critical-line firehose instability ($\epsilon \gtrsim 10^{-6}$)	$k_\parallel \rho_i \approx \sqrt{3/2}\, k_\perp \rho_i < 1$	$\mu_e \epsilon^{1/2}$	$\epsilon\beta \sim 1$
Critical-line firehose instability ($\epsilon \ll 10^{-6}$)	$k_\parallel \rho_i \approx \sqrt{3/2}\, k_\perp \rho_i \sim \epsilon^{1/12}$	$\mu_e \epsilon^{7/12}$	$\epsilon\beta \sim 1$
Slow-wave instability	$k_\perp \rho_i \lesssim k_\parallel \rho_i \sim 1$	$\mu_e^{5/4} \epsilon$	$\epsilon\beta \sim \mu_e^{-1/4}$
Long wavelength KAW instability	$k_\parallel \rho_i < k_\perp \rho_i \sim 1$	$\mu_e^{5/4} \epsilon k_\parallel / k_\perp$	$\epsilon\beta \sim \mu_e^{-1/4}$
CES whistler instability	$k_\perp \rho_e \lesssim k_\parallel \rho_e \sim 1$	$\mu_e^{1/2} \epsilon$	$\epsilon\beta \sim \mu_e^{-1/2}$
Electron mirror instability	$k_\parallel \rho_e \lesssim k_\perp \rho_e \sim 1$	$\mu_e^{1/2} \epsilon$	$\epsilon\beta \sim \mu_e^{-1/2}$
Parallel electron firehose instability	$k_\perp \rho_e \lesssim k_\parallel \rho_e \sim 1$	$\mu_e \epsilon$	$\epsilon\beta \sim \mu_e^{-1/2}$
Oblique electron firehose instability	$k_\parallel \rho_e \lesssim k_\perp \rho_e \sim 1$	$\mu_e^{1/2} \epsilon$	$\epsilon\beta \sim \mu_e^{-1/2}$
EST instability ($\epsilon\beta^{5/7} \lesssim \mu_e^{-1/2}$)	$k_\parallel \rho_e < 1 \lesssim k_\perp \rho_e \sim \epsilon^{1/2} \beta^{1/2} \mu_e^{1/4}$	$\mu_e^{5/4} \epsilon^{5/2} \beta^{3/2}$	$\epsilon\beta \sim \mu_e^{-1/2}$
EST instability ($\epsilon\beta^{5/7} \gtrsim \mu_e^{-1/2}$)	$k_\parallel \rho_e < 1 \ll k_\perp \rho_e \sim \epsilon^{1/5} \mu_e^{1/10}$	$\mu_e^{1/5} \epsilon^{2/5}$	—
Whisper instability	$k_\parallel \rho_e < 1 \ll k_\perp \rho_e \sim \epsilon^{1/2} \beta^{1/2} \mu_e^{1/4}$	$\mu_e^{3/8} \epsilon^{3/4} \beta^{1/4}$	$\epsilon\beta \sim \beta^{2/7} \mu_e^{-1/2}$
Parallel transverse instability	$k_\perp \rho_e \lesssim k_\parallel \rho_e \sim \epsilon^{1/2} \beta^{1/2} \mu_e^{1/4}$	$\mu_e^{3/4} \epsilon^{3/2} \beta^{1/2}$	$\epsilon\beta \sim \mu_e^{-1/2}$
Oblique transverse instability	$1 \lesssim k_\parallel \rho_e \lesssim k_\perp \rho_e \sim \epsilon^{1/2} \beta^{1/2} \mu_e^{1/4}$	$\mu_e^{3/4} \epsilon^{3/2} \beta^{1/2}$	$\epsilon\beta \sim \mu_e^{-1/2}$
Ordinary-mode instability	$k_\parallel = 0$, $k_\perp \rho_e \sim \epsilon^{1/2} \beta^{1/2} \mu_e^{1/4}$	$\mu_e^{3/4} \epsilon^{3/2} \beta^{1/2}$	$\epsilon\beta \sim \beta^{2/3} \mu_e^{-1/2}$
CET whistler instability	$k_\perp \rho_e \lesssim k_\parallel \rho_e \sim \epsilon^{1/5} \beta^{1/5} \mu_e^{1/20}$	$\mu_e^{1/4} \epsilon$	$\epsilon\beta \sim \mu_e^{-1/4}$

- Both heat-flux- or momentum-flux-driven instabilities!
- 'General' stability map for all instabilities:



Stabilisation conditions:

1. $\beta\lambda/L \sim 1$
2. $k\lambda \sim 1$

When is classical transport theory okay?

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1. **At sufficiently low β**
   - All CE microinstabilities either stabilised (or growth strongly suppressed) when $\beta \ll L/\lambda$

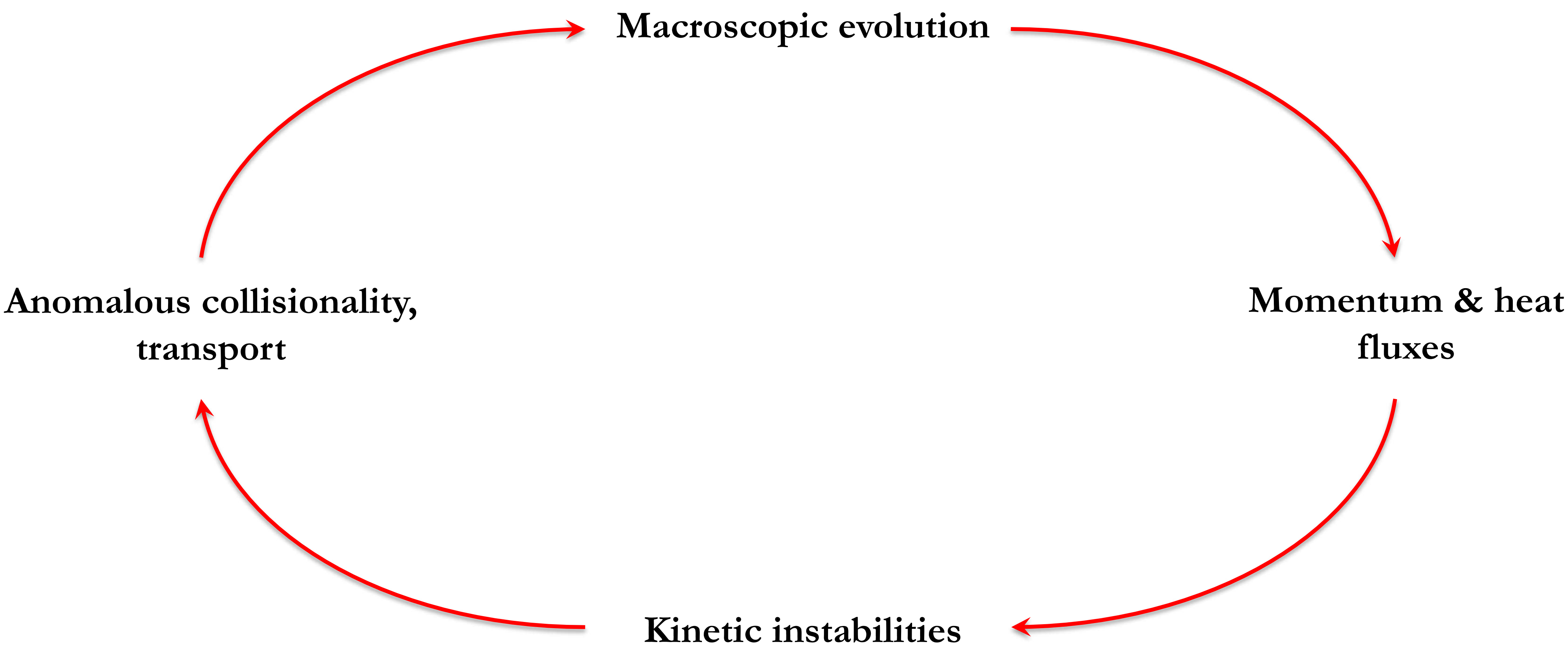
2. **When the separation between characteristic collisionless scales and the mean free path is insufficiently large**
   - Need microinstability wavenumber to satisfy $k\lambda \lesssim 1$ for effective collisional damping
   - ***This is never satisfied in magnetised plasma***

- In plasmas satisfying either of the above conditions, using Braginskii values (or corrections) for conductivity, resistivity, viscosity okay…

- …but if neither condition satisfied, then **<u>fluid equations with Braginskii transport coefficients cannot be assumed</u>**
   - Cannot rule out anomalous scattering of particles
   - Significant potential impact on the correct values of viscosity, conductivity and resistivity
   - Is fluid modelling of such plasmas even possible, or do we need to resort to kinetic theory?

Where does this leave us?

High-β, magnetised, weakly collisional plasmas → complicated interplay between macro/microscales



Studies on macro/micro interplay I

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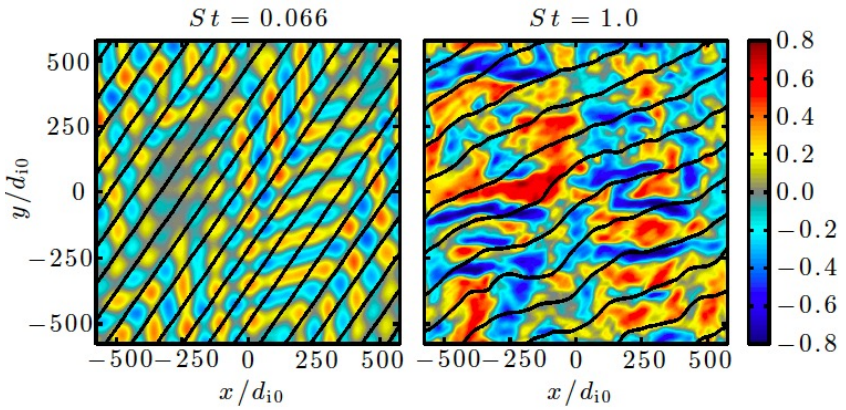
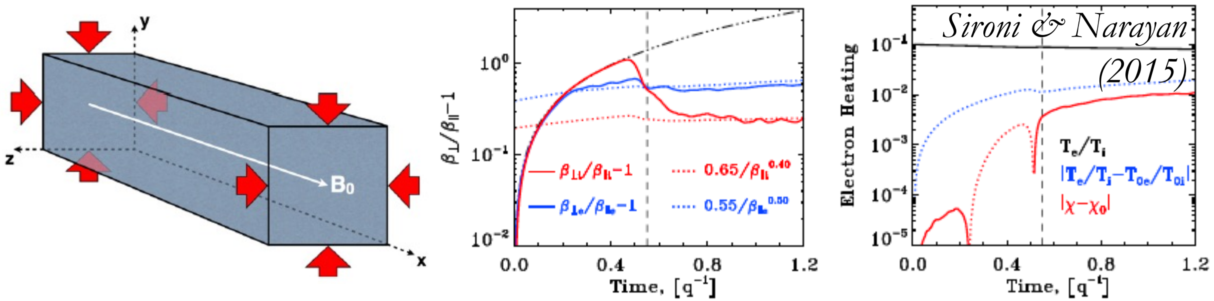
Numerous recent analytical + simulation studies in weakly collisional/collisionless high-β plasma... significant progress!

Microphysics



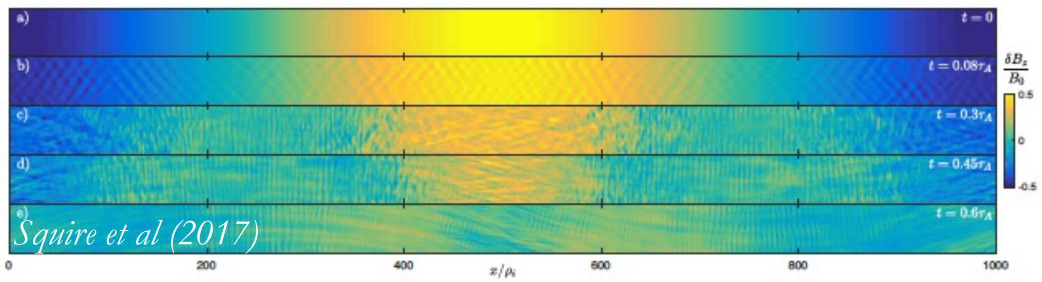
Anomalous collisionality (e.g. Kunz *et al* 2014)
Anomalous particle populations (e.g. Riquelme *et al* 2015)

Thermodynamics



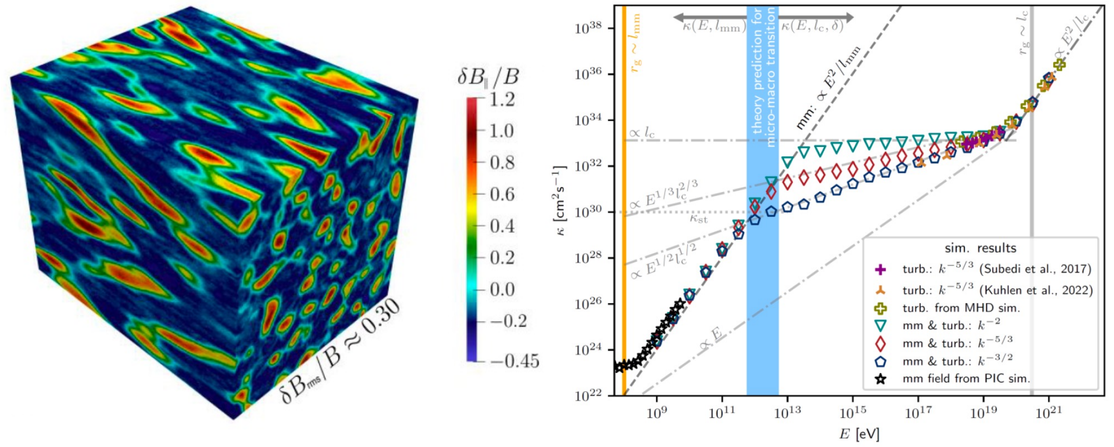
Pressure-anisotropy regulation (e.g. Camporeale & Burgess 2008)
Anomalous heating (e.g. Sironi & Narayan 2015)

Waves



Shear-Alfvén wave interruption (Squire *et al* 2017)
Sound-wave propagation (e.g. Kunz *et al* 2020)

Non-thermal particles

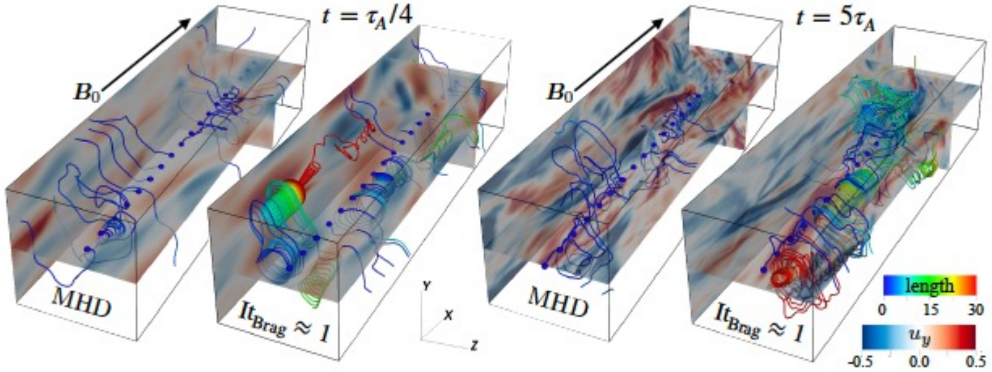


Generation of suprathermal particles (Ley *et al* 2019)
Anomalous transport of CRs (Reichherzer *et al* 2023)

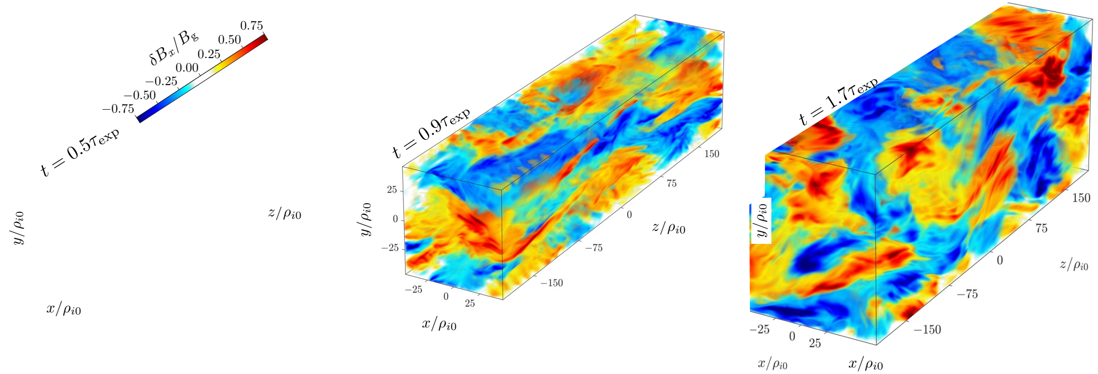
Studies on macro/micro interplay II

Astrophysical plasma turbulence: sometimes well described by collisional MHD models, but sometimes not!

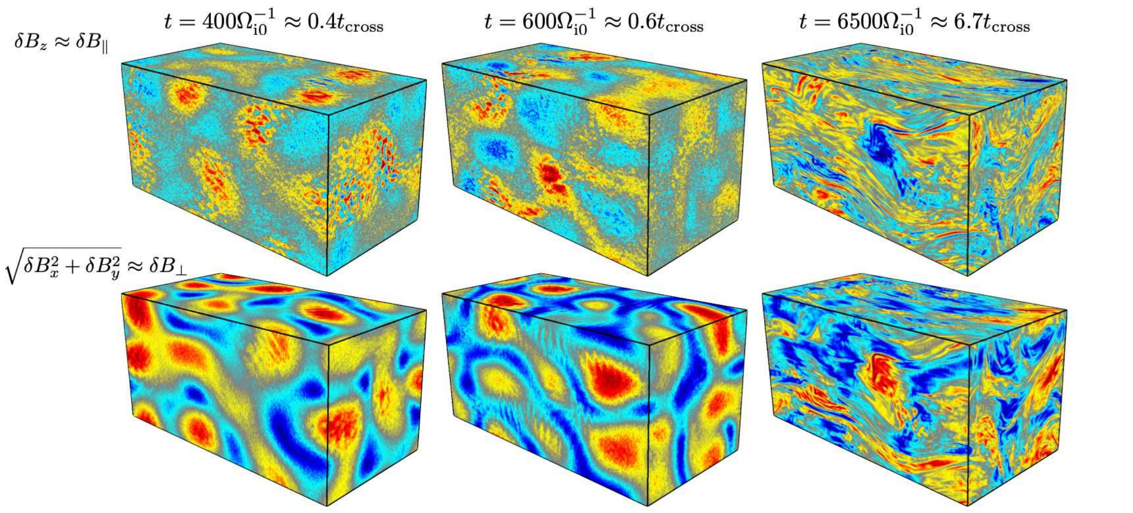
Magnetoimmutability (Squire *et al* 2019)

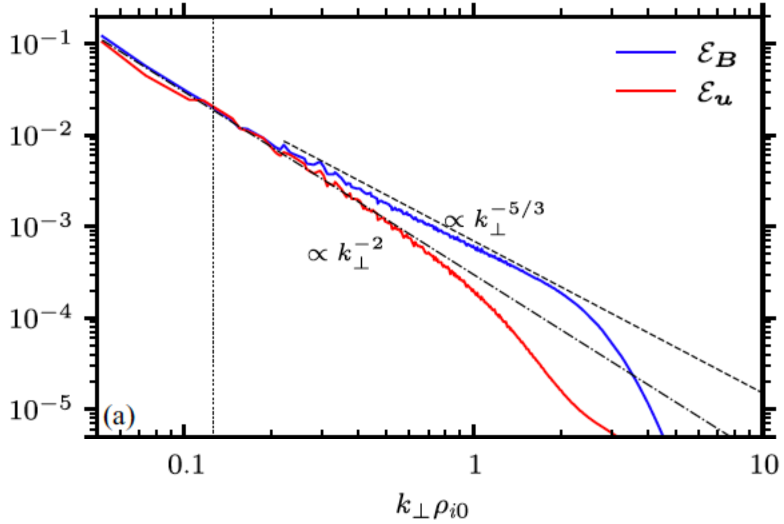


Adaptive critical balance in an expanding plasma (Bott *et al* 2021)



Anomalous viscous scale and spectral slopes (Arzamasskiy *et al* 2023)





Conclusions

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- The plasmas present in various astrophysical environments of interest are classical, magnetised, and weakly collisional (or collisionless)
- In such plasmas, fluid dynamical modelling that uses classical theories of collisional transport makes predictions that are often inconsistent with astronomical observations and recent experiments
- A plausible explanation for this discrepancy are kinetic plasma instabilities driven by momentum and heat fluxes that arise due to macroscopic gradients
- This gives rise to a rich interplay between macroscopic and microscopic scales in astrophysical plasmas, including anomalous collisionality and non-Newtonian transport

Open questions

1. When can weakly collisional and collisionless plasmas be modelled as fluids? If so, on what scales?
2. In what circumstances is kinetic modelling essential? In such cases, can we use reduced kinetic models (e.g. gyrokinetics) combined with an effective collision operator?
3. What insights can be gained on these questions from contemporaneous studies in galactic dynamics?

I hope we will make some progress towards answering these questions during this KITP program!