- **From plasmas to galaxies**

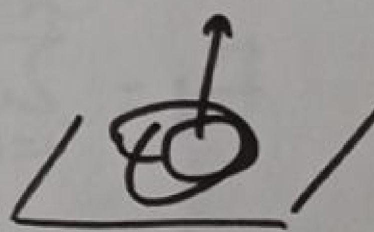
$$\text{Coulomb potential: } \pm 1/|\vec{R}-\vec{R}'|$$
$$\text{Newton potential: } - 1/|\vec{R}-\vec{R}'|$$

(Always attractive, cannot be screened)

Analogies go beyond the change of sign! goal of this program.

- **Examples of self-gravitating systems:**

  - **Disc Galaxies:** $N = 10^{11}$, $T_{dyn} = 10^8\,yr$, "Cold" & Young



  - **Globular Clusters:** $N = 10^5$, $T_{dyn} = 10^5\,yr$, "Hot" & Old

  - **Galactic Nuclei:** $N = 10^6$, $T_{dyn} = 10 - 1{,}000\,yr$, "Hot" & Very old

More generically, these are all long-range interacting systems. What is the dynamics of these?

- **Phase space:** $w = (x, v) = (q, p)$ (can be very general)
- N particles of mass: M_{tot}/N

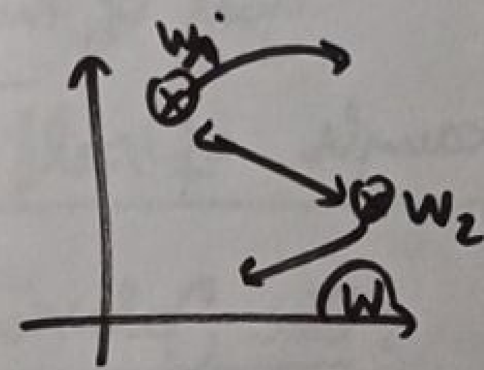
- **Hamiltonian:**

$$H_{\cancel{tot}}(w) = \underbrace{\sum_{i=1}^{N} m\, U_{ext}(w_i)}_{ex:\ \frac{1}{2}v^2} + \sum_{i<j} \underbrace{m^2\, U(w_i, w_j)}_{ex:\ -G/|x-x'|}$$

Can be (much) more generic

- **State of the system:**

$$F_d = \sum_{i=1}^{N} m\, \delta(w - w_i(t))$$



- **N-body problem:**

$$\frac{\partial F_d}{\partial t} + \underbrace{[F_d, H_d]}_{\substack{\text{Poisson bracket} \\ \text{Coordinate-free}}} = \underbrace{0}_{\text{Exact}}$$

$\Big\{$ Simple Space
Complicated DF

Average dynamics:

$$F_d = \underbrace{\langle F \rangle}_{\substack{\text{Mean} \\ \text{State}}} + \underbrace{\delta F}_{\text{Perturbations}}$$

e.g: $\delta F \propto 1/\sqrt{N}$ Poisson Shot noise

This leads to a whole range of dynamics

- **Mean dynamics:**

$$\frac{\partial \langle F\rangle}{\partial t} + [\langle F\rangle, \langle H\rangle] = 0$$

Vlasov eq.
CBE
Liouville

Collisionless dynamics. As a fluid in phase space

Ex: Violent relaxation & NFW profile

- **Dynamics of perturbations.**

$$+\frac{\partial \delta F}{\partial t} + \underbrace{[\delta F, \langle H\rangle]}_{\text{Phase Mixing}} + \underbrace{[\langle F\rangle, \delta H]}_{\text{Collective Effects}} = 0$$

with $\Delta \delta\Phi = 4\pi G \int dv\, \delta F$ (Poisson eq.)

Governs **Linear Response Theory:**

Ex. (Linear) Landau damping

- Collective dressing: 

Mass: $1/\epsilon(v)$
- Modal response: $\delta\Phi(t) \simeq e^{-i\omega_M t}$
↑Complex frequency

Ex: Spiral Arms,

- **Collisional dynamics**

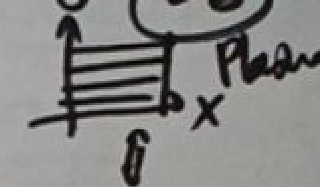
→ (Quasi)Equilibrium: $[\langle F\rangle, \langle H\rangle] = 0$

→ $\delta F \longrightarrow \delta F(t \to +\infty)$ "Time Separation"

$$\frac{\partial \langle F\rangle}{\partial t} = \underbrace{C[\langle F\rangle]}_{\text{BL equation}}$$ "Collision Operator"

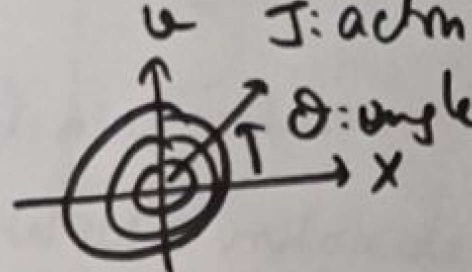
Ex: → Radial migration in discs
→ Gravothermal Catastrophe

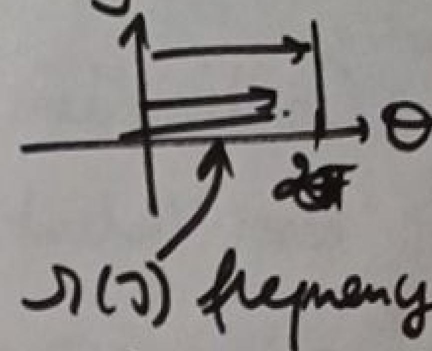
• Can we be more explicit?
A galaxy is a plasma in the good variable



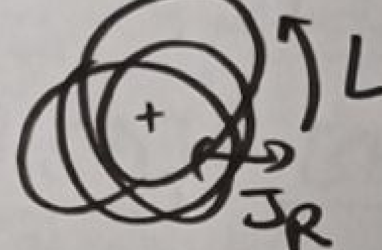
Harmonic oscillator

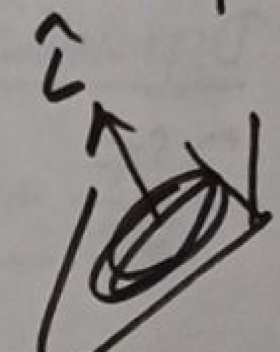
$$\mathcal{H} = \frac{1}{2}v^2 + \frac{1}{2}\omega^2 x^2$$





<u>Examples</u>: . <u>Disc</u>: (J_R, L)





. Globular Cluster $(J_R, L, \hat{L})$

. Galactic nuclei: $(a, e, \hat{L})$

Now, we can translate

Plasmas		Galaxies
(x, v)	Orbits	(θ, J)
v	Frequencies	$\Omega(J)$
$\mathcal{U}(x, x')$	Interaction	$\mathcal{U}([\theta, J], [\theta', J'])$
$F(v)$	Distribution	$F(J)$

3a

Translation of BL:

$$\frac{\partial F(J,t)}{\partial t} = - \frac{M_{tot}}{N} \frac{\partial}{\partial J} \cdot \Bigg[\sum_{k,k'}{}' k \int dJ' \left| \frac{U_{kk'}(J,J')}{\epsilon_{kk'}(J,J',k\Omega)} \right|^2$$

$$\delta(k\Omega - k'\Omega') \left\{ k' \frac{\partial}{\partial J'} - k \frac{\partial}{\partial J} \right\} F(J,t) F(J',t)$$

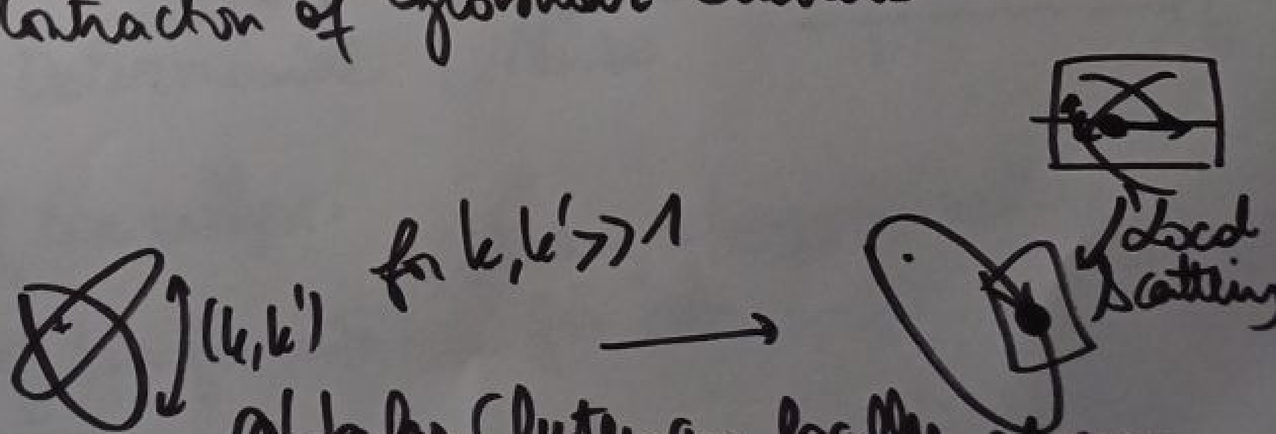
Elements

- Resonance condition, i.e. Scattering in AA
- $\propto M_{tot}/N$ sourced by Poisson shot noise
  $\Leftrightarrow 1/N$
- Long-range interaction $U_{kk'}(J,J') \Leftrightarrow 1/k^2$ in plasma
- Dressing: $1/\epsilon(k.\Omega)$: Equivalent of Debye Shielding
- $\underbrace{\text{Friction}}_{\text{Dynamical friction of stars}}$ & $\underbrace{\text{Diffusion}}_{\text{Thickening of the MW disc}}$

Does it work? Yes, it works where it works.

→ Radial Migration in discs
→ Eccentricity relaxation in Nuclei
→ Contraction of Globular Clusters

Side-note



Globular Clusters are locally plasmas.

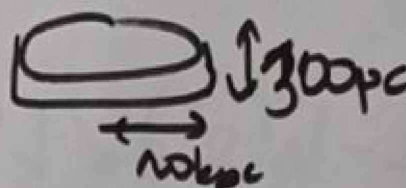
Should it work? No.

This is the topic of this program.

Will conclude with examples of limits

$\boxed{\text{Integrability}}$: Absence of A&A

Ex: Thick disc of the MW



Oblate halos :

~~Orbits are~~

Orbits explore phase-space much more efficiently

$\boxed{\text{Degeneracies}}$

$\delta(k\Omega - k'\Omega')$ Resonance condition

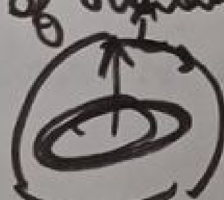
Ex: Harmonic cores: $\Omega(J) = \Omega_0$

$$\frac{\delta([k-k'])}{\Omega_0}.$$

System is too resonant. Relaxation in cores at the center of halos "stalling of dynamical friction"

Anomalo

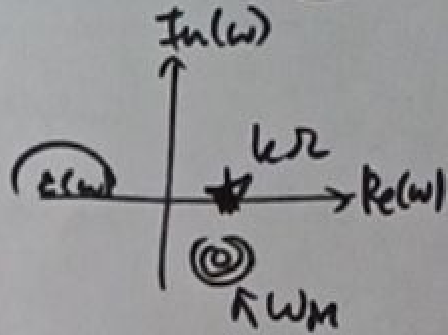
Ex: Relaxation of orientation $\Omega(J) = 0$



Linked to
- Strong turbulence
- Gyrokinetic
- Renormalisation theory

Marginal Stability

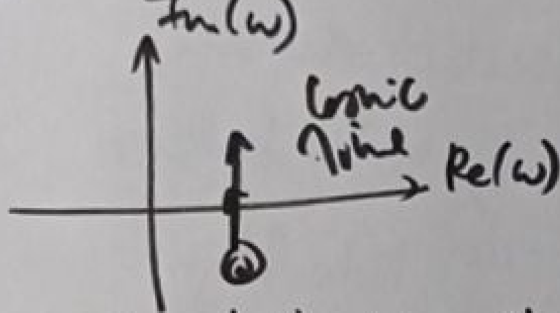
Theory: $U \to \dfrac{U}{|\epsilon|}$ Wake: $1/\epsilon(\omega)$



$$\delta\Phi(t) = \underbrace{\frac{e^{-ik\Omega(J)t}}{\epsilon(k\Omega)}}_{\text{Wake}} + \underbrace{e^{-i\omega_M t}}_{\text{Mode}}$$

Mode : Landau damping

Approaching instability



$\left(\partial F/\partial t\right)_{BL} \propto \dfrac{U^2}{|\epsilon|^2}$ diverges !

What is happening

Linked to:
- Non-linear trapping
- Mode particle
- Weak turbulence
- Bifurcation

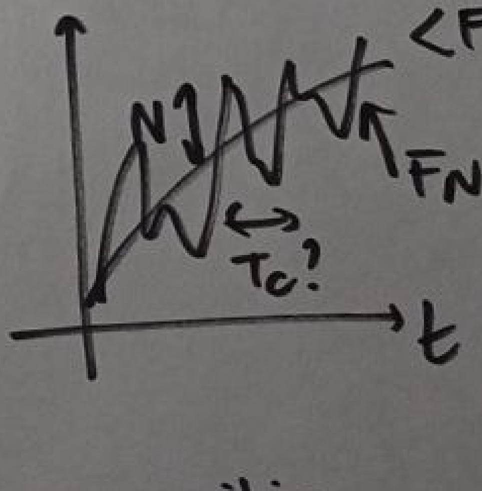
$\boxed{\text{Large deviations}}$

$\dfrac{\partial \langle F \rangle}{\partial t} = C[\langle F \rangle]$ Deterministic.

What about one galaxy? $F_d = \sum_i \delta[w - w_i(t)]$

$\underbrace{F_N}_{\text{Stochastic}} = \xi \int \dfrac{d\theta}{(2\pi)^3} F_d$ (like $\int dx$ in plasma)

$$\frac{\partial F_N}{\partial t} = \underbrace{C[F_N]}_{\text{Deterministic}} + \underbrace{\eta[F_N]}_{\text{Noise}}$$



Ex: Time of core collapse:
Time of bar formation:

Obtaining a Langevin writing for PDF

To conclude:

Plasmas & galaxies share deep similarities.

Could also mention:

→ Branch cuts in linear response
→ Orbit Average & gyrokinetics
→ N-body simulation
→ Kinematic diversity
→ Phase-Space Turbulence
→ N-body simulations

Interconnections have to be strengthened