

Kinetics for gravity

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Gravity

vs.

plasma

$$\ddot{\mathbf{r}} = -\frac{G m_1 m_2}{m_1 |\mathbf{r}|^2}$$

single kind of charge

equal inertial and gravitating mass

no screening; Jeans length comparable to the size of the system

$$\ddot{\mathbf{r}} = \frac{q_1 q_2}{m_1 |\mathbf{r}|^2}$$

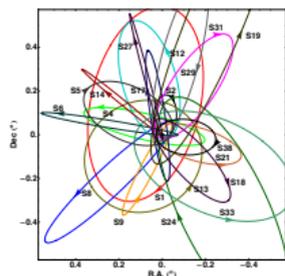
opposite charges

different charge/mass ratio

Debye screening length much smaller than the size of the system

In both cases $n\lambda^3 \gg 1 \Rightarrow$ nearly collisionless dynamics;
non-Maxwellian and possibly anisotropic velocity distributions.

Characteristic scales of self-gravitating systems



galactic nuclei



star clusters



galaxies

Size

$$N \sim 10^3 - 10^7$$

$$N \sim 10^6 - 10^{11}$$

$$\sim 1 - 10 \text{ pc}$$

$$\text{stars: } \sim 0.1 - 10 \text{ kpc}$$

$$\text{halo: } \sim 1 - 100 \text{ kpc}$$

Velocity

$$\sim 10 \text{ km/s}$$

$$\sim 10 - 200 \text{ km/s}$$

Dynamical time

$$\tau \equiv (4\pi G \rho)^{-1/2}$$

$$\sim 10^6 \text{ yr}$$

$$\sim 10^8 \text{ yr}$$

dynamically old

Relaxation time

$$T_{\text{rel}} \sim N\tau / \ln \Lambda$$

$$\sim 10^9 - 10^{10} \text{ yr}$$

$$\gg 10^{10} \text{ yr}$$

thermodynamically evolving

$$\frac{1 \text{ pc}}{1 \text{ Myr}} \approx 0.98 \text{ km/s}$$

Key features of kinetics of self-gravitating systems

Kinetic equation for $f(\mathbf{x}, \mathbf{v}, t)$:
$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \frac{\partial f}{\partial \mathbf{v}} = \mathfrak{C}_{\text{coll}}[f].$$

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better use *orbits* J and *locations on the orbit* θ as phase-space variables.
(instead of *velocity* and *position* as in spatially uniform plasma).

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For a phase-mixed system, $f = f(\mathbf{J})$ only, and could be almost arbitrary.

Solution of the CBE + Poisson equation:

$$\rho = \iiint f(\mathbf{J}) d^3\mathbf{v}$$
$$\mathbf{J} = \mathbf{J}(\mathbf{x}, \mathbf{v}; \Phi)$$
$$\nabla^2 \Phi = 4\pi G \rho$$

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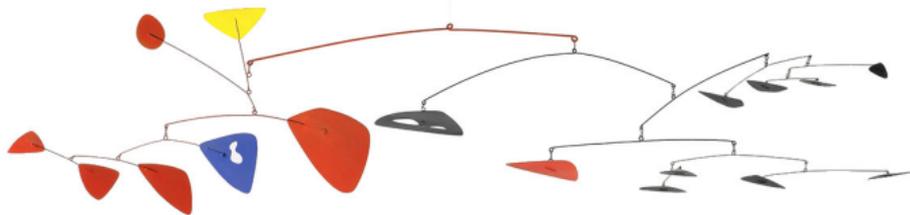
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Alternative methods for constructing
equilibrium models: $\rho(\mathbf{x}) + \Phi(\mathbf{x}) \Rightarrow f(J)$

$$\mathbf{J} = \mathbf{J}(\mathbf{x}, \mathbf{v}; \Phi)$$

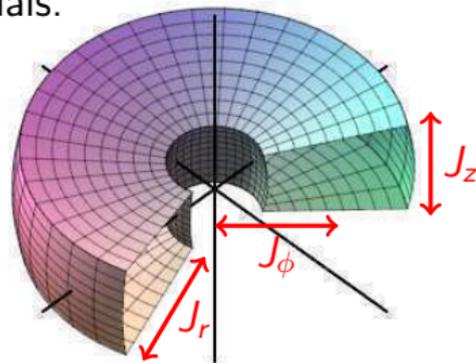
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Action–angle variables in galactic dynamics

Typically we consider spherical or axisymmetric potentials.

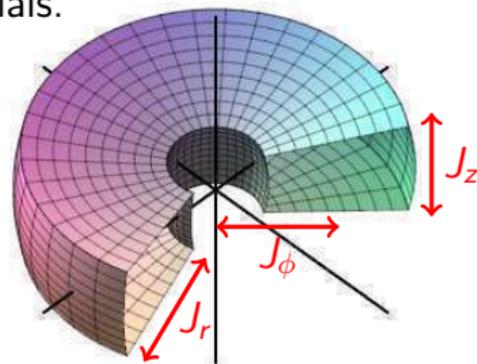
- ▶ radial action J_r ;
- ▶ vertical (or polar) action J_z ;
- ▶ azimuthal action $J_\phi = L_z$
(conserved component of angular momentum).



Action-angle variables in galactic dynamics

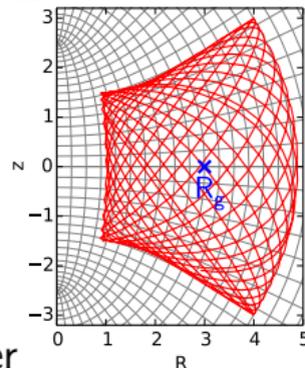
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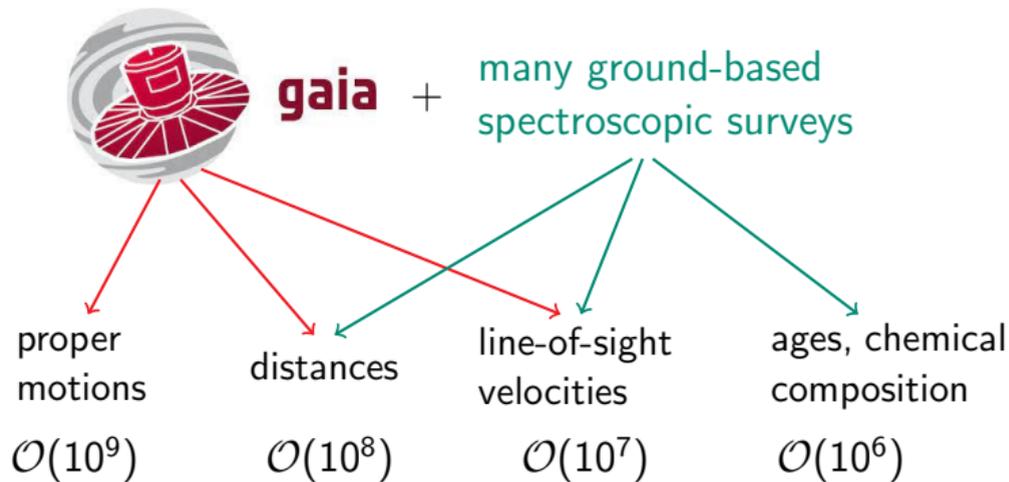


Transformation $\{\mathbf{x}, \mathbf{v}\} \Leftrightarrow \{\mathbf{J}, \boldsymbol{\theta}\}$:

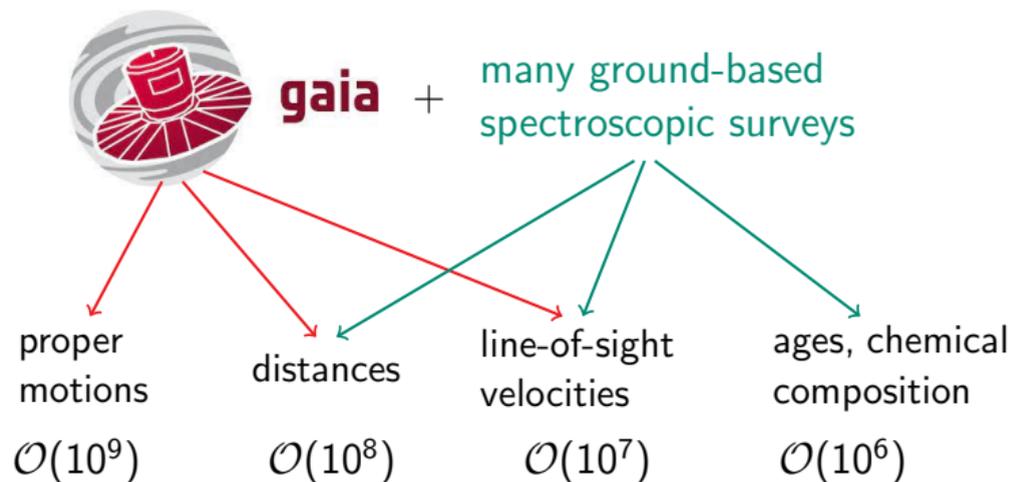
- ▶ for spherical potentials – (almost) analytic, only 1d numerical integrals for J_r ;
- ▶ for axisymmetric potentials close to the equatorial plane – epicyclic approximation (separable motion in R and z);
- ▶ —” —, Stäckel approximation (spheroidal coordinate system);
- ▶ most general (but tricky to use): ”torus mapping” using Fourier series for generating functions of the canonical transformation.



Observational developments in recent years



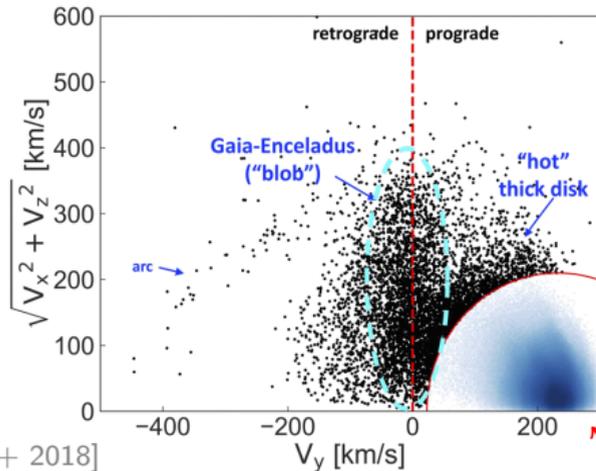
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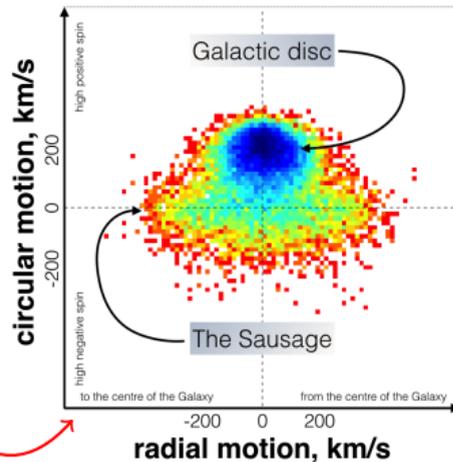
Available data: significant part of the Galactic disc (\sim few kpc);
central region, outer halo, some satellites...

Challenges: patchy coverage; not all objects have 6d phase-space coords...

Recent discoveries: Galactic assembly history

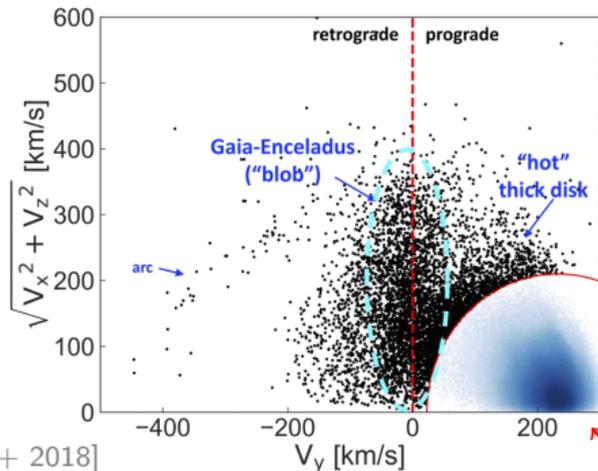


[Helmi+ 2018]

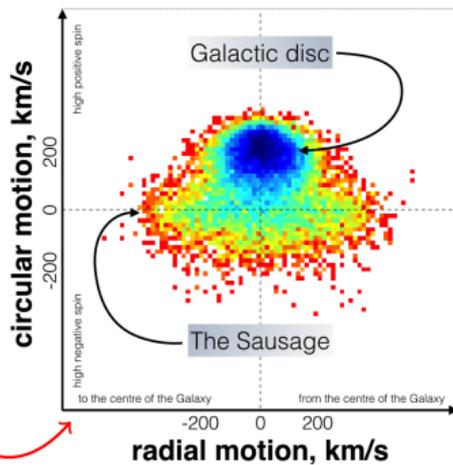


[Belokurov+ 2018]

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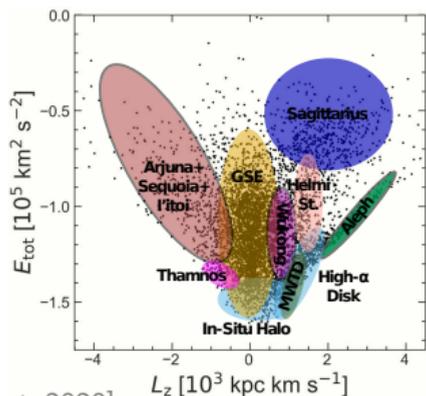


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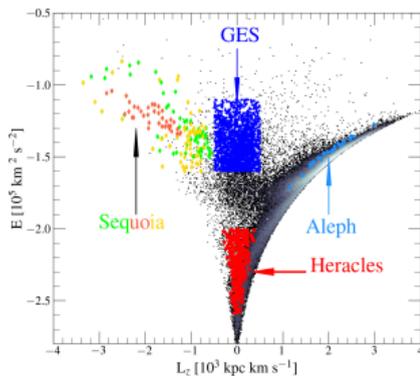


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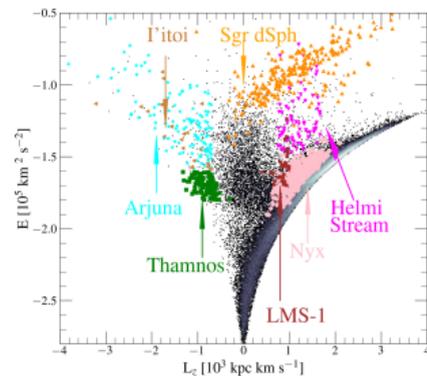
turn 90°



[Naidu+ 2020]



[Horta+ 2023]

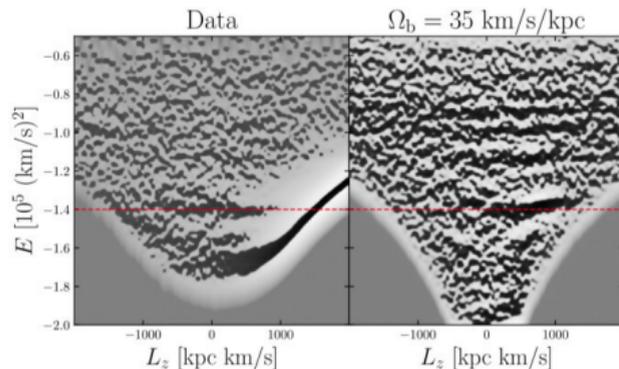


and many others...

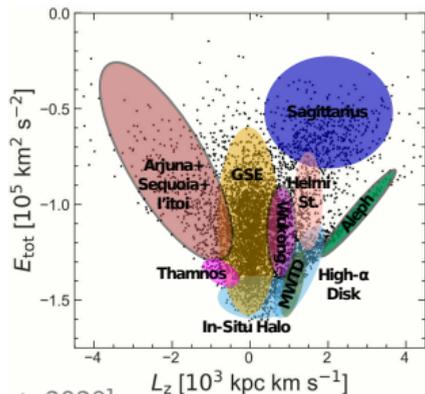
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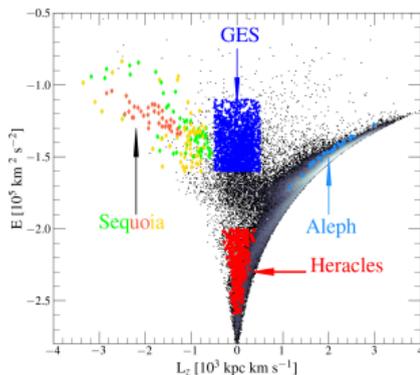
- overlapping debris from multiple progenitors;
- incomplete phase mixing;
- limited spatial coverage;
- blurring of substructures by later perturbations;
- dynamical creation of structures;
- lack of consistent nomenclature...



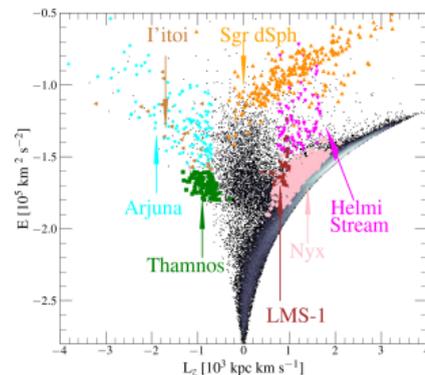
bar-induced stripes [Dillamore+ 2023, 2024]



[Naidu+ 2020]



[Horta+ 2023]



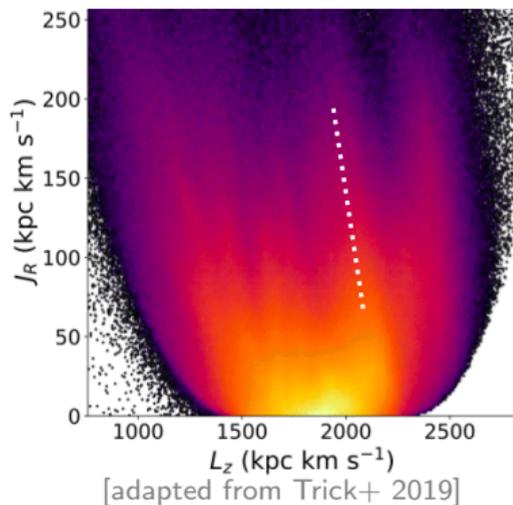
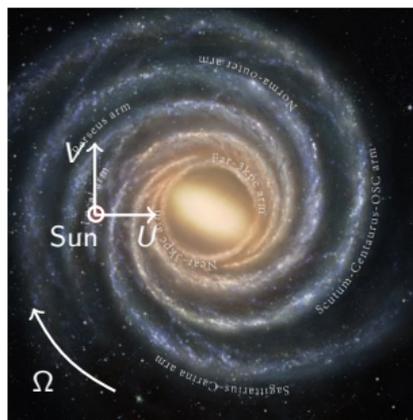
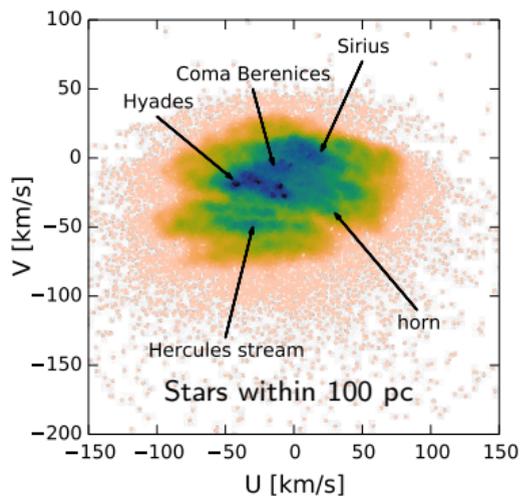
and many others...

Recent discoveries: imprint of Galactic bar and spiral arms

Velocity distribution in the equatorial plane (U, V) contains various structures associated with resonantly trapped orbits [Dehnen 2000; Quillen & Minchev 2005].

In a more extended region, trapped orbits show up as lines in the $J_R - L_Z$ plane [e.g., Sellwood 2010; Binney 2018; Monari+ 2017, 2019; Trick+ 2019; Hunt+ 2019].

Their location depends on the pattern speed of the bar Ω_b and spiral arms Ω_s . A slowing-down bar transports trapped objects outwards [e.g., Dillamore+ 2024] and creates age-dependent structures in the resonant islands [e.g., Chiba+ 2021].



Recent discoveries: radial migration and disc heating

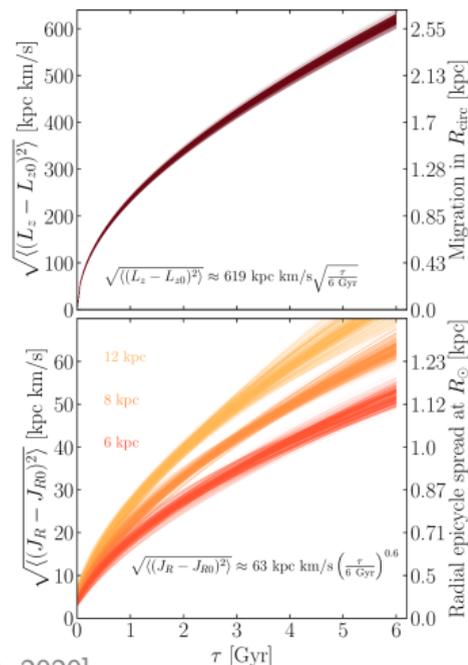
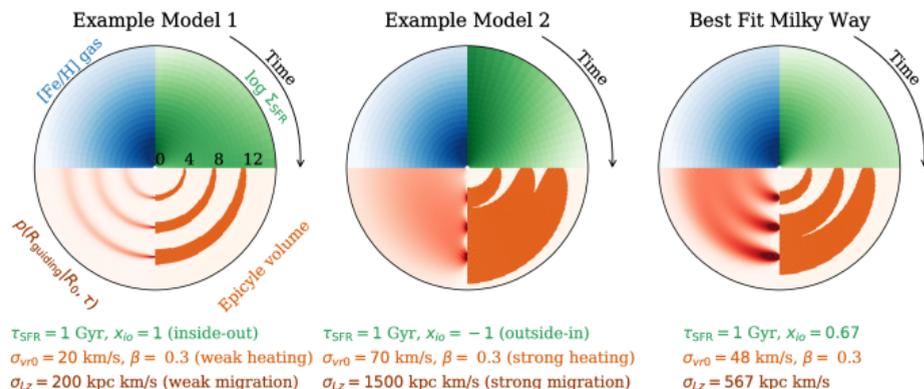
Stars do not stay at the same near-circular orbits where they were born due to a combination of two effects [Sellwood & Binney 2002; Roškar+ 2008; Minchev & Famaev 2010]:

- radial migration (“churning” – change in L_z while conserving J_r);
- heating (“blurring” – diffusion in J_r).

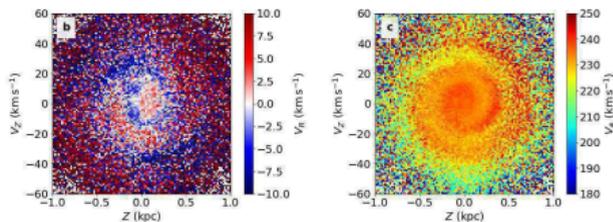
Mechanisms:

- resonances with the bar and spiral arms;
- molecular clouds and other massive perturbers;
- external perturbations (e.g., satellite flybys).

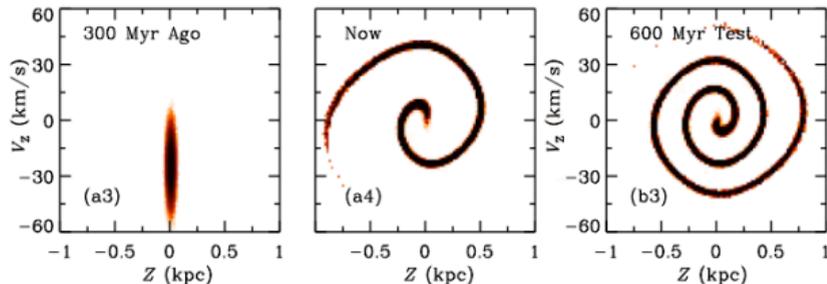
Churning appears to be much stronger than blurring.



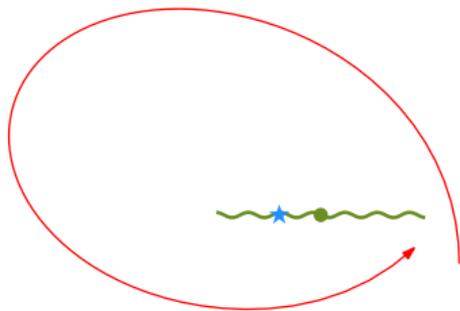
Recent discoveries: vertical perturbations in the Galactic disc



Gaia DR2 [Antoja+ 2018]



[Li & Shen 2019]

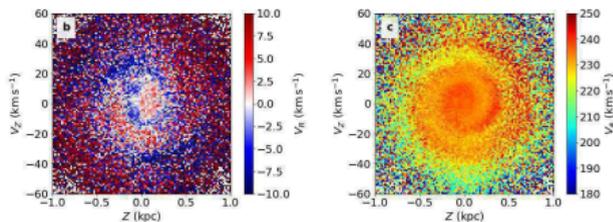


Leading theory: ripples after the impact of a massive satellite (implying Sgr dSph) through the disc [Widrow+ 2012; Laporte+ 2018,2019; Binney & Schönrich 2018; Li & Shen 2019; Bland-Hawthorn & Tepper-García 2021, etc.]

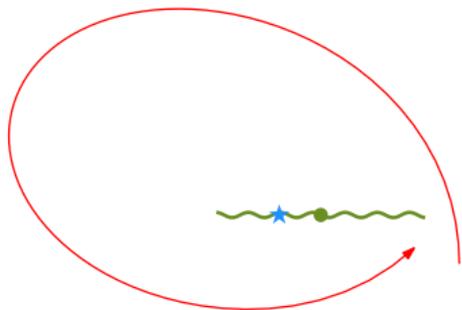
Caveat: Sgr was likely not massive enough at the time of the previous passage through the disc (1 Gyr ago) [Vasiliev & Belokurov 2020; Bennett+ 2022].

Counter-caveat: Sgr may have excited long-lived oscillations in the MW halo, which in turn perturb the disc [Grand+ 2022].

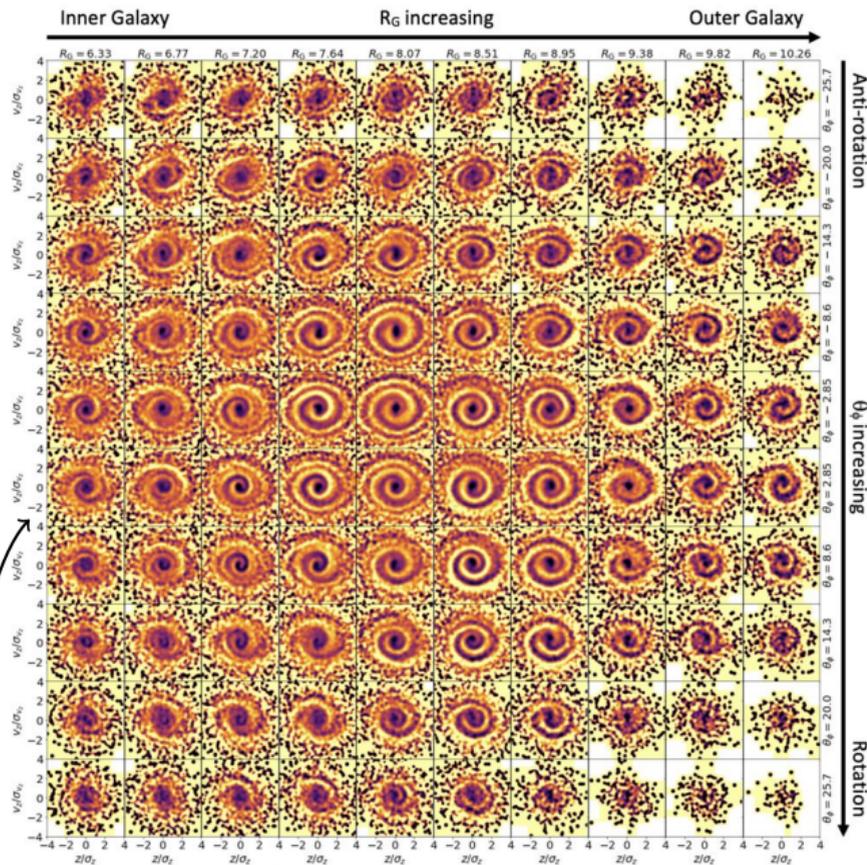
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Gaia DR2 [Antoja+ 2018]



two-arm spiral in the inner Galaxy:
breathing perturbation due to the bar?



Gaia DR3 [Hunt+ 2022]

Recent discoveries: precessing warp in the outer Galactic disc

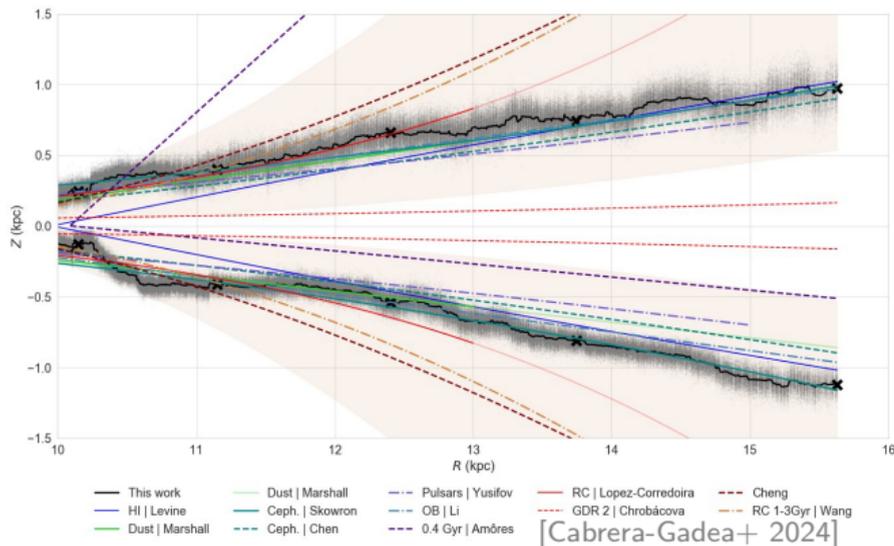
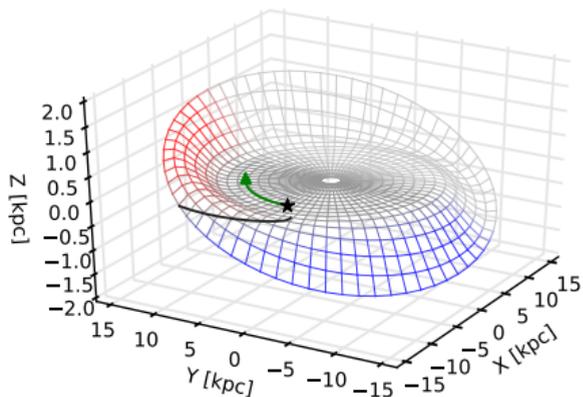
The warp is a coherent large-scale vertical perturbation of the disc beyond $\sim 10-12$ kpc.

Formation theories:

- impact of satellites (Sgr, LMC);
- misalignment between the disc and the dark halo;
- cold gas accretion from a misaligned direction.

Challenges:

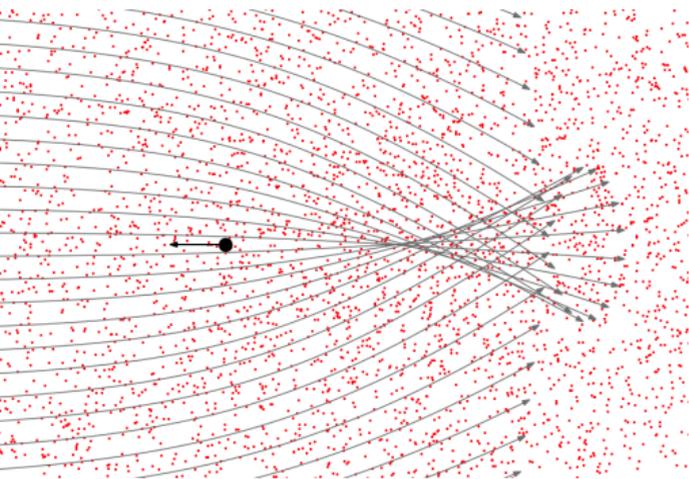
- disagreement in amplitude and precession rate between populations of different ages.



Recent discoveries: LMC–Milky Way encounter

The Large Magellanic Cloud is only $5 - 10\times$ less massive than the Milky Way, and just passed its pericentre at 50 kpc. The LMC-induced perturbation is twofold:

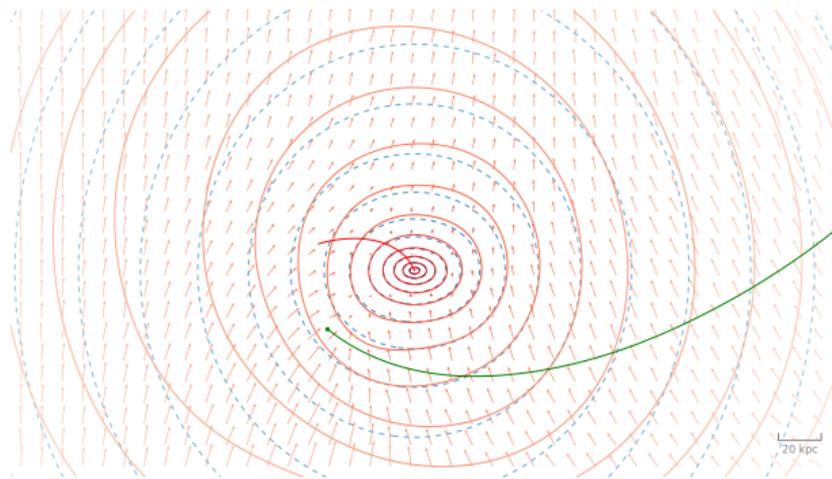
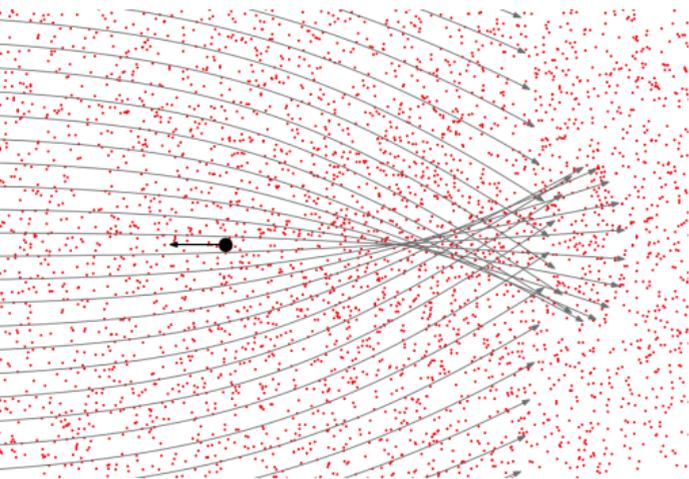
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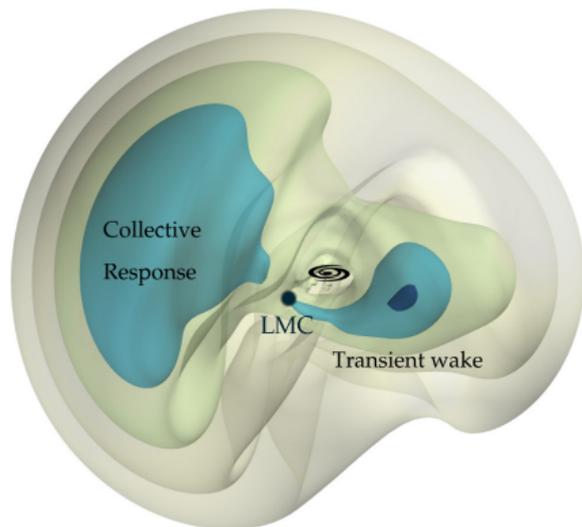
1. Stars in the vicinity of the moving LMC are deflected into a trailing density wake, creating a dynamical friction force [Chandrasekhar 1942].
2. The two galaxies move around the common centre of mass, but not as rigid bodies. In the MW-centred reference frame, outer halo appears to move up (dipole perturbation).



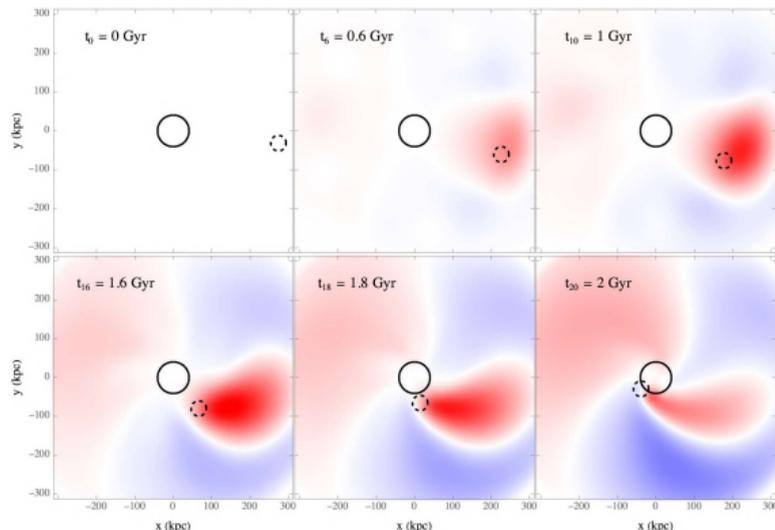
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N-body sims [Garavito-Camargo+ 2020]



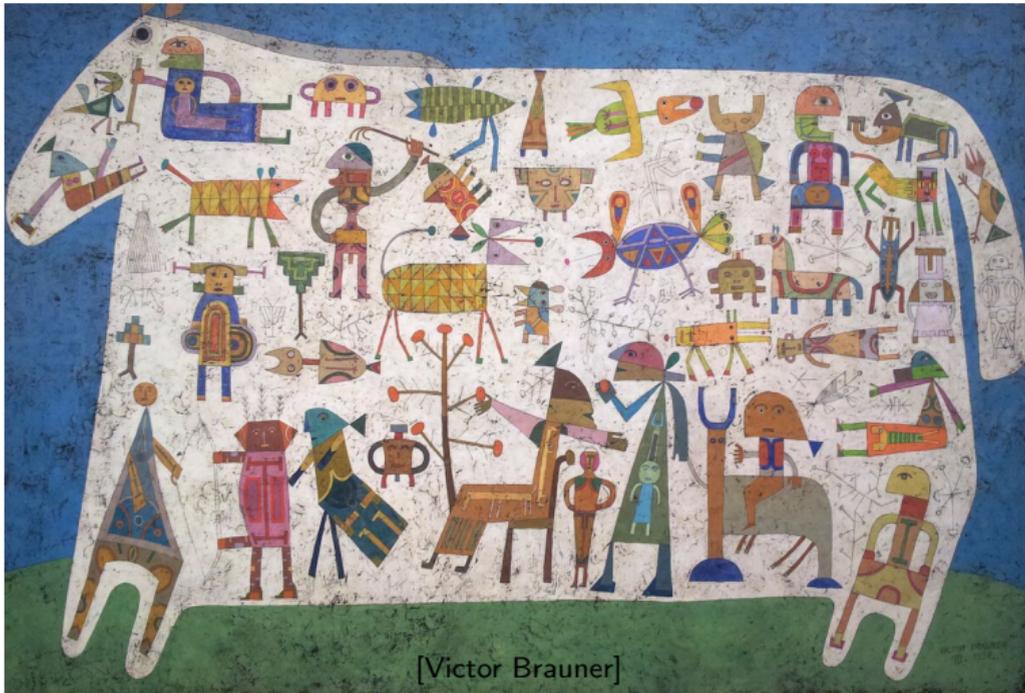
perturbation theory [Rozier+ 2022]

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[Victor Brauner]