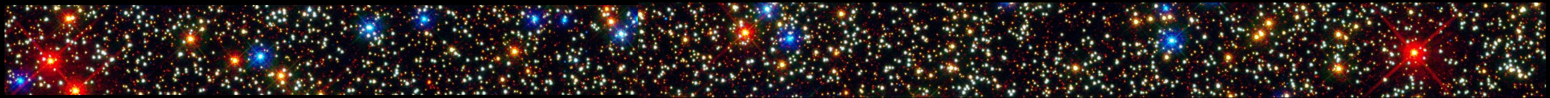
Star clusters in (phase) space

Anna Lisa Varri

University of Edinburgh

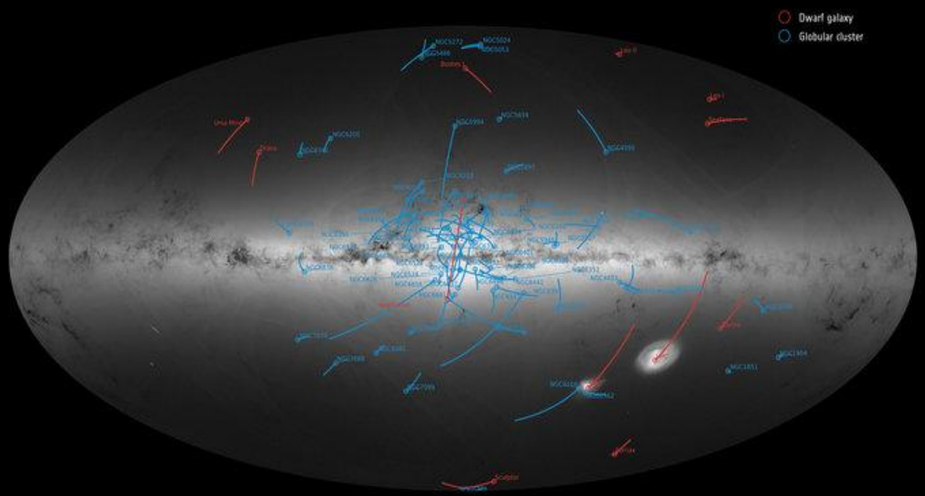
in collaboration with many, to be gratefully acknowledged along the way - and thanks to UKRI FLF



NASA, ESA, J. Anderson and R. van der Marel (STScI)

1. New *observational* landscape

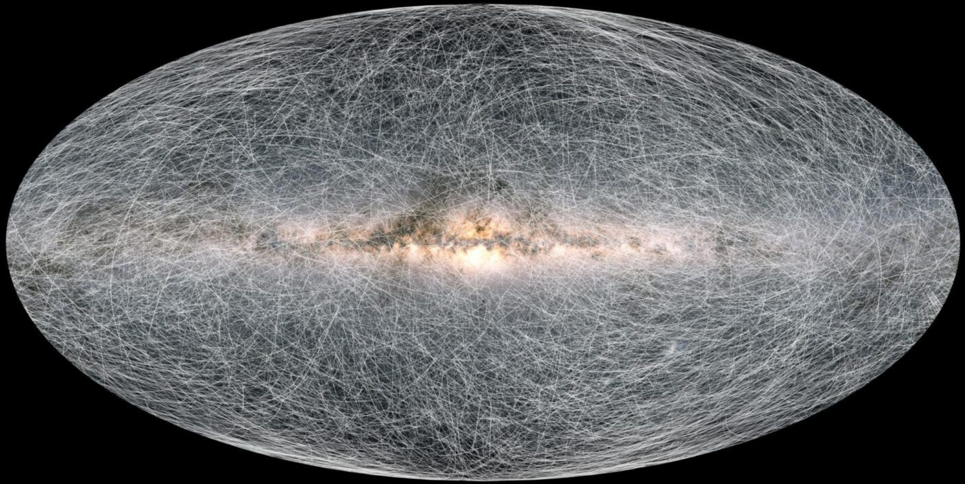
Precision Galactic Astrometry
(Gaia DR2)



ESA/Gaia/DPAC

1. New *observational* landscape

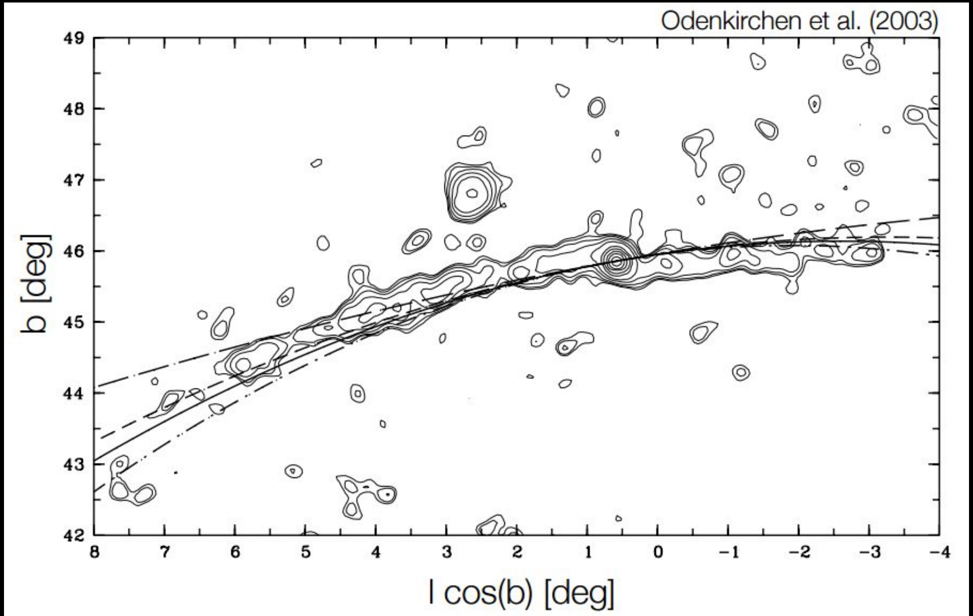
Precision Galactic Astrometry
(Gaia DR3)



[Eugene Vasiliev's seminar in week 1]

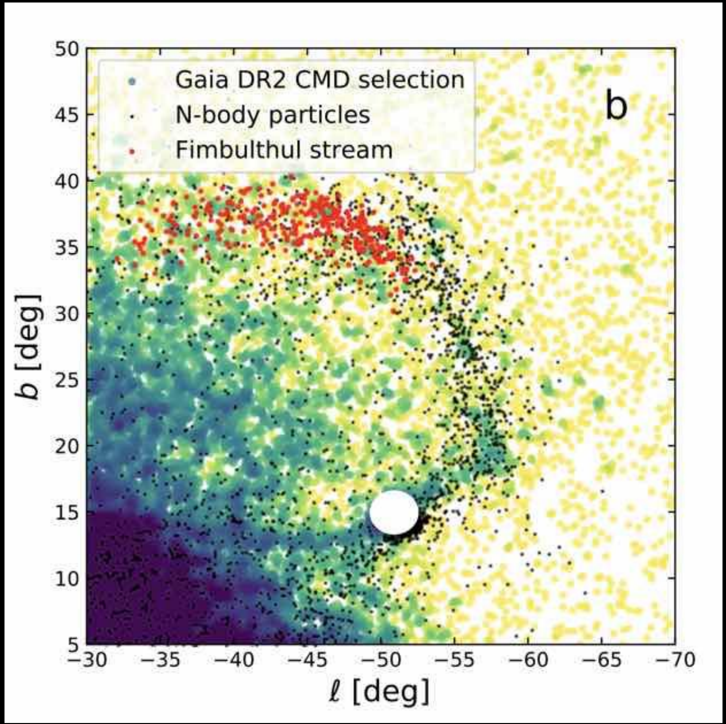
ESA/Gaia/DPAC

Many tidal streams to come



Palomar 5 | for detailed modelling: Kuepper+ 2015 MNRAS

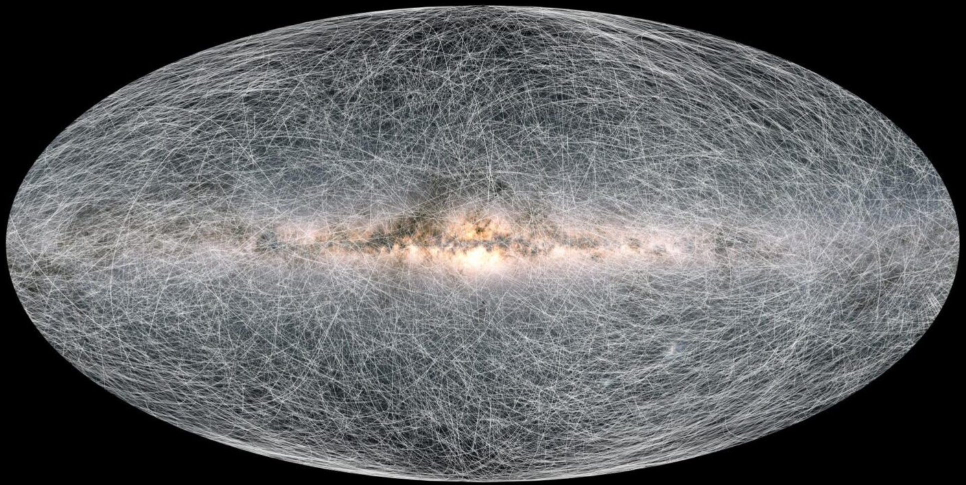
Soon, also extragalactic (via Euclid, Roman)
[Several items in darkmatter24 program & conference]



The long stream of omega Cen | Ibata+ 2019 Nature

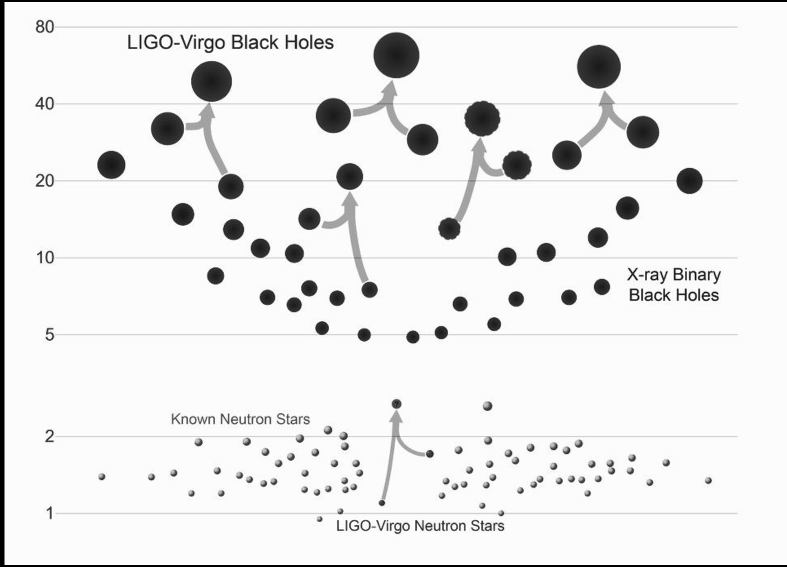
1. New *observational* landscape

Precision Galactic Astrometry
(Gaia DR3)



ESA/Gaia/DPAC

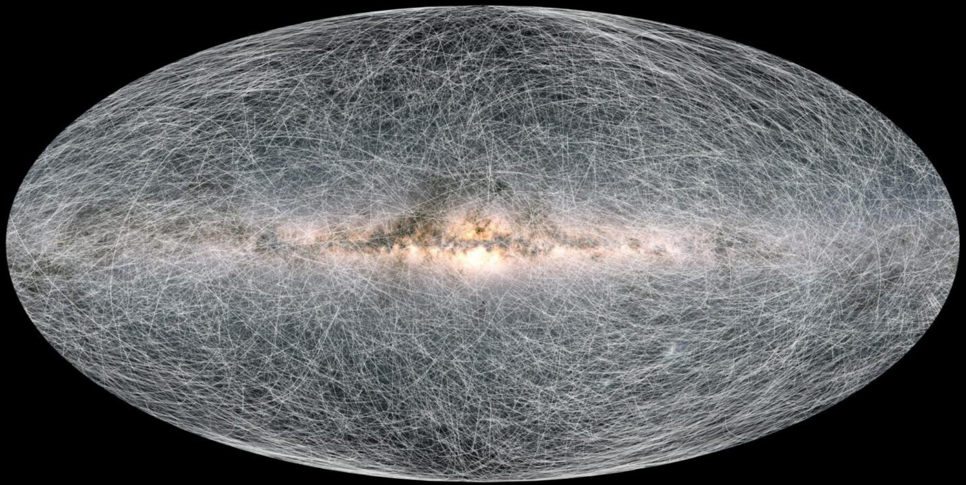
Gravitational Wave Astronomy
(February 2016 -)



LIGO-Virgo / Northwestern

1. New *observational* landscape

Precision Galactic Astrometry
(Gaia DR3)



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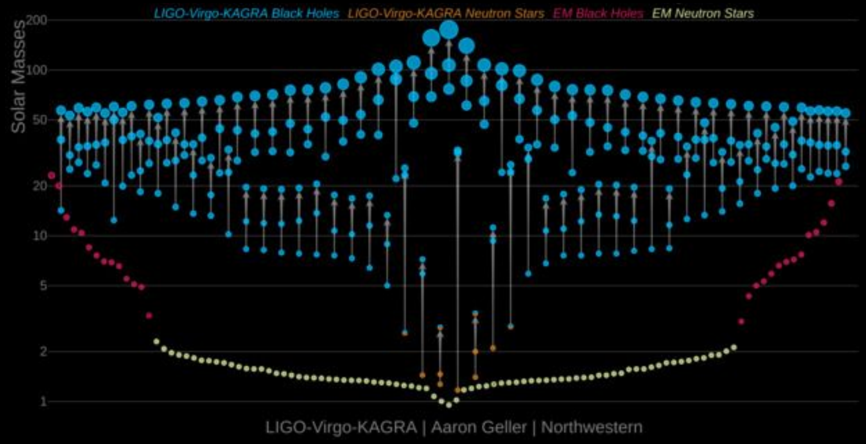
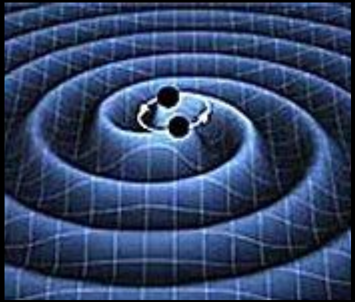
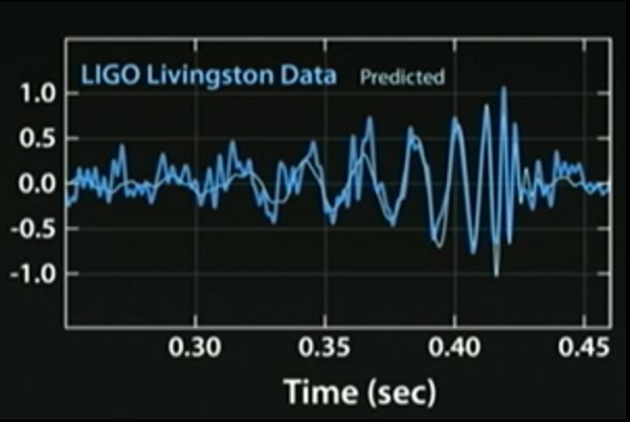
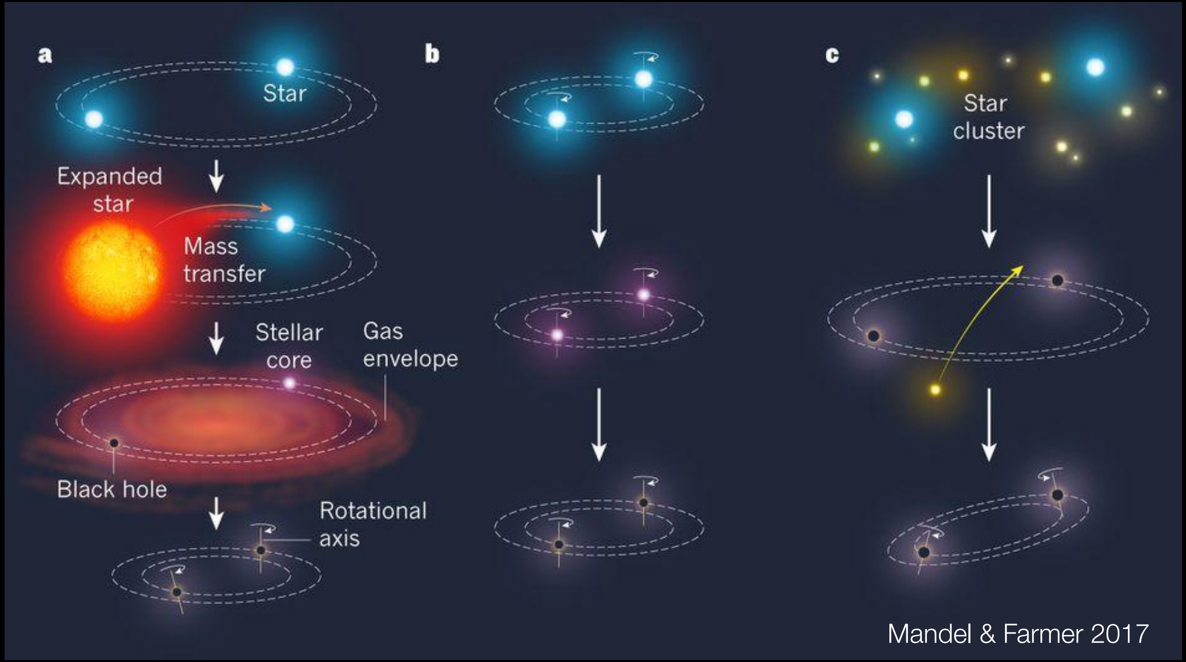


LIGO-Virgo-KAGRA / Northwestern

Many black holes to come





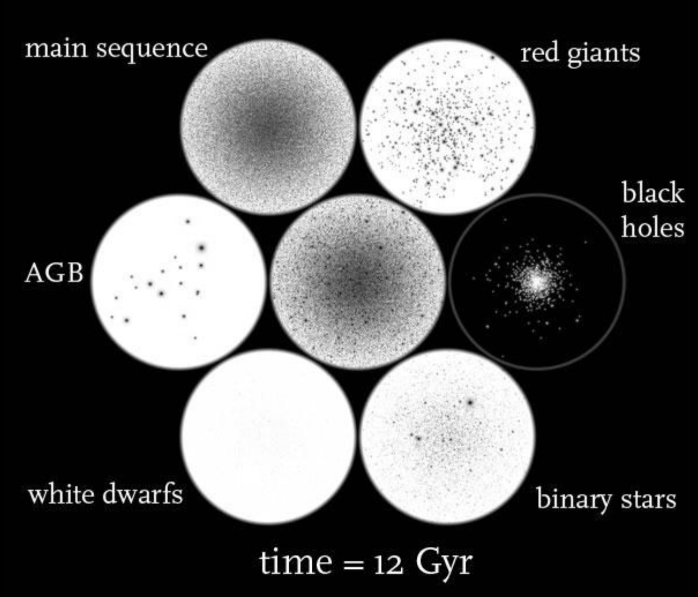


[Carl Rodriguez's seminar in week 5]

2. New *computational* landscape

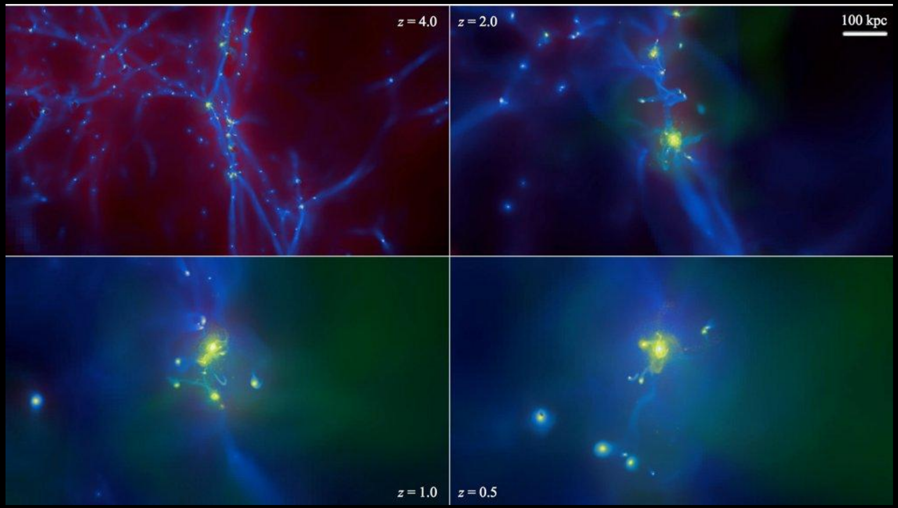
[Rainer Spurzem's' seminar in week 2]

Gravitational million-body problem 'solved'



DRAGON series of N-body simulations | Wang+ 2016 ApJ
N-body model of M4 (N=484710) | Heggie 2014 MNRAS
… thanks to NBODY6-GPU (Nitadori & Aarseth 2012)
and decades of GRAPE development by Makino and team.

Towards GC formation in a cosmological context



Renaud+ 2017 MNRAS; Carlberg 2017 ApJ; Li, Gnedin^2 2017 ApJ,
E-MOSAICS project and, now, many others ...

… also, role during reionization? Ricotti 2004, Boylan-Kolchin 2017a,b ...

3. New *conceptual* landscape

New richness
of old globular clusters

STScI/Hubble

Baseline: globular clusters as quasi-isothermal spheres

$$f_K(E) = \begin{cases} A\left[\exp(-aE) - \exp(-aE_0)\right] & \text{if } E \leq E_0 \\ 0 & \text{if } E > E_0 \end{cases}$$

$$E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Phi,$$

$$\psi(r) = a[E_0 - \Phi(r)]$$

$$\rho_K(\psi) = \hat{A} e^{\psi} \gamma\left(\frac{5}{2}, \psi\right) \equiv \hat{A}\hat{\rho}(\psi).$$

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$$

Vlasov-Poisson initial value problem

$$\frac{1}{\hat{r}^2}\frac{d}{d\hat{r}}\left(\hat{r}^2 \frac{d\psi}{d\hat{r}}\right) = -9\frac{\hat{\rho}(\psi)}{\hat{\rho}(\Psi)}$$

$$\psi(0) = \Psi$$

$$\psi'(0) = 0$$

$$\hat{r} = r/r_K.$$

$$r_K = \left(\frac{9}{4\pi G \rho_0 a}\right)^{1/2}$$

$$\psi(\hat{r}_t) = 0.$$

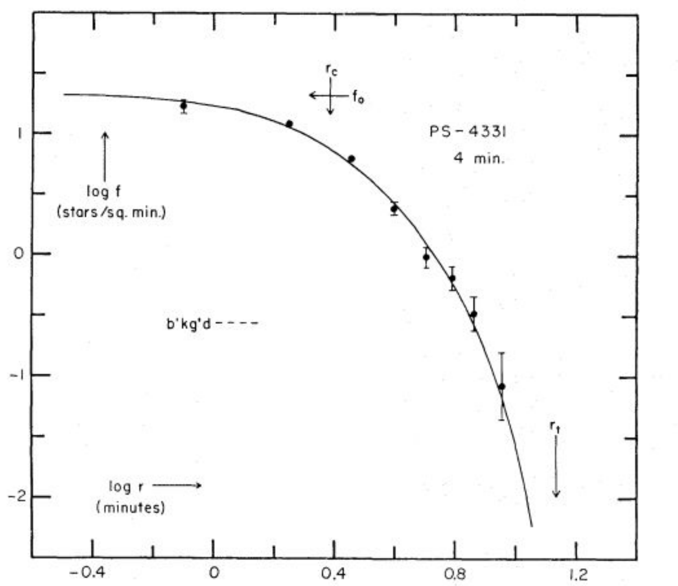
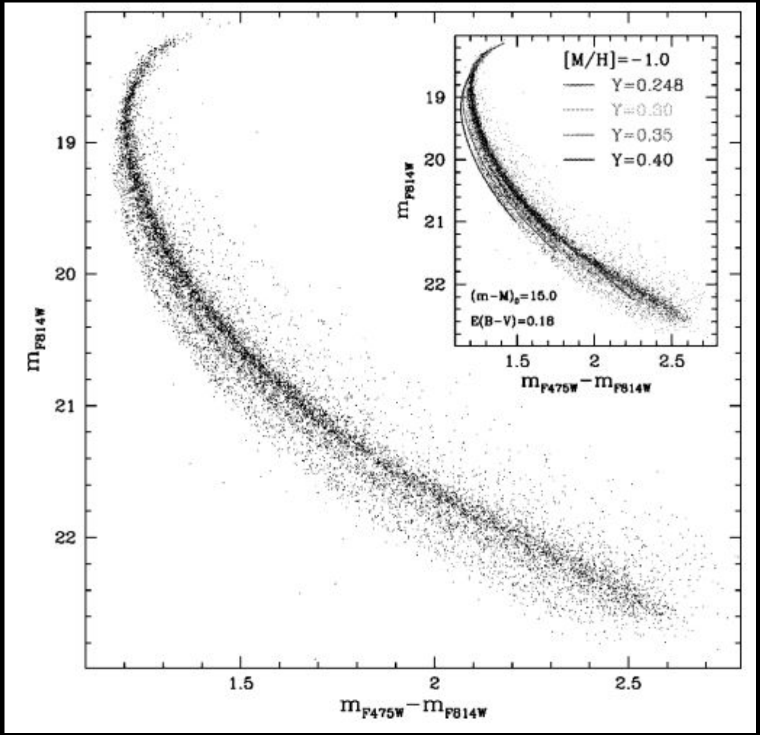


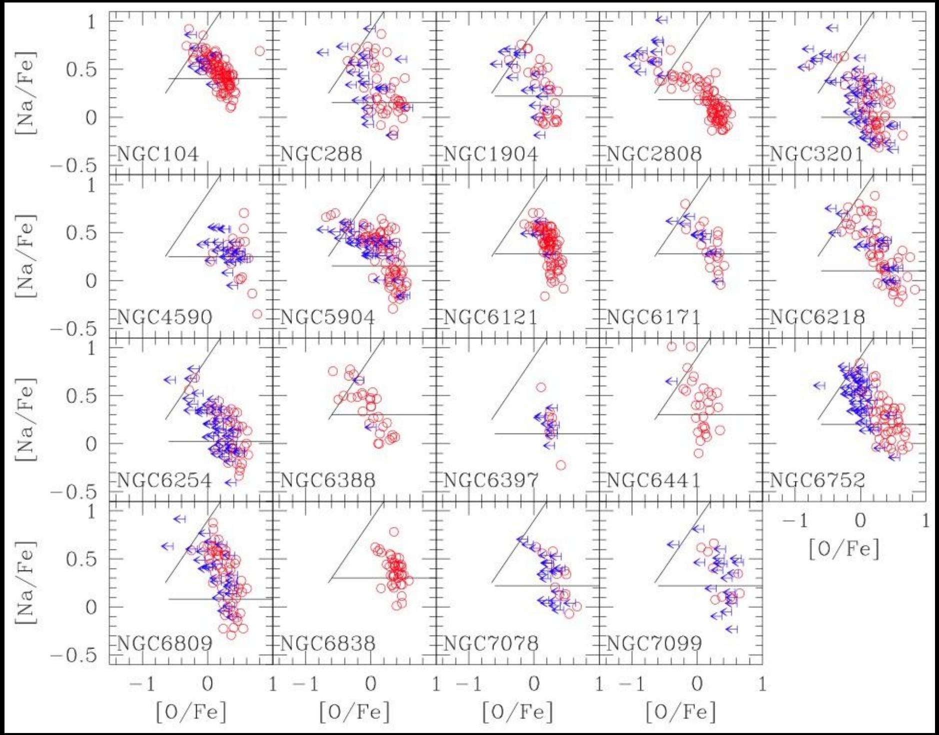
FIG. 3. Comparison of star counts in NGC 5053 with theoretical curve for $\log(r_t/r_c) = 0.75$. Medium-exposure 48-in. Schmidt plate.

King 1966 AJ
(see also King 2008 IAUS 246)

New chemical richness



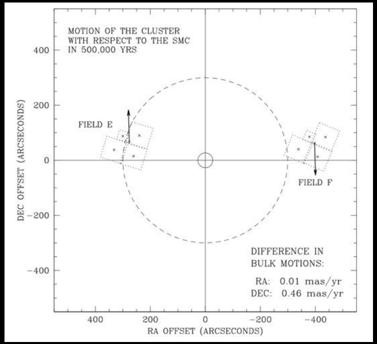
NGC 2808 | Piotto+ 2007 MNRAS



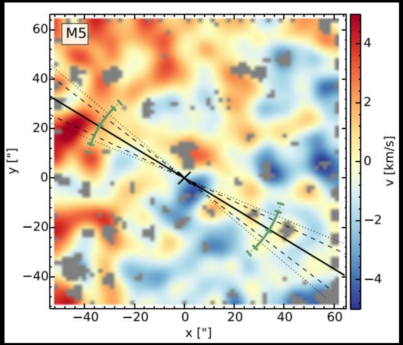
Several Galactic globulars | Carretta+ 2009 A&A

New kinematical richness

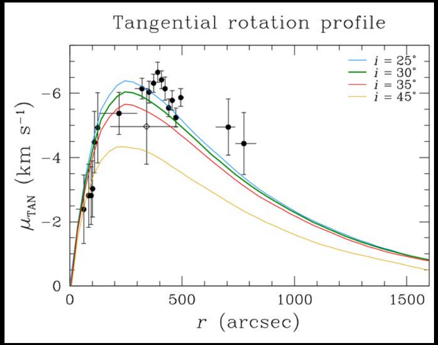
Internal rotation



47 Tuc | Anderson & King 2003 AJ

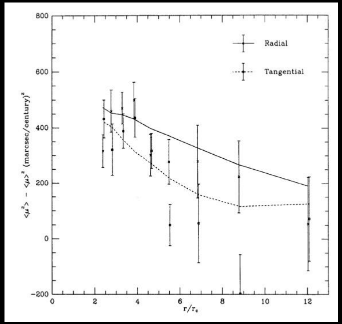


M5 | Fabricius et al. 2014 ApJL

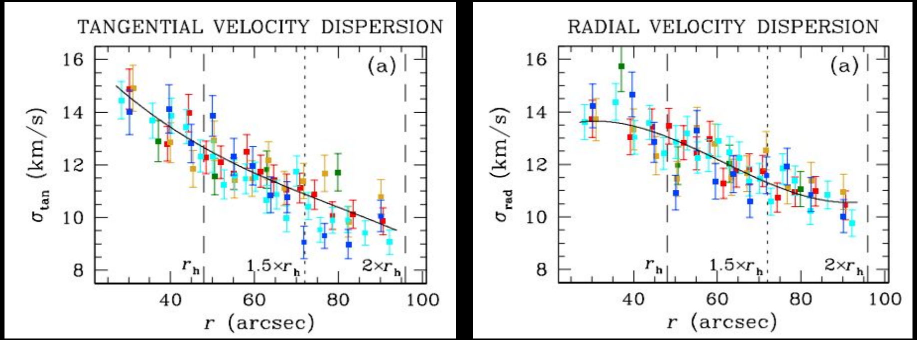


47 Tuc | Bellini et al 2017 ApJ (HSTPROMO)

Velocity anisotropy



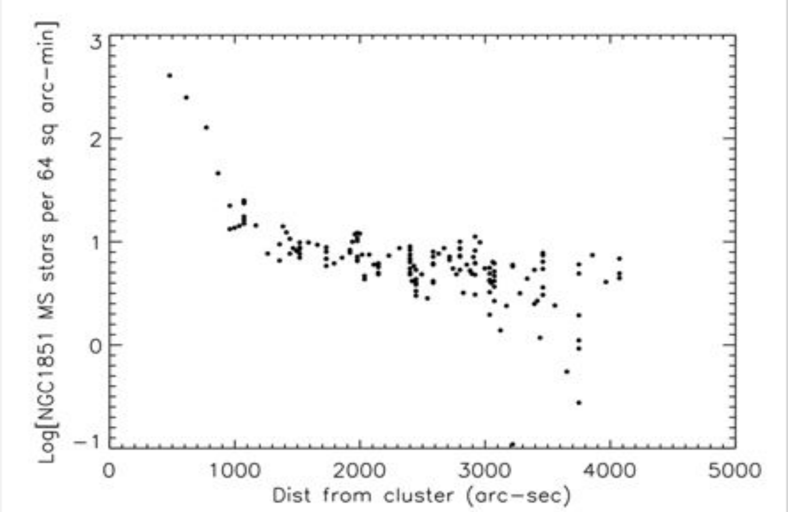
M13 | Lupton, Gunn, Griffin ApJ 1987



NGC 2808 | Bellini et al. 2015 ApJL, see also Watkins et al 2015a,b ApJ (HSTPROMO)

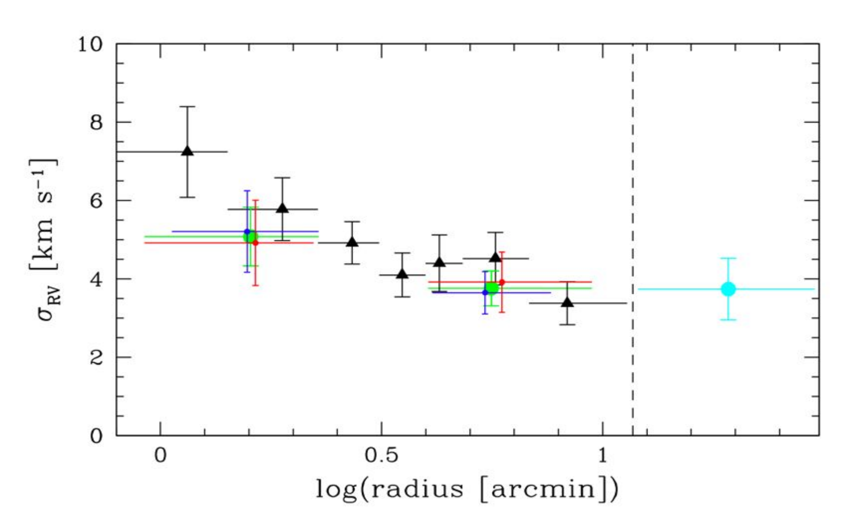
And some puzzles

Tenuous, extended stellar 'envelopes'?



NGC 1851 | Olszewski+ 2009 ApJ

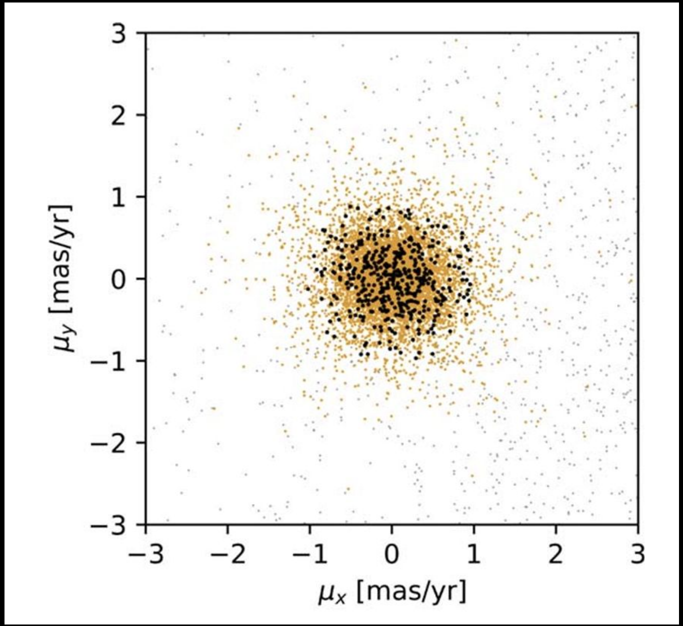
Stars at the edges moving too quickly?

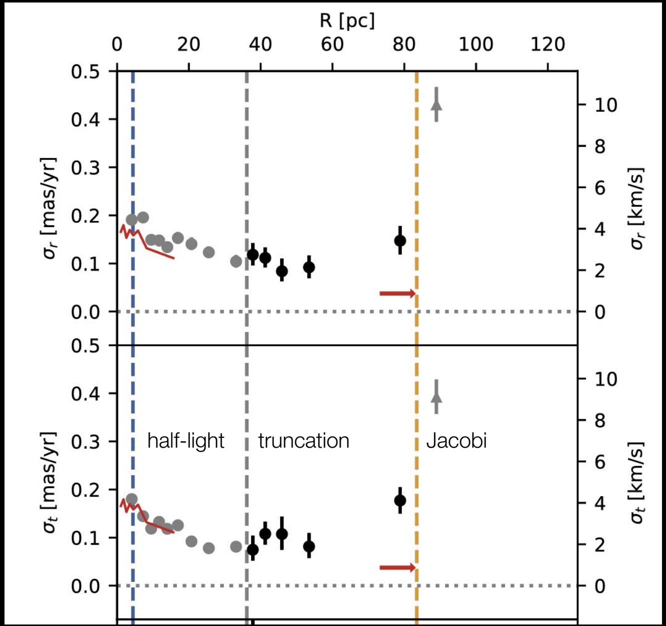


NGC 1851 | Marino+ 2014 MNRAS

And some puzzles

Stars at the edges moving too quickly?

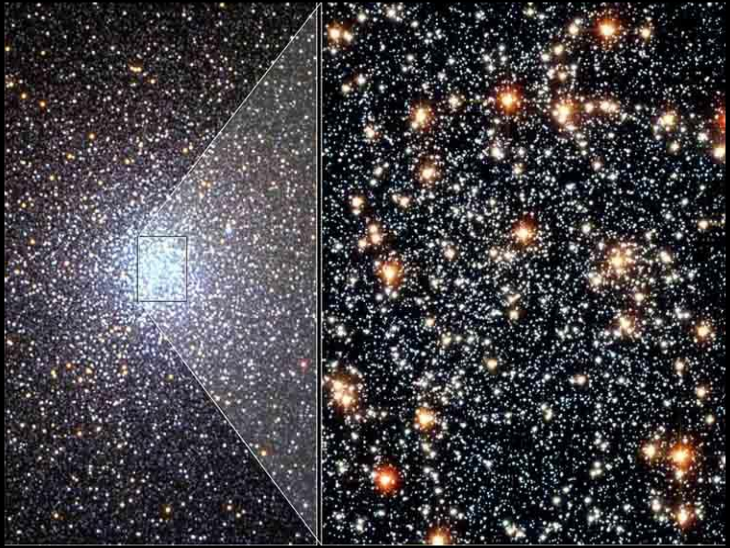




NGC 3201 | Bianchini+ 2019 ApJL

3. New *conceptual* landscape

New richness
of old globular clusters

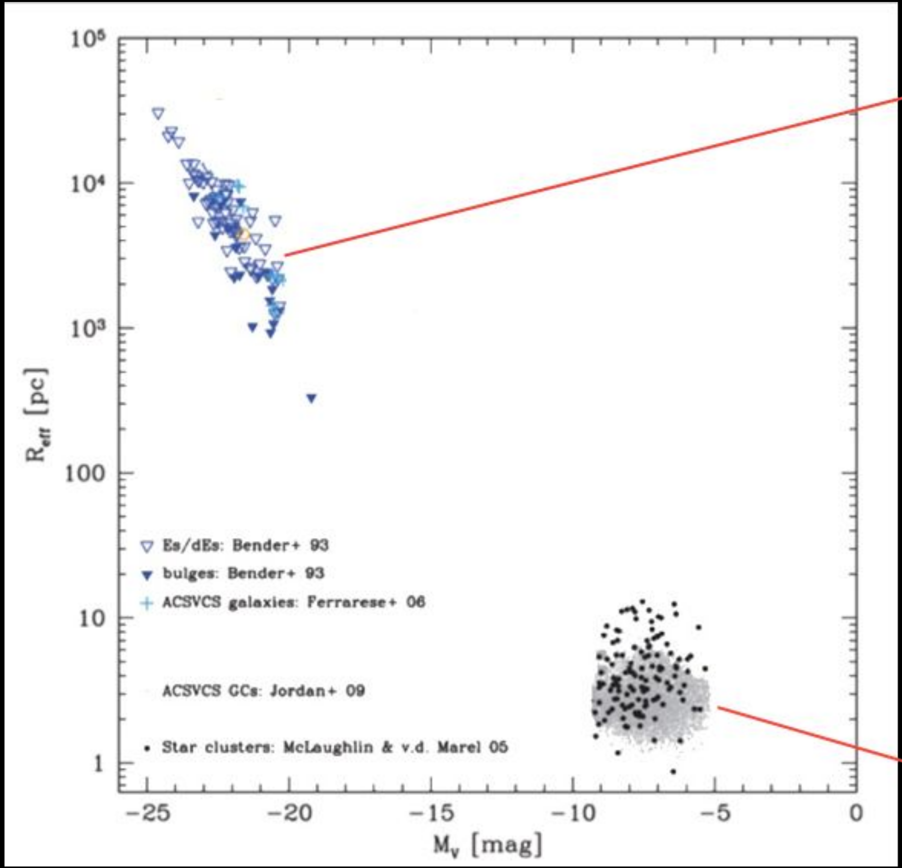


STScI/Hubble

Enigmatic small satellites:
star clusters or galaxies?

Garrison-Kimmel

When (a dynamicist's) life was simple

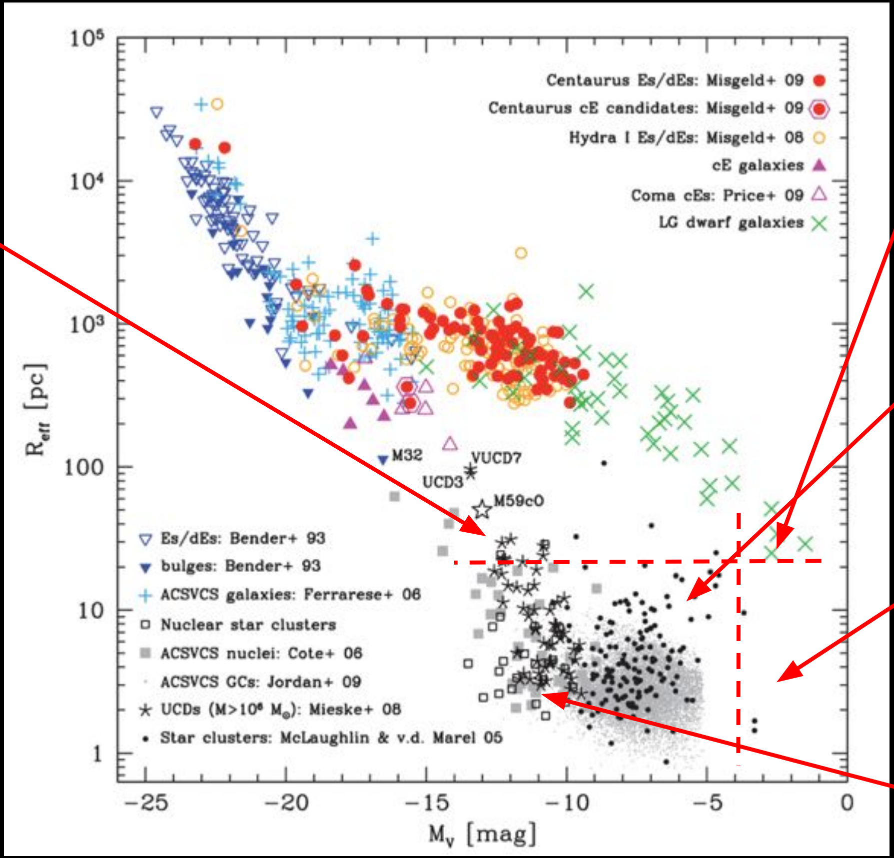


Ellipticals and bulges
R_eff ~ 5 kpc; M ~ -22

Star clusters
(Galactic and extragalactic)
R_eff ~5 pc; M ~ -7

When (a dynamicist's) life is a nightmare

Misgeld & Hilker 2011 MNRAS, Norris & AIMSS Coll. 2014 MNRAS



Ultra-faint 'satellites' *[lots of DM?]*
r_eff > 20pc; M < -3.5

Hydra II, Laevens 2, Pegasus III, Ret II, Eridanus II, Tucana II, Horologium I, Pictoris I, Phoenix II, Draco II, Sagittarius II, Horologium II, Grus II, Tucana III, Columba I, Tucana IV, Reticulum III, Tucana V, Crater 2, Acquarius 2, Pictoris II, Segue 1

Extended clusters and 'faint fuzzies' *[no DM?]*
10pc < r_eff < 20pc

Discovered in outskirts of MW, M31, M33, many Local dwarfs ...

Ultra-faint star clusters *[no DM?]*
r_eff < 20pc; M < -3.5

Segue 3, Munoz 1, Balbinot 1, Laevens 1/Crater, Laevens 3, Kim 1, Kim 2, Eridanus III, DES 1, Kim 3

Ultra-compact dwarfs *[DM? central BH?]*
massive globulars or stripped dwarfs?

Two *old* questions
on the *new* “kinematic richness”

Internal rotation and velocity anisotropy

Old question #1

What are the stability properties of rotating, anisotropic spheroidal equilibria?

Old question #1

$$F_q(E,L) = \frac{3\Gamma(6-q)}{2(2\pi)^{\frac{5}{2}}\Gamma(q/2)} E^{\frac{7}{2}-q} H\left(0, \frac{1}{2}q, \frac{9}{2}-q, 1; \frac{L^2}{2E}\right)$$

Dejonghe 1987 MNRAS

Equilibria have the *same* (Plummer) structure, and ‘controlled’ kinematics:

$$\sigma_r^2(r) = \frac{1}{6-q}\frac{1}{\sqrt{1+r^2}}$$

$$\beta = 1 - \frac{\sigma_\varphi^2}{\sigma_r^2} = 1 - \frac{\sigma_\theta^2}{\sigma_r^2} = \frac{q}{2}\frac{r^2}{1+r^2}$$

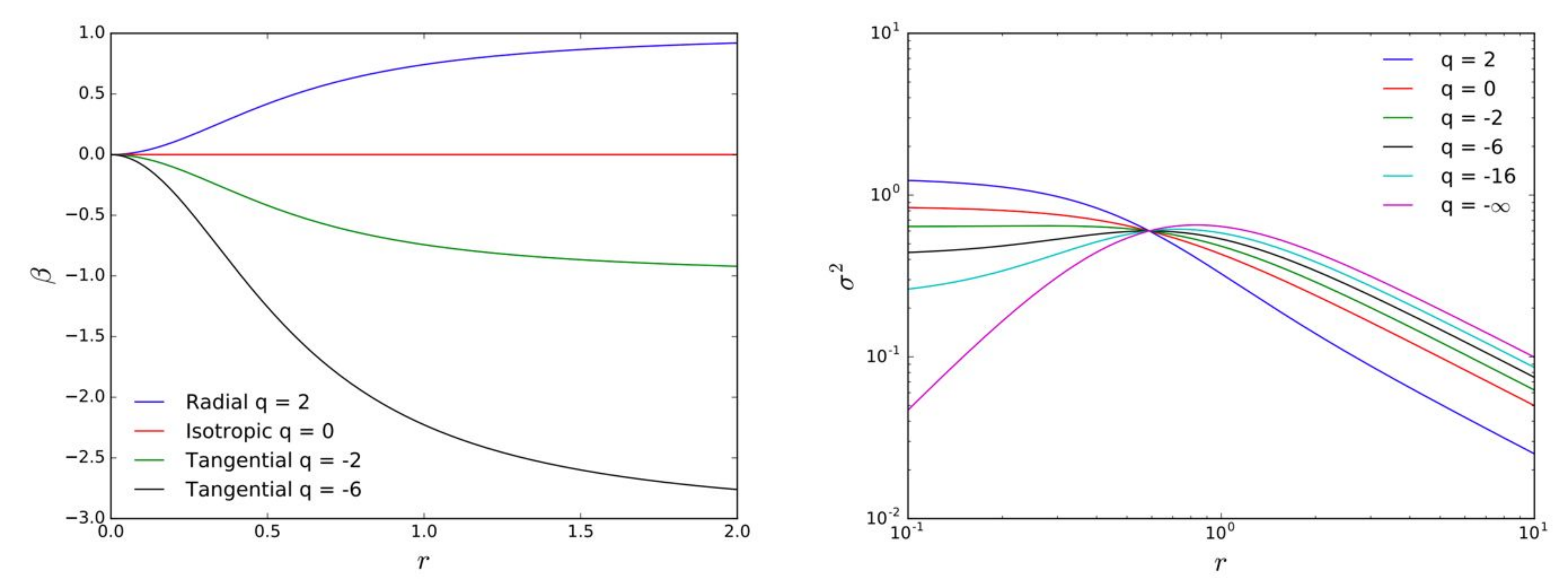
$$\sigma_\varphi^2(r) = \sigma_\theta^2(r) = \frac{1}{6-q}\frac{1}{\sqrt{1+r^2}}\left(1 - \frac{q}{2}\frac{r^2}{1+r^2}\right)$$

q>0 Radial
q=0 Isotropic
q<0 Tangential

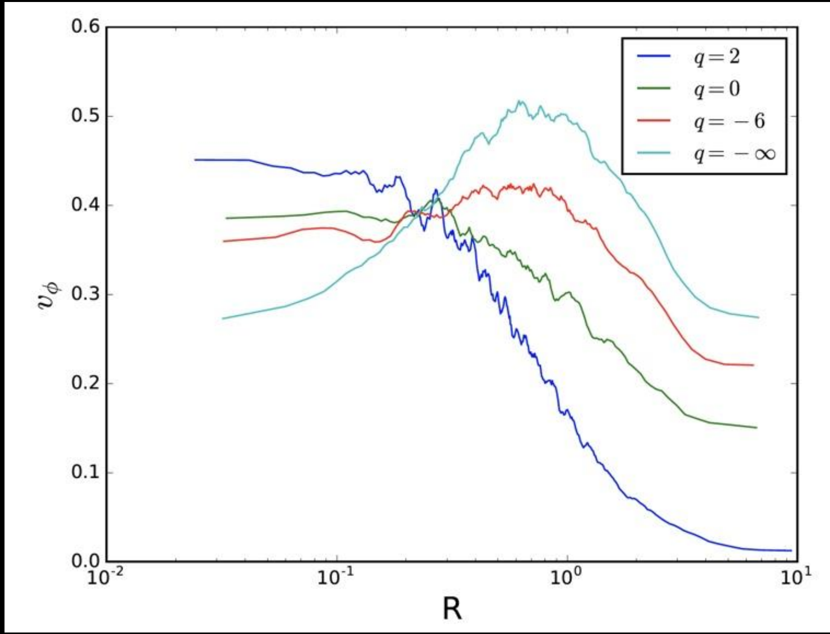
Limiting case (fully tangential): ‘Einstein sphere’
Radial regime may be extended (q>2) with Osipkov-Merritt’s Plummer spheres (but ROI unstable).

Old question #1

$$F_q(E,L) = \frac{3\Gamma(6-q)}{2(2\pi)^{\frac{5}{2}}\Gamma(q/2)} E^{\frac{7}{2}-q} H\left(0, \frac{1}{2}q, \frac{9}{2}-q, 1; \frac{L^2}{2E}\right)$$



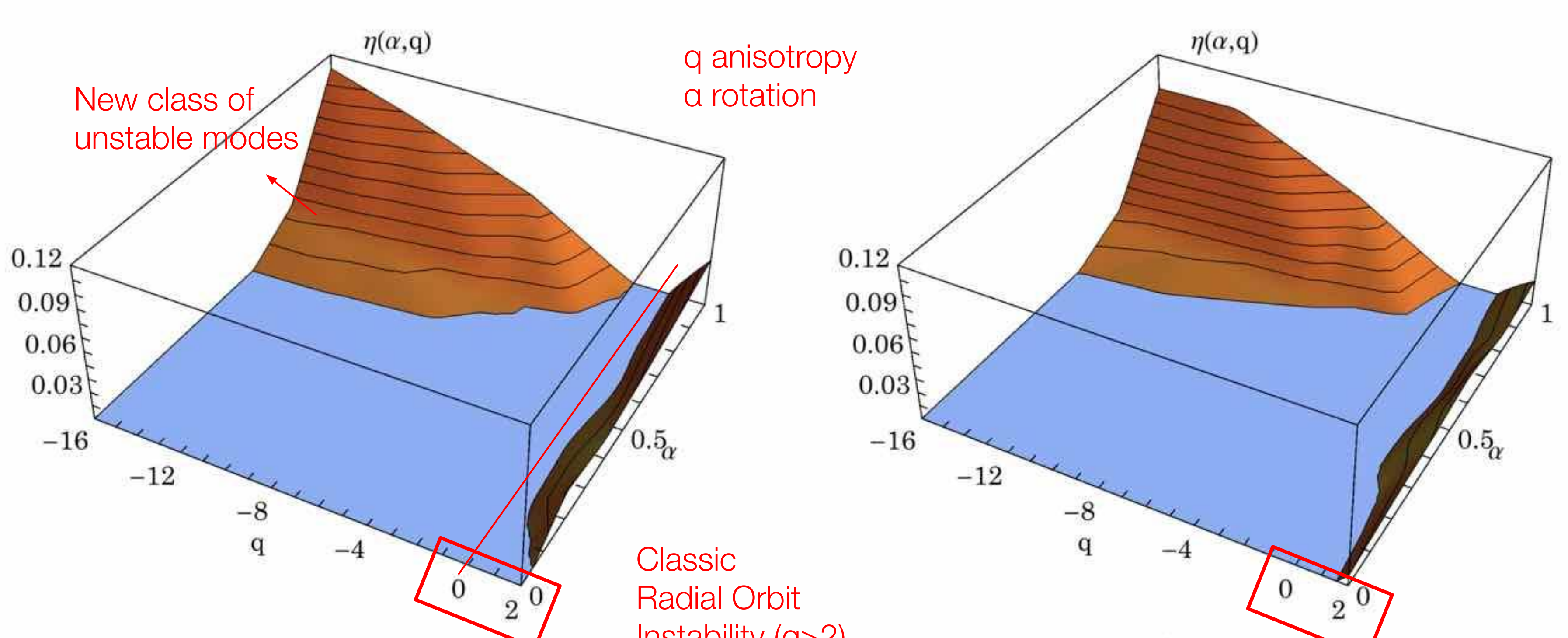
Old question #1



Internal rotation introduced via the “Lynden-Bell’s deamon”

$$f(E, L_z) = \alpha(E, L)\mathcal{H}(L_z)f(E) - (1 - \alpha(E, L))\mathcal{H}(-L_z)f(E) \qquad |\alpha| \leqslant 1$$

Old question #1



(a) Growth rate, matrix method. (b) Growth rate, N-body measurements.

Rozier, Fouvry, Breen, Varri, Pichon, Heggie 2019 MNRAS
Rozier, Breen, Heggie, Varri 2021 MNRAS

Old question #2

What are the implications of this “kinematic richness” on the long-term evolution of collisional systems?

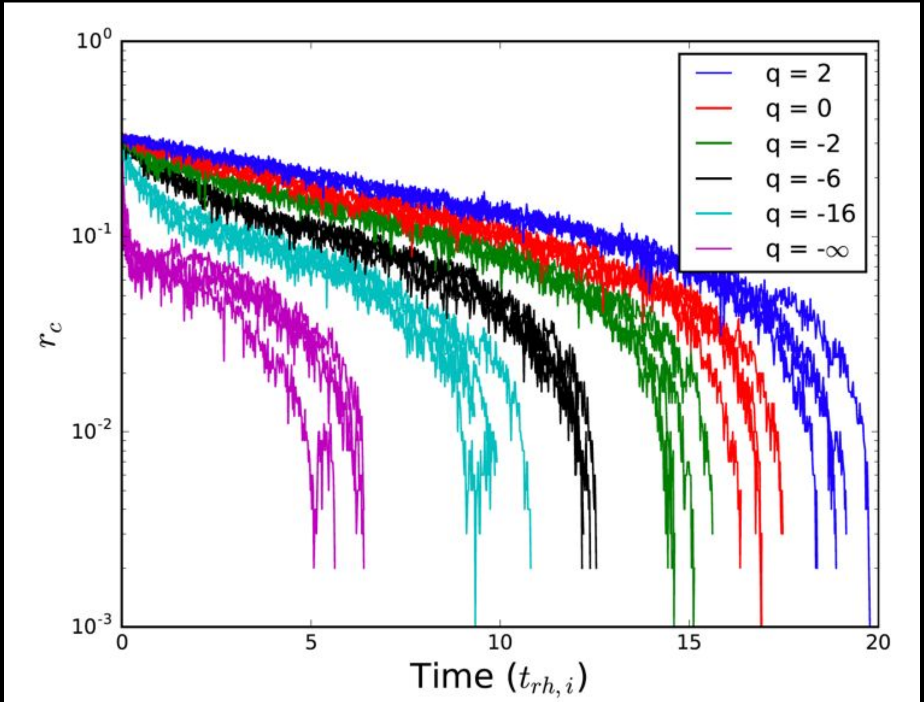
Old question #2

Tangentially (radially) anisotropic equilibria* reach core collapse earlier (later) than isotropic ones!

Catastrophic behaviour for highly tangential models

* with the same spatial properties and same initial half-mass relaxation time (Anisotropic Plummer, Dejonghe 1987)

Non-rotating anisotropic spheres



Breen, Varri, Heggie 2017 MNRAS

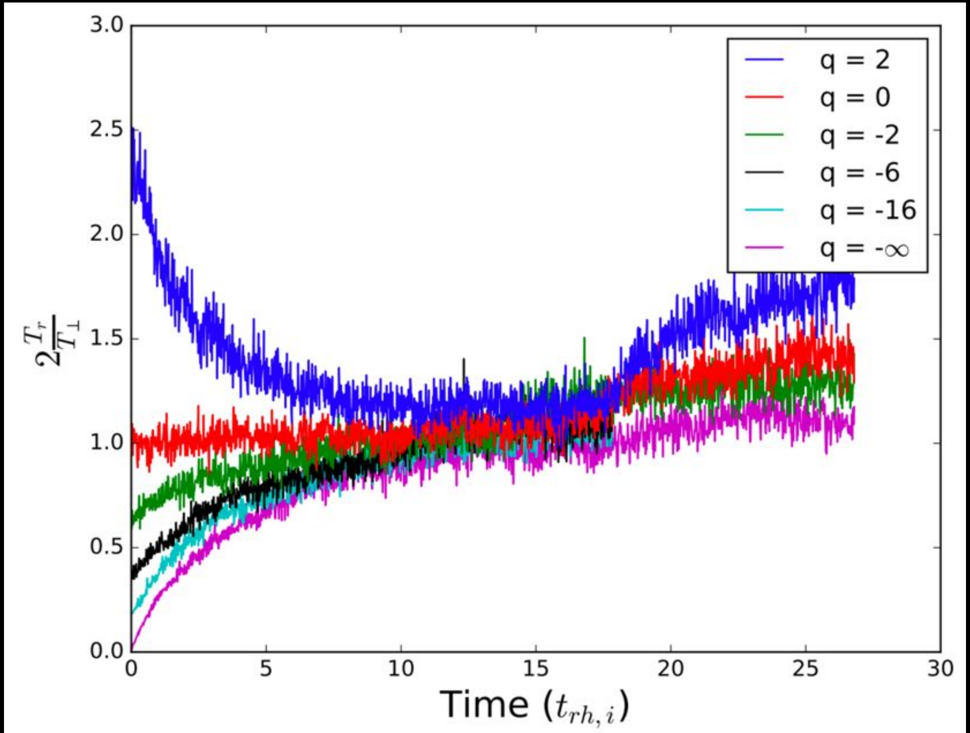
Side note: ‘primordial’ vs ‘evolutionary’ anisotropy

Star clusters in isolation: evolution towards isotropy, then generation of ‘evolutionary’ radially-biased anisotropy, mainly in the outer regions

On ‘evolutionary’ anisotropy, as driven by two-body relaxation:
Hénon 1971, Spitzer & Shapiro 1972, Bettwieser & Spurzem 1986, Spurzem 1991, Giersz & Heggie 1994, Takahashi 1995 ... (in isolation);
Giersz & Heggie 1997, Takahashi et al. 1997, Baumgardt & Makino 2003, Tiongco, Vesperini, Varri 2016a,b, 2018 (in a tidal field).

On ‘primordial’ anisotropy, as imprinted by various processes, e.g., ‘violent relaxation’:
van Albada 1982; Trenti, Bertin, van Albada 2005 ... (in isolation);
Vesperini, Varri, McMillan, Zepf 2014; Tiongco, Vesperini, Varri 2017, 2021, 2022 (in a tidal field).

Any impact of mass segregation? Yes, plenty.



Breen, Varri, Heggie 2017 MNRAS

Don’t ask me about rotation

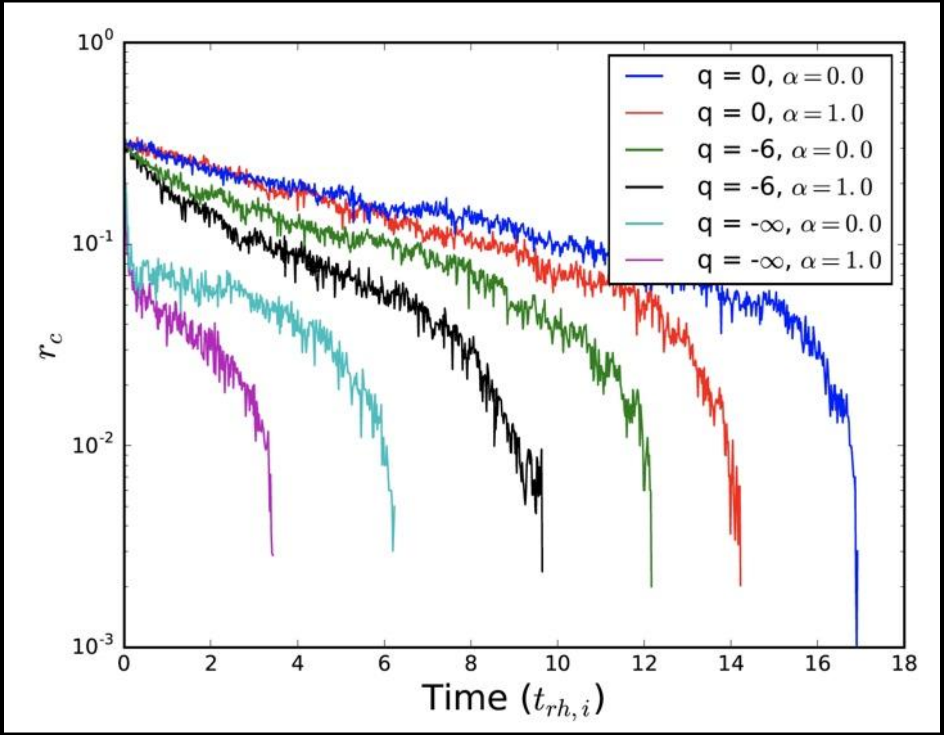
Rotating equilibria* reach core collapse earlier than their non-rotating counterpart

Previous investigations by Rainer Spurzem [discussion in week 4] and Hyung Mok Lee, with their collaborators (Fokker-Planck and N-body approaches), see also Kerwann Tep’s PhD thesis.

* with the same spatial properties and same initial half-mass relaxation time (Anisotropic Plummer, Dejonghe 1987)

Any impact of mass segregation? Yes, plenty.

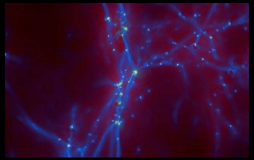
Rotating anisotropic spheres



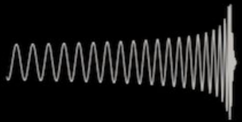
Breen, Varri, Heggie - only exploratory (unpublished)

Why is this *still* relevant?

Three big questions



How did the first stellar aggregates form?



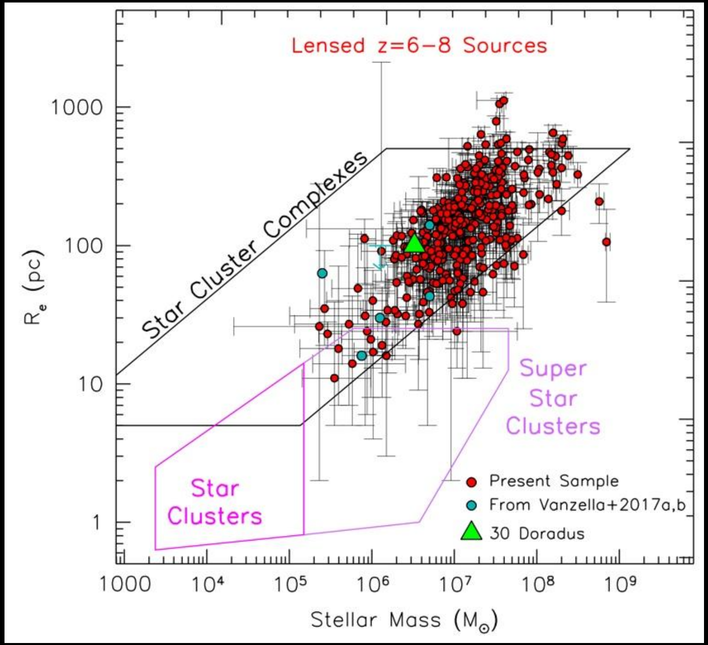
What links stellar and supermassive black holes?

Limit for dark matter in the smallest stellar systems?

Q#1. How did the first stellar aggregates form?

Direct approach

Star-forming 'blobs' in the early Universe

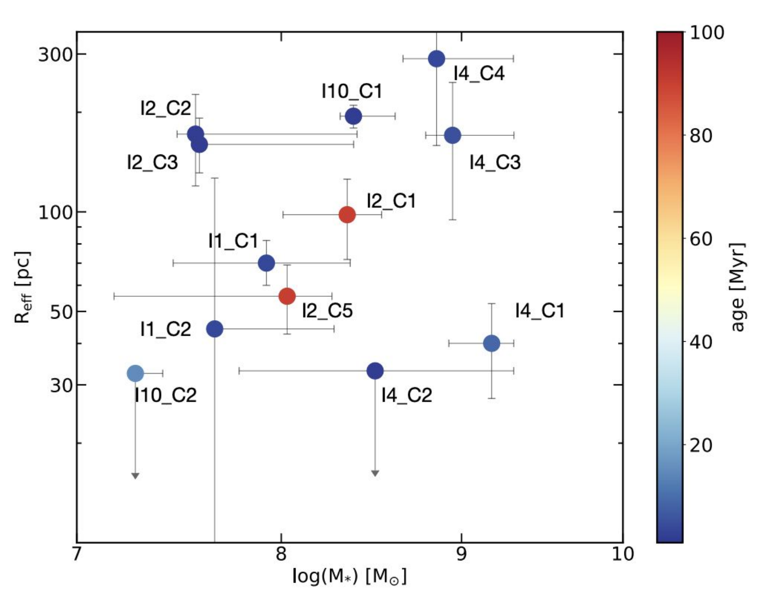


Small systems in Hubble Frontier Fields | Bouwens+ 2017 (2021)

Q#1. How did the first stellar aggregates form?

Direct approach

Star-forming ‘blobs’ in the early Universe

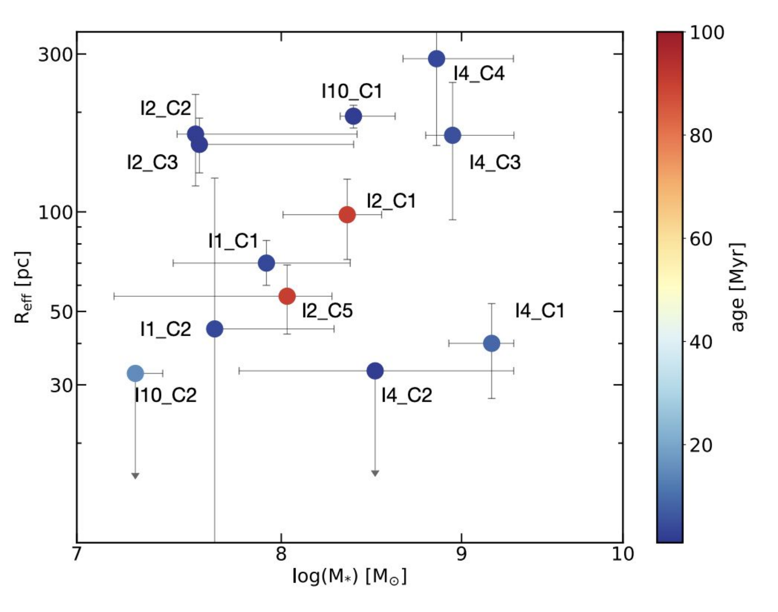


A JWST Cycle 1 example:
Clump population in SMACS0723 | Claeyssens+ 2023

Q#1. How did the first stellar aggregates form?

Direct approach

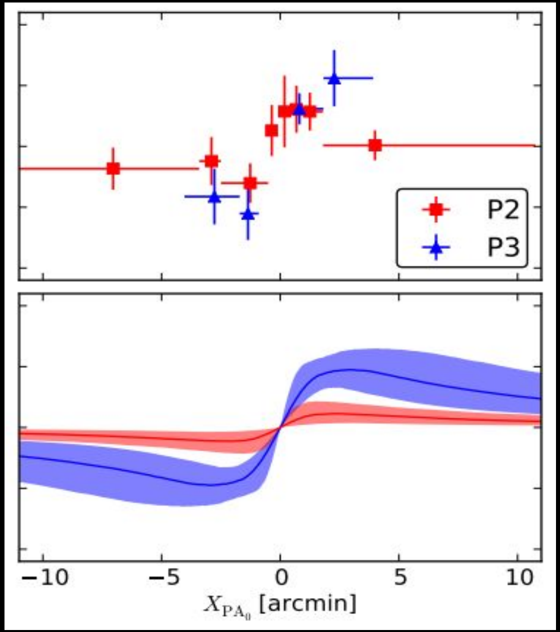
Star-forming ‘blobs’ in the early Universe



A JWST Cycle 1 example:
Clump population in SMACS0723 | Claeyssens+ 2023

Inverse approach

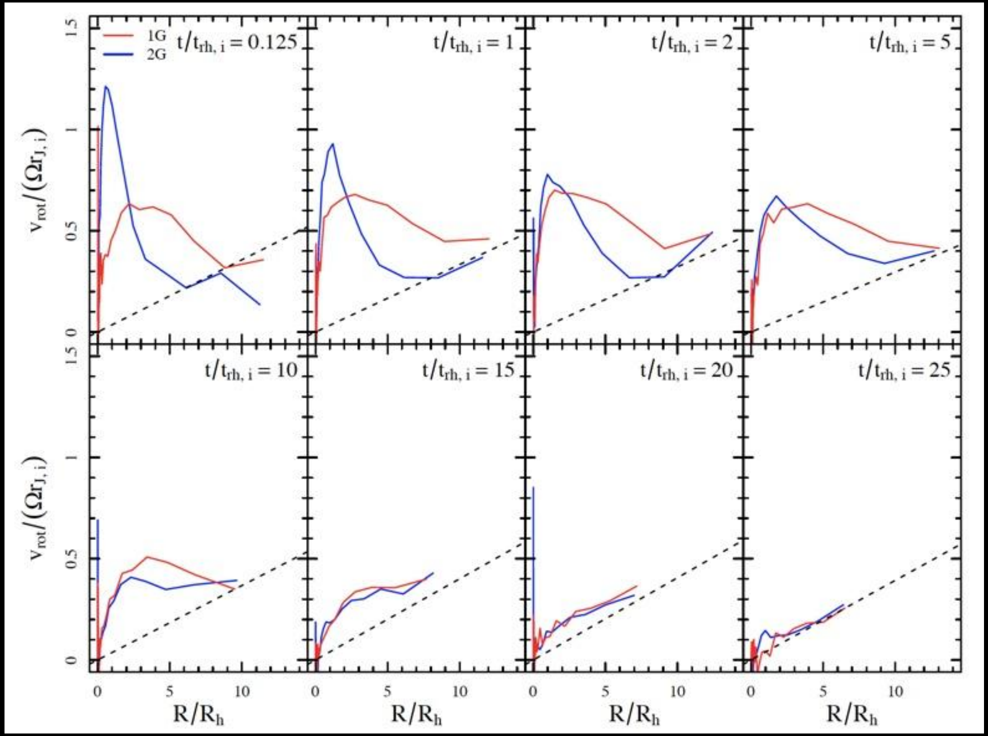
Dynamical ‘hysteresis’ of old star clusters



Two rotation curves in M13 |
Cordero+ 2017 MNRAS

Q#1. How did the first stellar aggregates form?

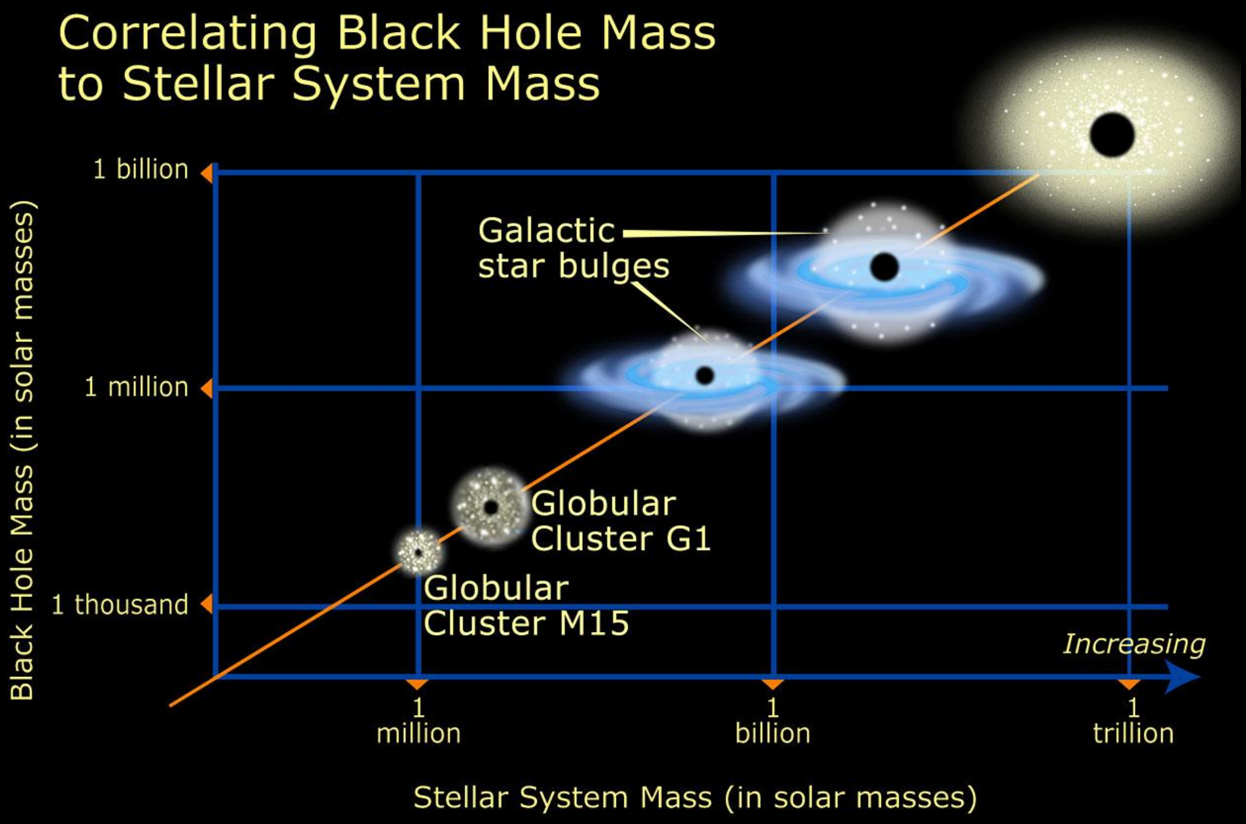
Theoretical: the memory of kinematic differences can survive into the life of the clusters



Maria Tiongco's PhD thesis,
Indiana Univ. Bloomington, USA

Tiongco, Vesperini, Varri 2019 MNRAS

Q#2. What links stellar and supermassive black holes?



Q#2. What links stellar and supermassive black holes?

New family of self-consistent isotropic models with central black holes (also axisymmetric, rotating)

$$f_K(E) = \begin{cases} A\left[\exp(-aE) - \exp(-aE_0)\right] & \text{if } E \leq E_0 \\ 0 & \text{if } E > E_0 \end{cases}$$

$$E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Phi_c$$

$$\psi(r) = a[E_0 - \Phi_c(r)]$$

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_{BH}}{r^2} - \frac{4\pi G}{r^2}\int_{r_{\min}}^{r} s^2\rho(s)\mathrm{d}s$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\hat{r}} = -\frac{9\mu}{4\pi\hat{r}^2} - \frac{9}{4\pi\hat{r}^2}\int_{\epsilon}^{\hat{r}} 4\pi s^2 \frac{\hat{\rho}}{\hat{\rho}_0}\mathrm{d}s$$

$$\mu \equiv \frac{M_{BH}}{\rho_0 r_K^3}$$

$$\epsilon \equiv \frac{r_{\min}}{r_K}$$

Huntley & Saslow 1975 (“Loaded polytropes”)

Q#2. What links stellar and supermassive black holes?

New family of self-consistent isotropic models with central black holes (also axisymmetric, rotating)

$$f_K(E) = \begin{cases} A\left[\exp\left(-aE\right) - \exp\left(-aE_0\right)\right] & \text{if } E \leq E_0 \\ 0 & \text{if } E > E_0 \end{cases}$$

$$E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Phi_c$$

$$\psi(r) = a[E_0 - \Phi_c(r)]$$

Vlasov-Poisson initial value problem

$$\nabla^2\psi = -9\frac{\hat{\rho}(\psi)}{\hat{\rho}(\Psi)}$$
$$\psi(\epsilon) = \Psi$$
$$\psi'(\epsilon) = -\frac{9\mu}{4\pi\epsilon^2}$$

- Ψ central potential depth
- μ black hole mass
- ϵ : inner boundary radius

Q#2. What links stellar and supermassive black holes?

New family of self-consistent isotropic models with central black holes (also axisymmetric, rotating)

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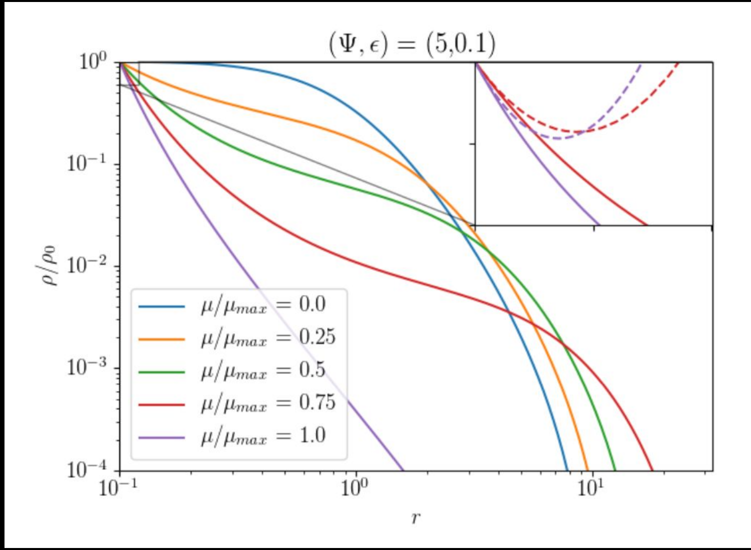

Bonsor, Varri, Vanneste - about to be submitted

Sam Bonsor's PhD thesis, Univ Edinburgh

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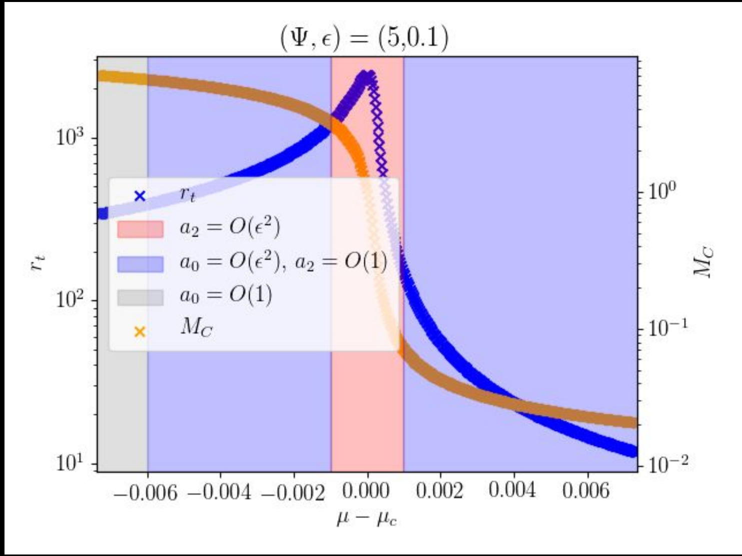

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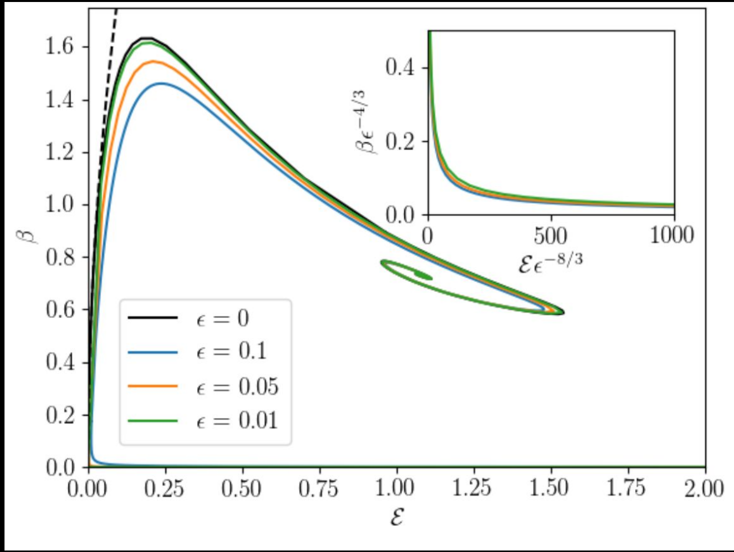
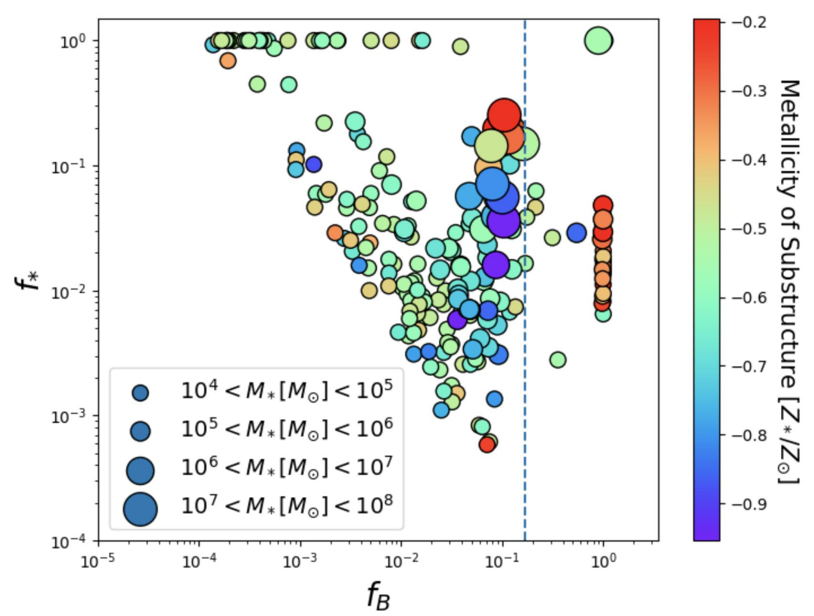


Bonsor, Varri, Vanneste - about to be submitted

Sam Bonsor's PhD thesis, Univ Edinburgh

Q#3. Which are the smallest stellar systems containing dark matter?

Theoretical: Small systems at early times: who's who?



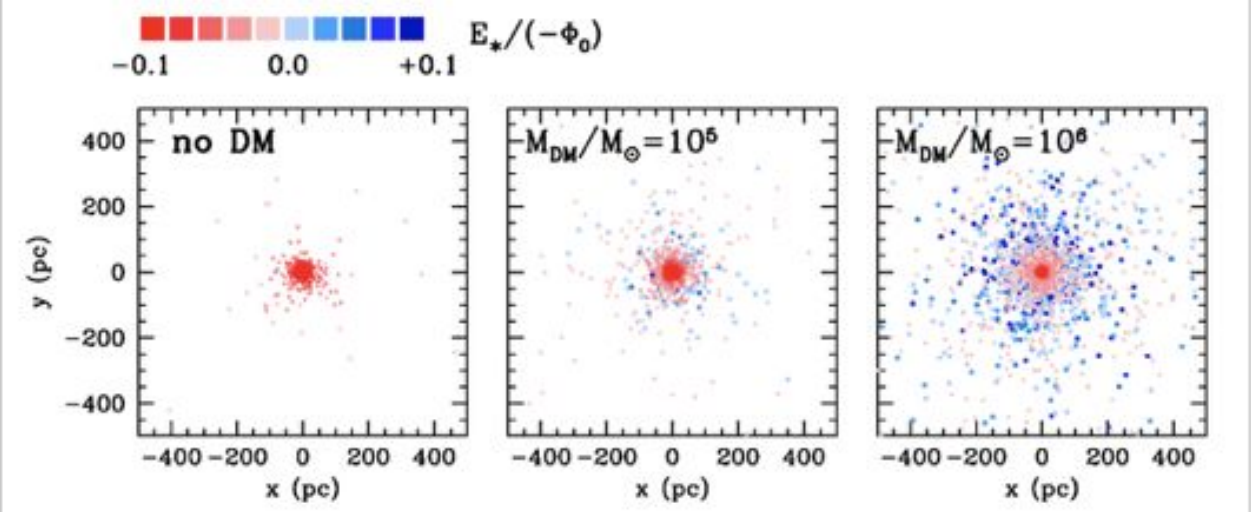
Frederika Phipps' PhD thesis,
Univ. Edinburgh, UK

Infant GCs in FiBY Simulations | Phipps, Khochfar, Varri+ 2020 A&A, 2023 MNRAS

Q#3. Which are the smallest stellar systems containing dark matter?

Theoretical: "Gravothermodynamics" in the presence of dark matter

Dynamical evolution of collisional stellar systems changes significantly.
Implications on the "star cluster - galaxy divide" (and dark matter clustering scale).



Penarrubia, Varri, Breen+ 2017 MNRAS

Q#3. Which are the smallest stellar systems containing dark matter?

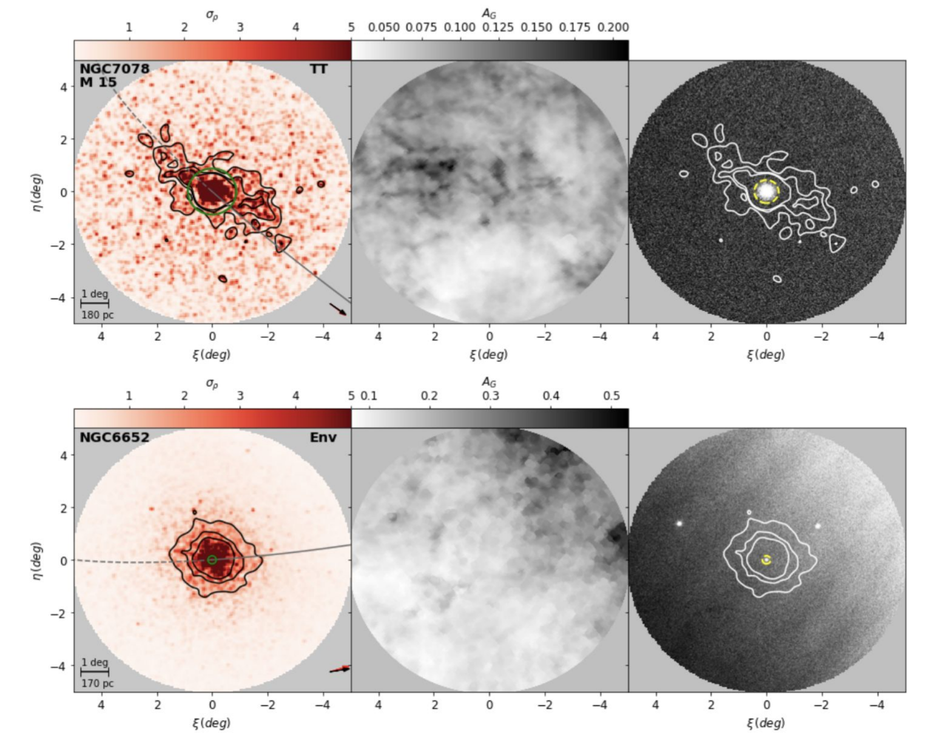
Observational: Systematic morphological characterisation of GC peripheries w/ DR3

Bayesian framework to assess membership
via photometry + astrometry,
with cluster / extended / contaminant population

Validation over synthetic data

Three classes:

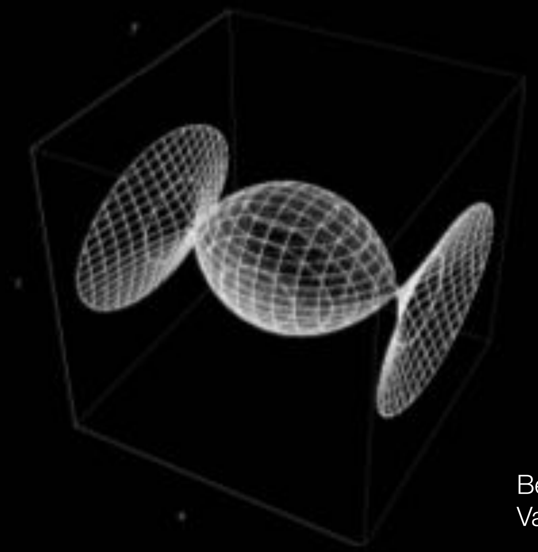
(1) Tidal Tails
(2) Envelope
(3) None



Kuzma, Varri, Ferguson et al - in prep (60 GCs)

Tidal field (triaxially) stretches star clusters

truncation radius ≠ Jacobi radius



Bertin & Varri 2008
Varri & Bertin 2009

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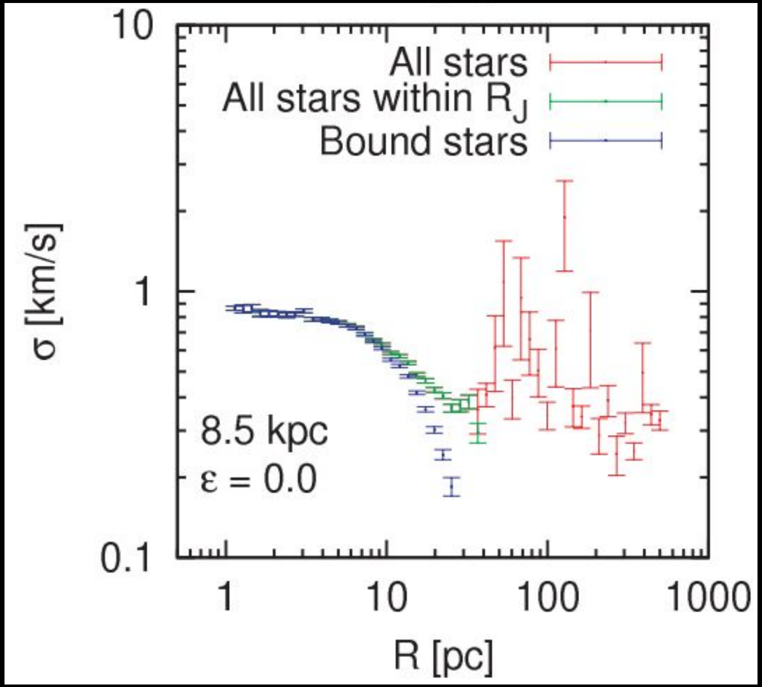
Singular perturbation problem, as for rigidly rotating polytropes à la Chandrasekhar

Rix & White 1989, Weinberg 1993, Heggie & Ramamani 1995

"Potential escapers"

energetically unbound, yet spatially confined stars



Kuepper, Kroupa, Baumgardt, Heggie 2010 MNRAS
Claydon, Gieles, Zocchi 2017 MNRAS

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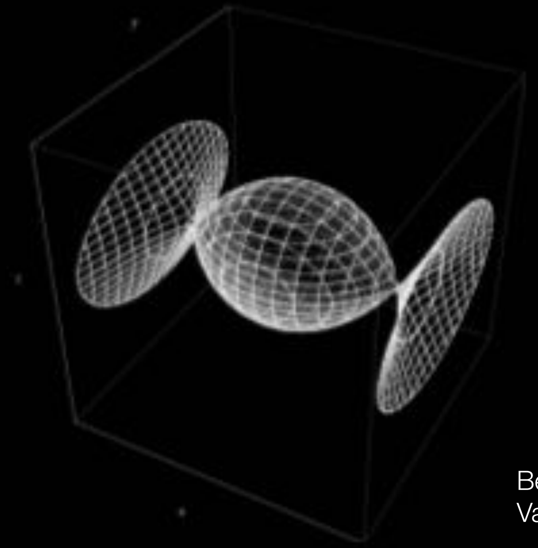

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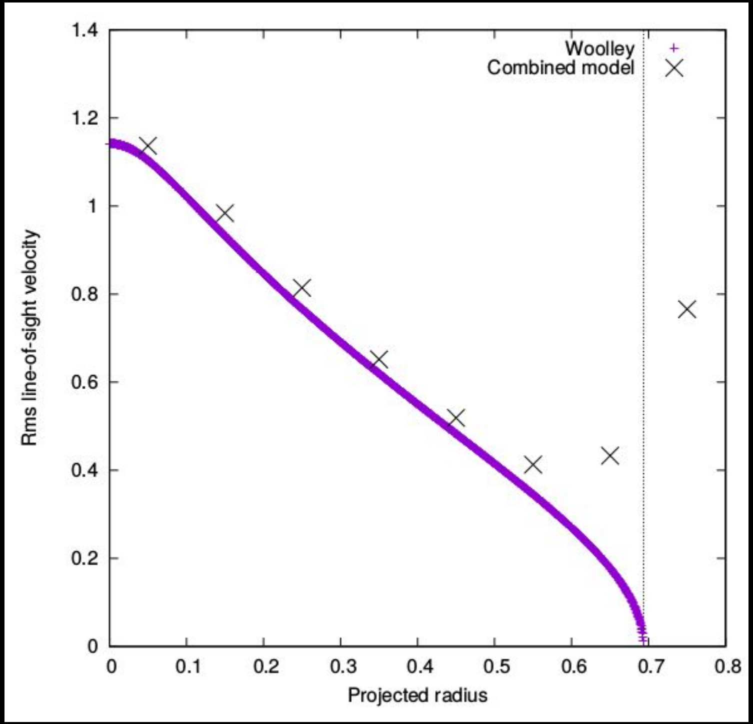
Singular perturbation problem, as for rigidly rotating polytropes à la Chandrasekhar

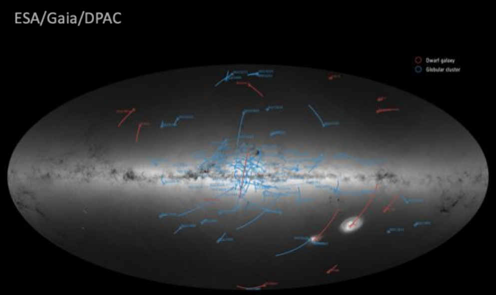
Rix & White 1989, Weinberg 1993, Heggie & Ramamani 1995

Equilibria with potential escapers

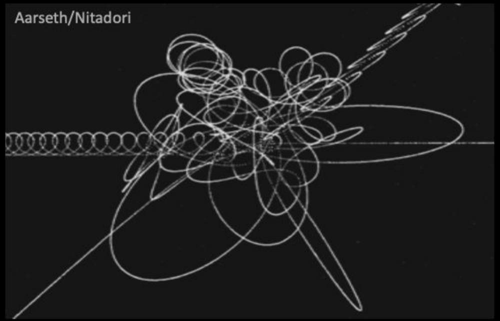
Inspired by Henon's 'family f' of RC3BP, constructed with Lidov-Kozai (quadrupole) theory



Daniel, Heggie, Varri 2017 MNRAS
see also Claydon, Gieles, Varri et al. 2019 MNRAS.

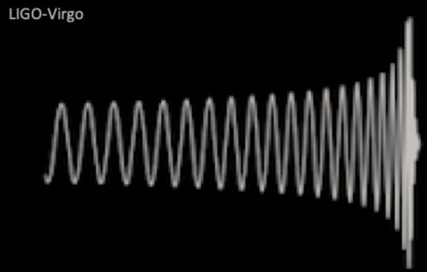


Precision astrometry
in our Milky Way

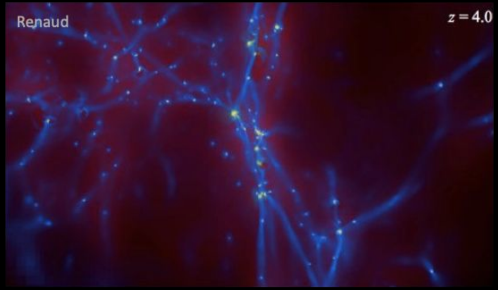


Gravitational million-body
problem ‘solved’

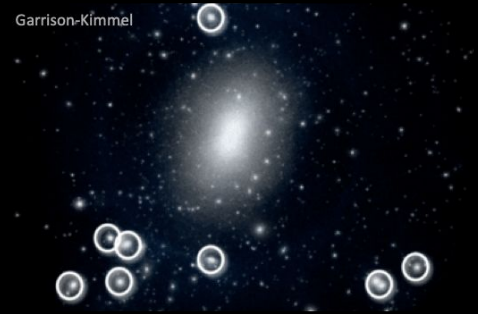
Emerging complexity
of old star clusters



Gravitational waves from
binary black holes



Towards small scales in
cosmological simulations



Enigmatic small satellites:
star clusters or galaxies?

Finale

#1 - Interesting times for the study of the dynamics of small stellar systems.

They have much to say about some of the biggest astrophysical questions.

#2 - Their emerging phase space richness requires a proper treatment of physical ingredients traditionally considered as ‘2nd-order complications’.

Synergy between ground-based spectroscopic surveys and HST + Gaia proper motions is transformative. Access to phase space, finally.

#3 - (New) science often lives at unexplored intersections.

Rotation ∩ tides, rotation ∩ anisotropy, anisotropy ∩ tides, collisional ∩ collisionless stellar dynamics, gravity ∩ plasma?

Investigation of the role of ‘classical’ physical ingredients is the essential foundation for understanding *any* dynamical signature of more complex phenomena (MSPs, IMBHs?, DM?)