Rotation drive by high-frequency plasma waves: dephasing and momentum conservation

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Kinetics of Plasmas and Self-Gravitating Systems

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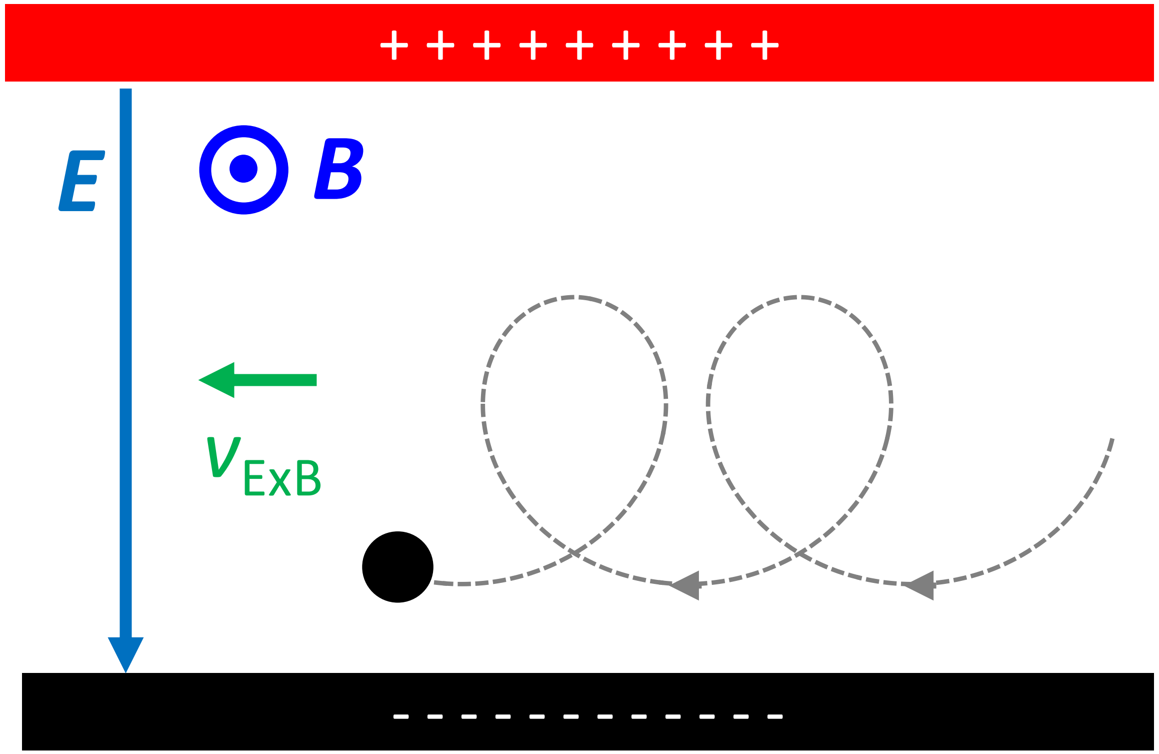
Perpendicular flows require charge transport

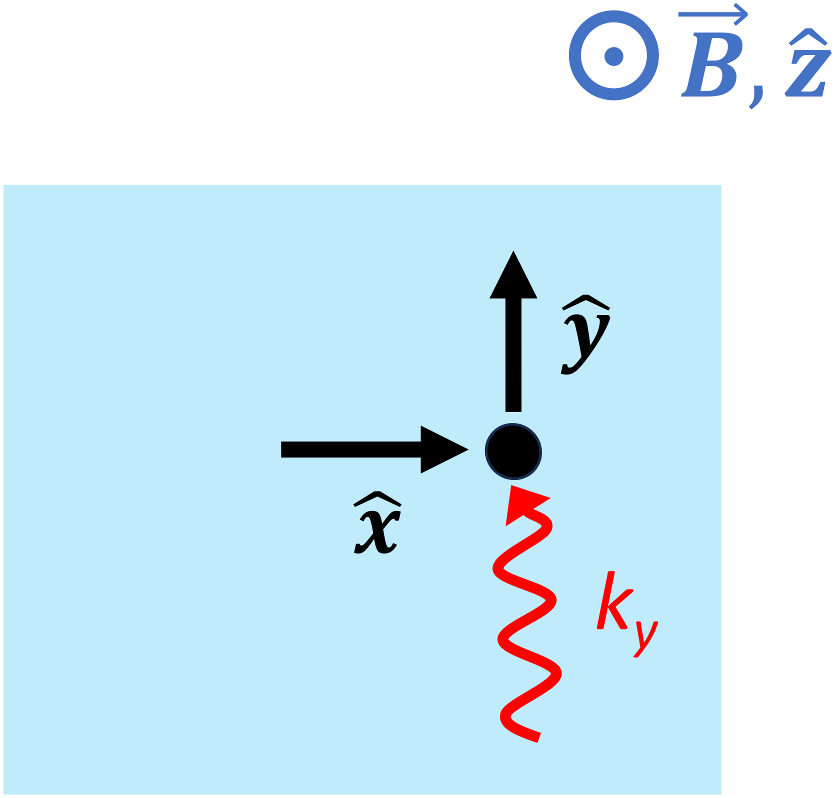
- How does charge move across field lines?



When can waves move charge across field lines?

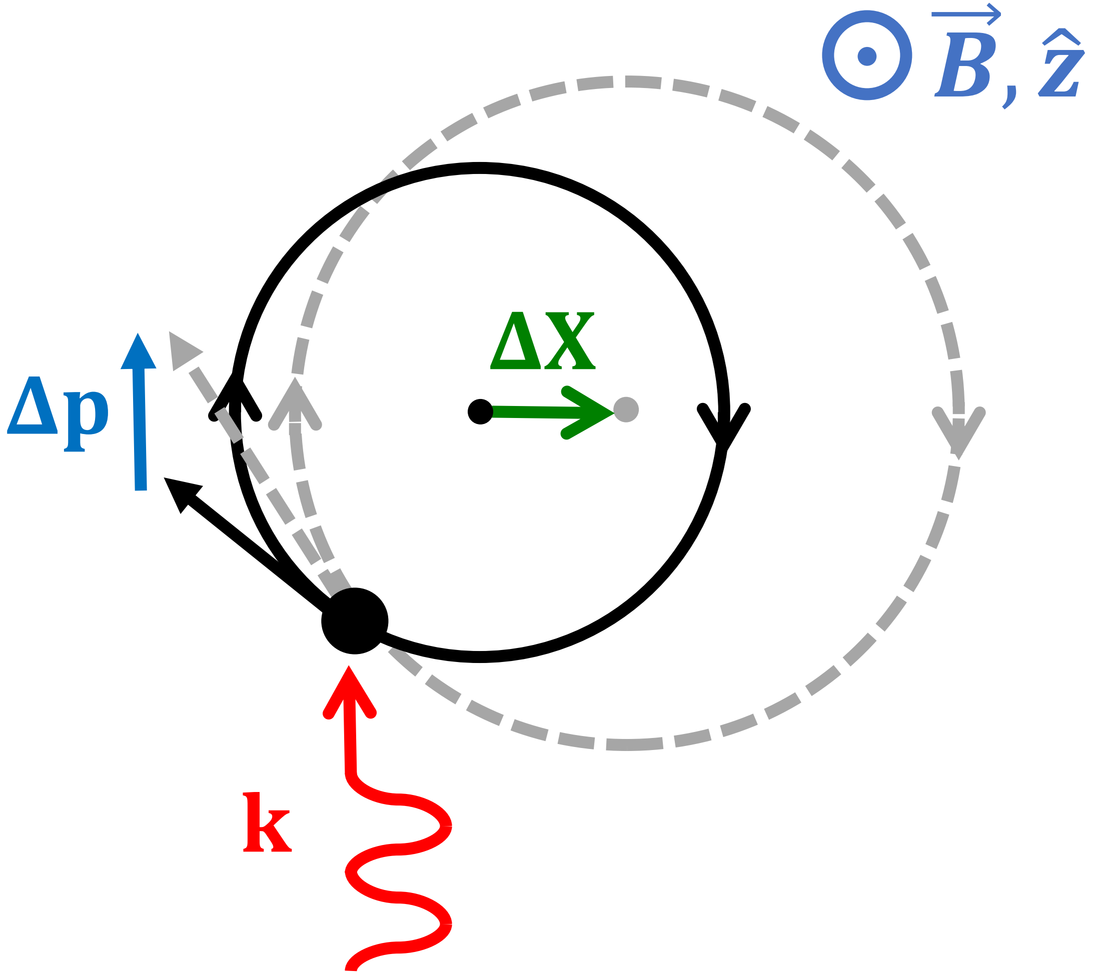
- Infinite plasma slab with **B** || **z**
- **Electrostatic** high-frequency wave in **y**
- Is charge moved (in **x**) by wave?

- This would produce a x-directed E field, driving ExB flow along y



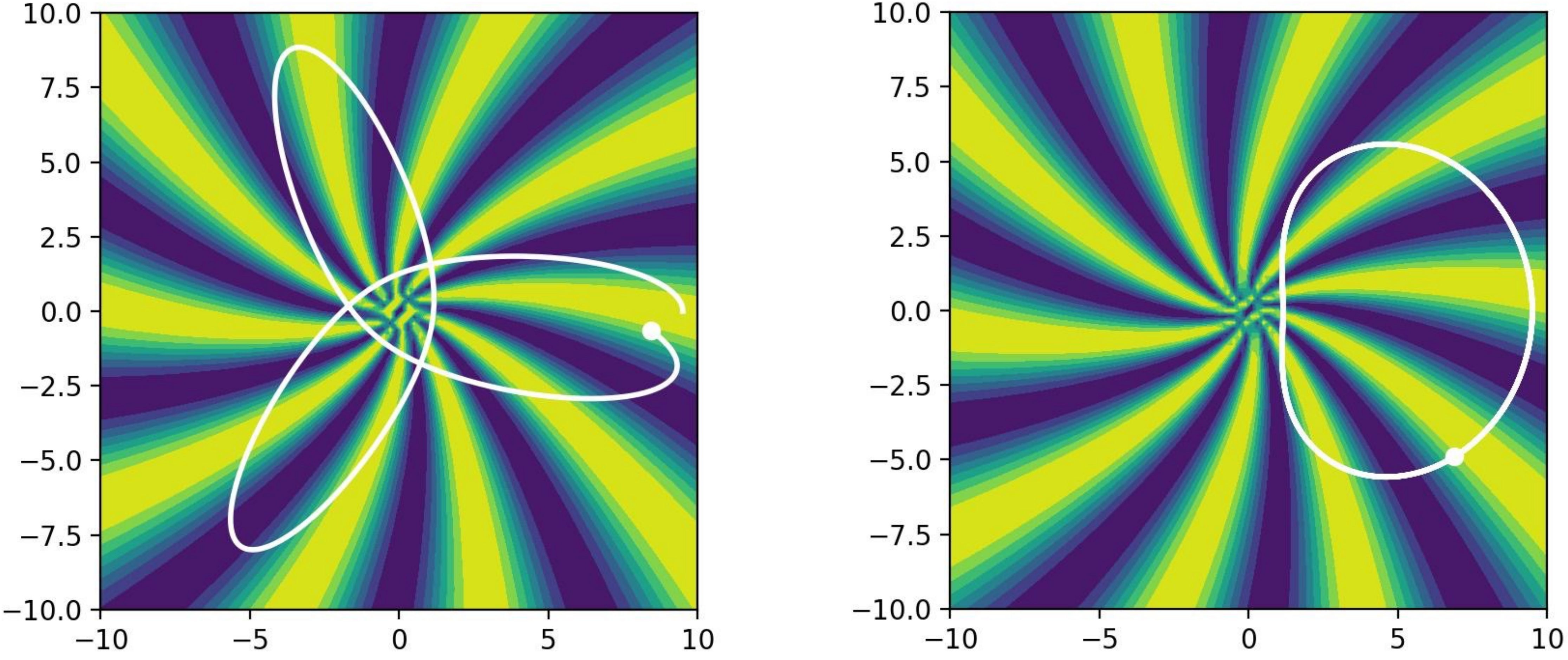
Resonant particles clearly move across field

- **Resonant particles** absorb momentum
- Moves gyrocenter in $\Delta\mathbf{p}\times\mathbf{B}$ direction
- Basis for **alpha channeling instability**
  - Coupled **diffusion** in energy / space
  - Hot alpha particles give up energy to wave while leaving the plasma



[N. J. Fisch and J.-M. Rax, PRL, 69(4), 612 (1992)]
[A. J. Fetterman and N. J. Fisch, PRL, 101(20), 205003 (2008)]

Astrophysics relevance: high-eccentricity orbit encountering local corotation resonance?



KITP Insertion: Diffusion Theory with Dephasing

Resonant particles “clearly” move across field

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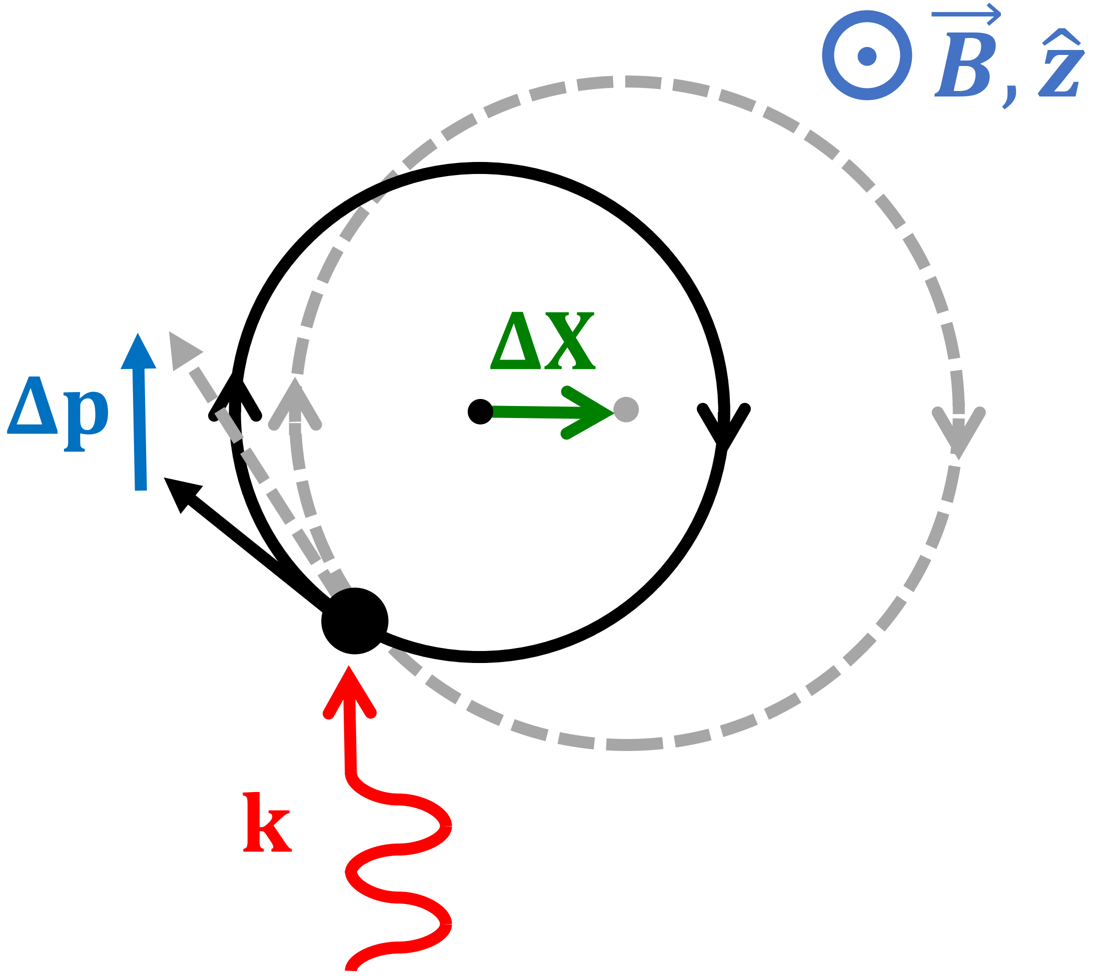

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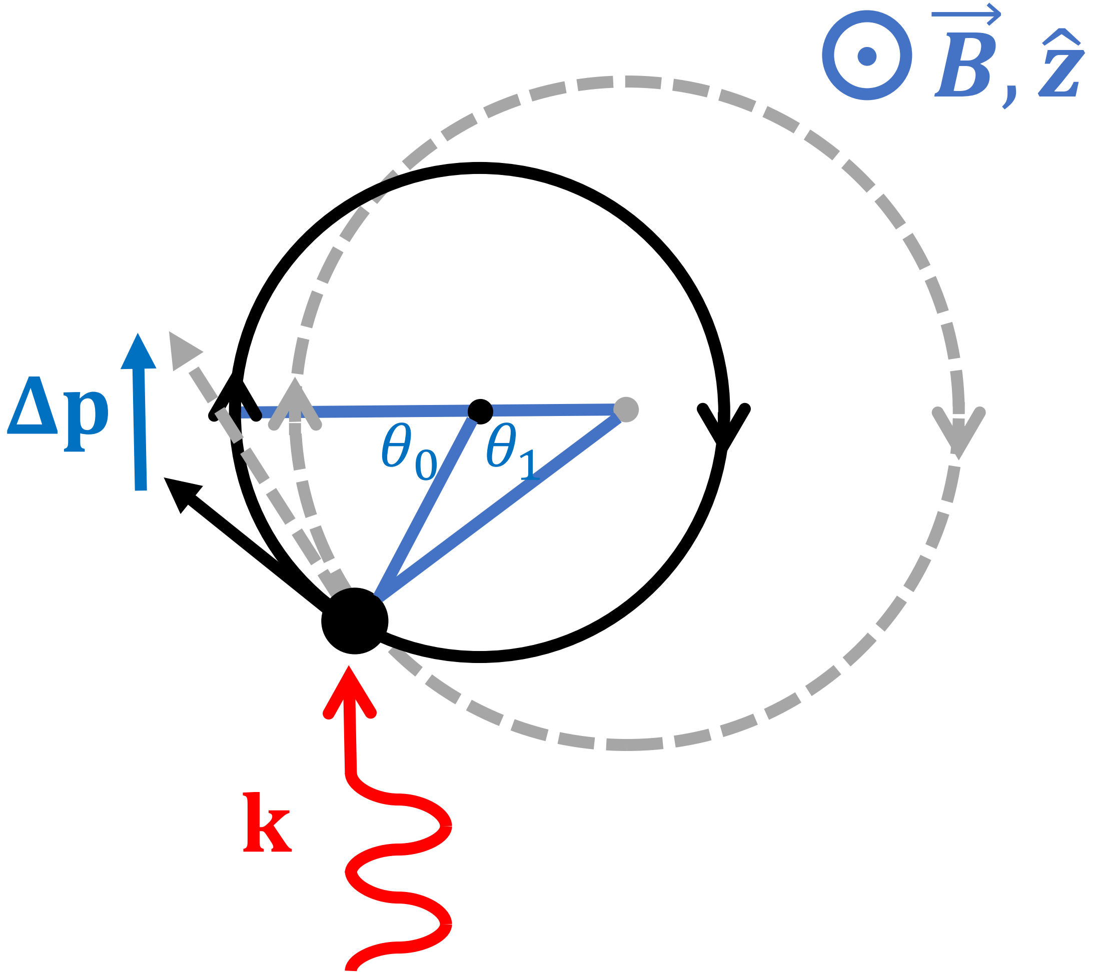
Diffusion Requires Phase Memory Loss

- Landau resonance—**not** a cyclotron resonance
- Bernstein-Landau paradox
- Requires sufficient "kick" to forget phase after one gyration
- "Stochastic Threshold"—Karney 1979

- $\phi = \omega \Delta t = n \Omega \Delta t + \text{"kick"}$
  - Once "kick" is O(1) on one orbit, cyclotron resonance is lost



Nonlinear Diffusion Coefficient

- Karney calculated diffusion coefficient laboriously from jump moments
- This formed a diffusion equation in perpendicular energy space
- Fisch, Rax (1992) included coupled gyrocenter-energy diffusion

- Nonstandard approach; normally linear theory -> quasilinear theory

Linear Theory

- Take particles to be “locally unmagnetized”
  - Start with kinetic unmagnetized theory of quasilinear diffusion
  - i.e. start with theory of bump-on-tail instability / Landau damping
- Transform theory to gyrocenter coordinates
  - $(x, y, v_x, v_y) \to (X, Y, K, \theta)$
- Invoke dephasing
  - Demand $F(X, Y, K, \theta) = F(X, Y, K)$
  - Average all equations [linear evolution and quasilinear diffusion] over θ
- Targets:

$$\left(\frac{\partial}{\partial K} + \frac{\mathbf{k} \times \hat{b}}{m_\alpha \omega \Omega_\alpha} \cdot \frac{\partial}{\partial \mathbf{X}}\right) F_{\alpha 0} > 0$$

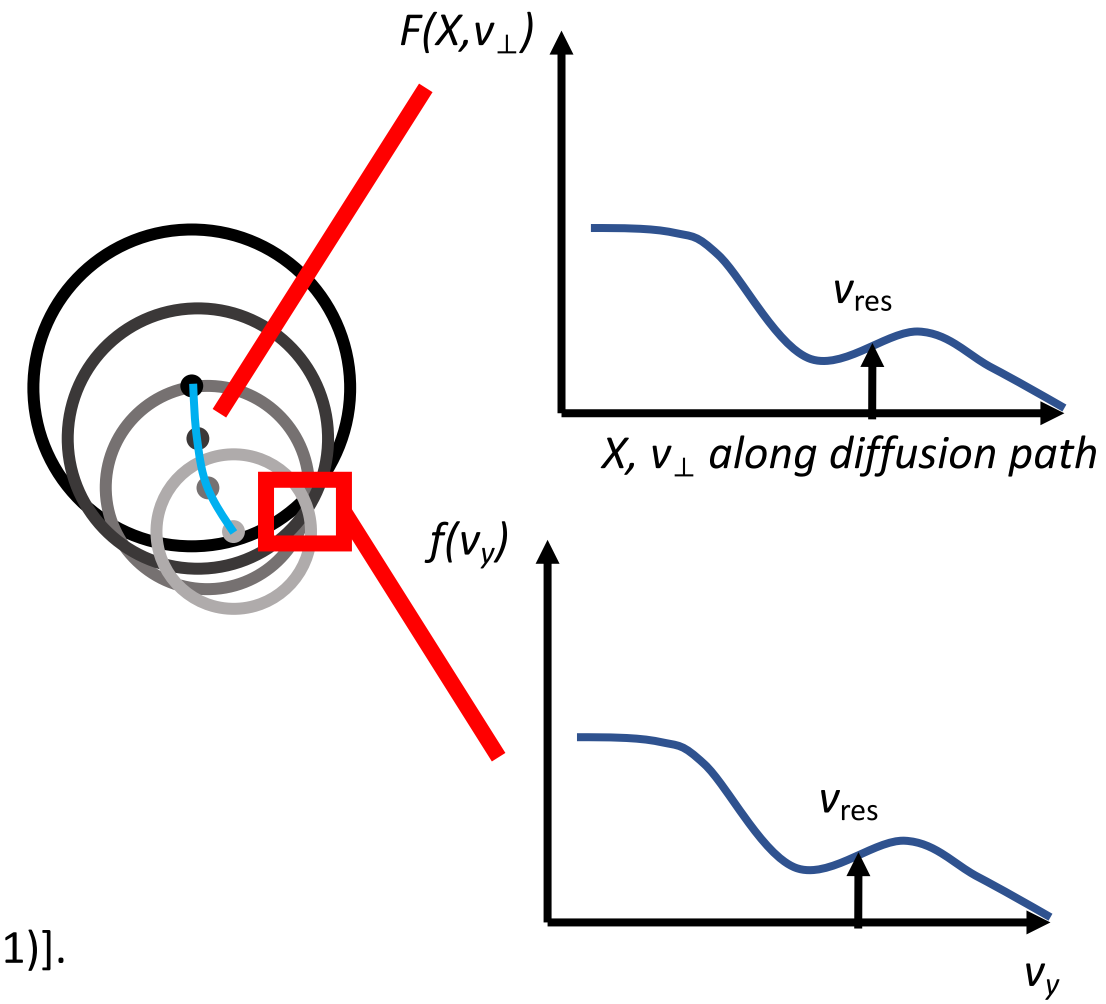
Channeling Amplification Condition

$$D^{KK} \equiv \frac{m_\alpha^2}{2} \left(\frac{q_\alpha E_0(y)}{m_\alpha}\right)^2 \frac{v_p^2}{\sqrt{k^2 v_\perp^2 - \omega_r^2}} H\left(v_\perp - v_p\right)$$

Diffusion Coefficient for Resonant Particles

Linear theory for alpha channeling instability

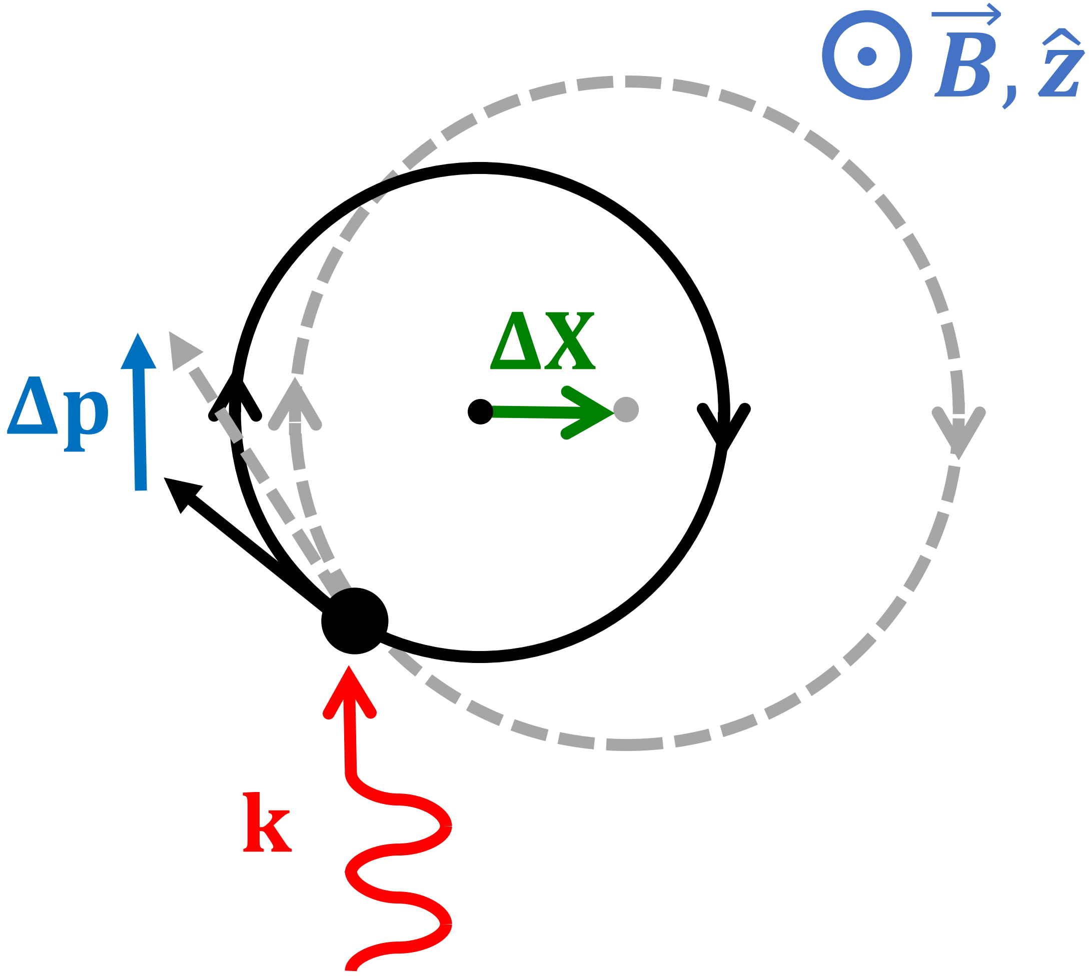
- Needed linear theory to apply general quasilinear theory
- Yields unexpected result:
  - Alpha channeling condition is same as bump on tail instability!
  - Coordinate change between gyrocenter and local coordinates



[I.E. Ochs and N.J. Fisch, Physical Review Letters, 127(2), 025003 (2021)].

Resonant particles clearly move across field

- Is there associated flow drive from this resonant particle current?
- "Free" way to drive rotation?



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Do the resonant particles tell the whole story?

- What happens to the nonresonant particles?
- Do they do nothing? Do they create an equal and opposite current?
- Need a self-consistent theory to see if flow drive is possible.

Outline: do waves drive plasma flows?

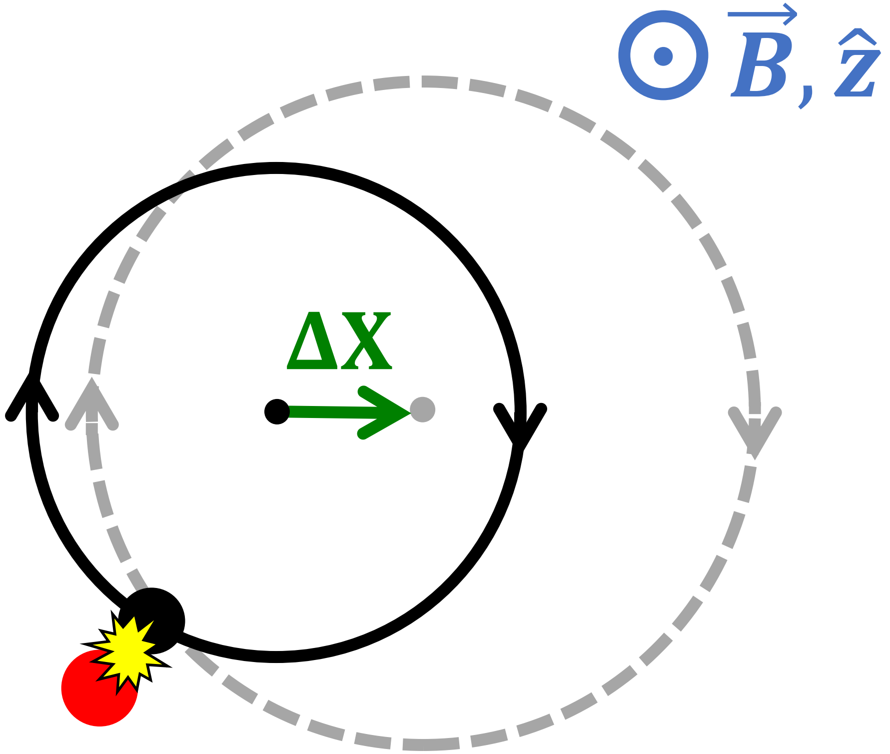
- Resonant Particle Diffusion

- Collisional Transport: connection between charge and momentum
- Wave Momentum: Minkowski momentum vs. Field momentum
- Problem Types: Evolution in Time vs. Space
- Theoretical Frameworks: Eulerian Averaging vs. Oscillation Centers

- Large differences in time- vs space-dependent waves

Collisional transport: a diffusion process

- Each collision puts particle on new Larmor orbit
- Individual charge moves across field lines



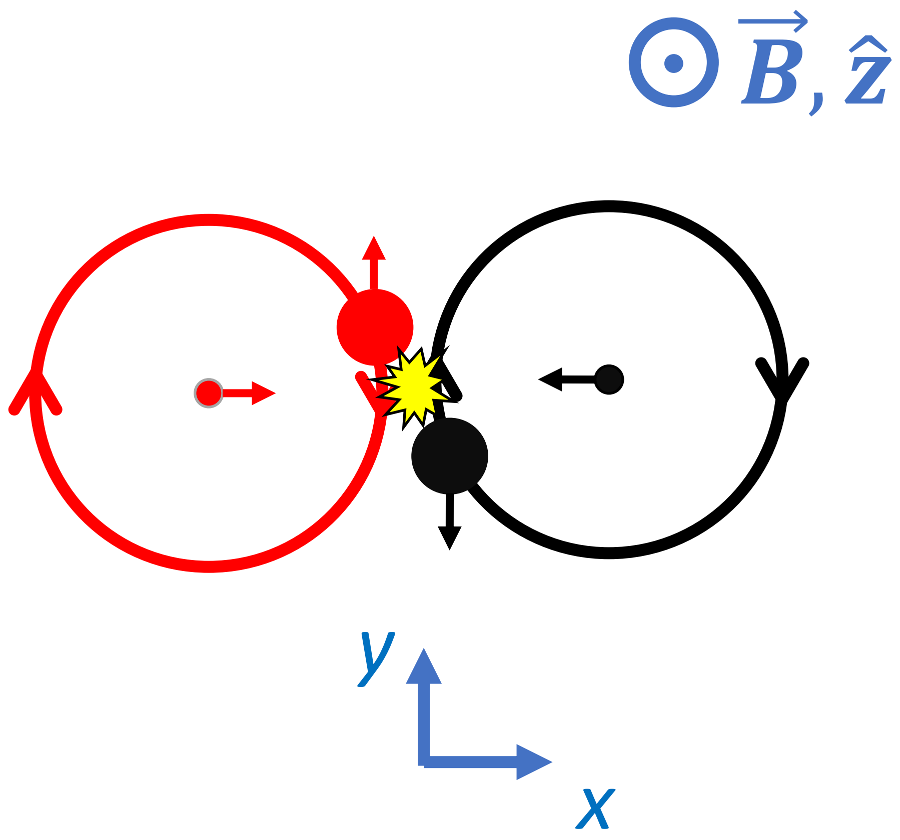
Collisional transport: no charge net moves

- Equal and opposite reactions from collision lead to equal and opposite charge transport
- Why? Canonical momentum – gyrocenter relation:

$$p_y = m\cancel{\langle v_y\rangle} + \frac{q}{c}\langle A_y(x)\rangle = \frac{q}{c}B_z\langle x\rangle$$

- Leads to:

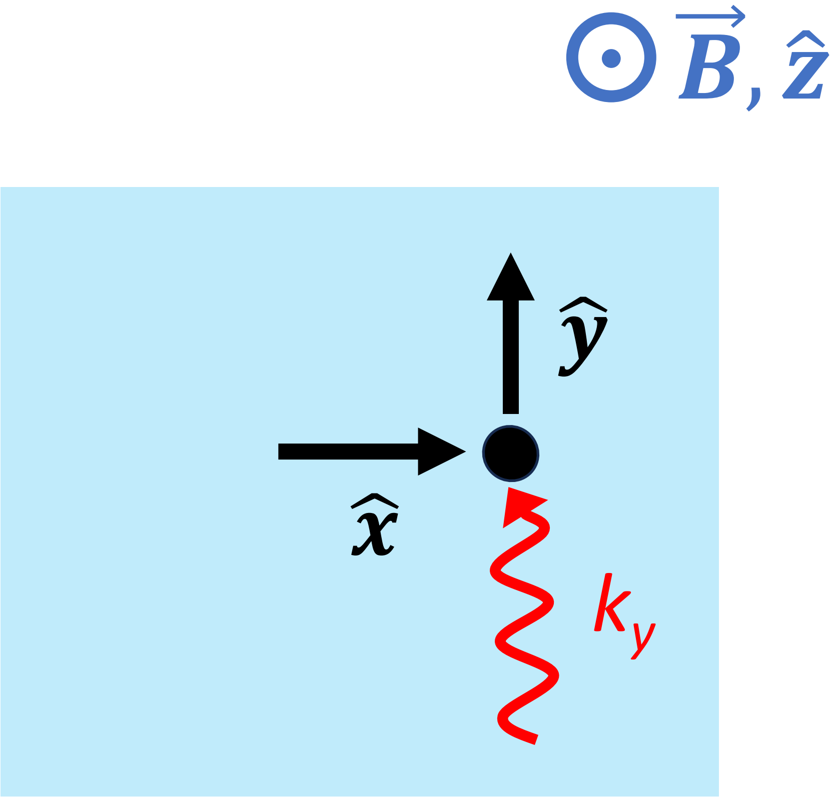
$$\sum_s q_s \Delta\langle x_s\rangle = \frac{c}{B_z}\sum_s \Delta p_{y,s} = 0$$



Lesson: charge transport requires momentum input

- Or: you have to push on the plasma to make it move

Transport lesson points to conundrum



- ExB flow drive requires **momentum** input
- Electrostatic plane wave has no momentum
    - Impossible to drive flow with **damped wave?**

What do we mean by “momentum” and “damping”?

Wave momentum is important—but which one?

- Minkowski (“Plasmon”, “GO”)

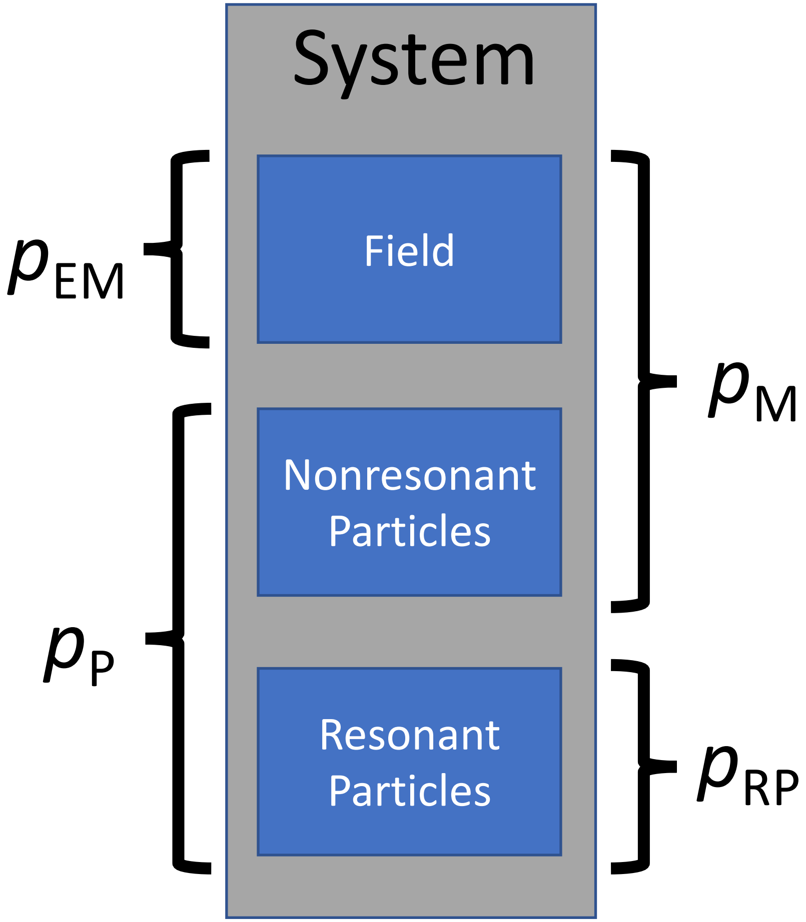
$$\mathbf{p}_M = \frac{\mathbf{k}}{16\pi\omega}\tilde{\mathbf{E}}^* \cdot \frac{\partial(\omega\boldsymbol{\epsilon})}{\partial\omega} \cdot \tilde{\mathbf{E}}$$

Exchanged with **Resonant Particles**

- Electromagnetic

$$\mathbf{p}_{EM} = \frac{\mathbf{E}\times\mathbf{B}}{4\pi c}$$

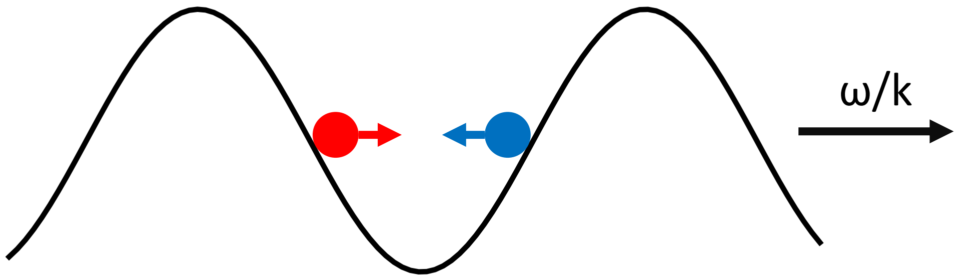
Exchanged with **All Particles**

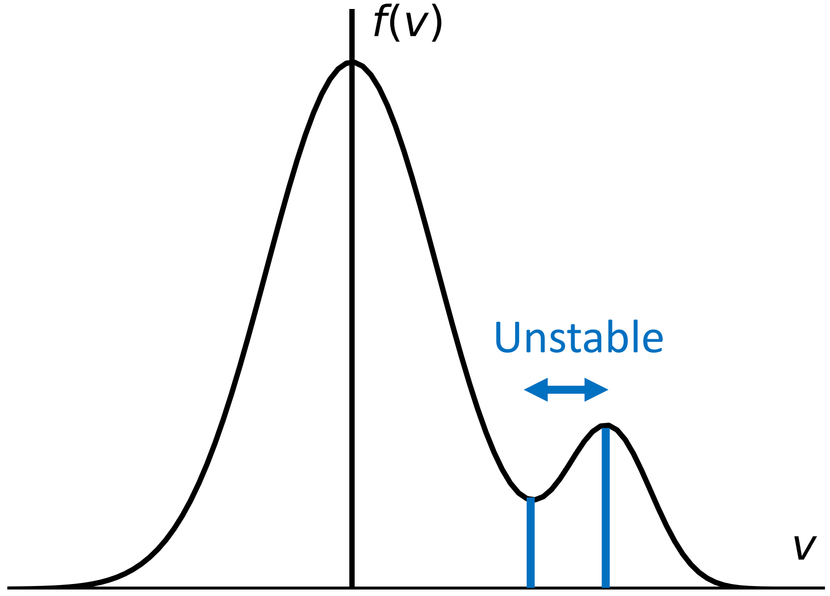


[I. Y. Dodin and N. J. Fisch, *Phys. Rev. A*, 86, 053834 (2012)]

“Familiar” example: the bump on tail instability

- **Electrostatic** (ES) plane wave in unmagnetized plasma
- Quasilinear diffusion transfers energy from resonant particles into wave

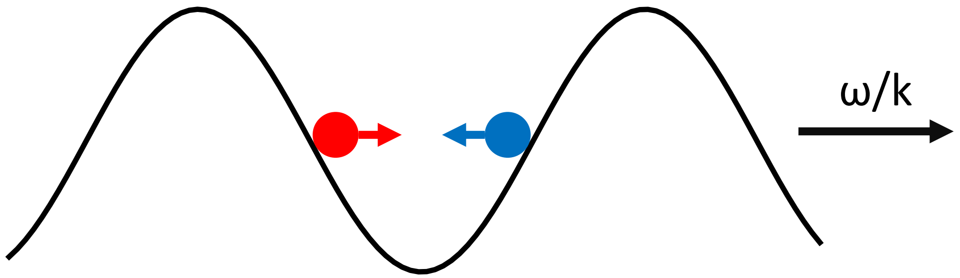


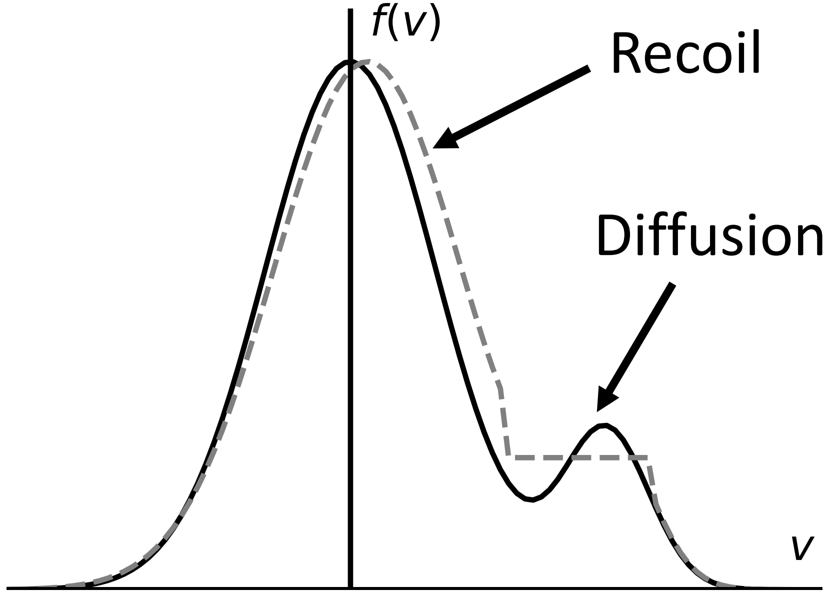


[A. N. Kaufman, J. Plasma Physics, *8*(1), 1-5 (1972)]
[R. C. Davidson, R. C. and J. E. Scherer, *Methods in Nonlinear Plasma Theory* (1972)]
[N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (1973)]
[V. N. Tsytovich, *Theory of Turbulent Plasma* (1977)]

Role of momentum in bump on tail instability

- Resonant particles lose momentum $\Delta \mathbf{p}_R$
- Wave gains Minkowski momentum $\Delta \mathbf{p}_M = -\Delta \mathbf{p}_R$
- Wave has no electromagnetic momentum
- Nonresonant particles enforce conservation: $\Delta \mathbf{p}_{NR} = -\Delta \mathbf{p}_R$
- We call this the "ponderomotive recoil"

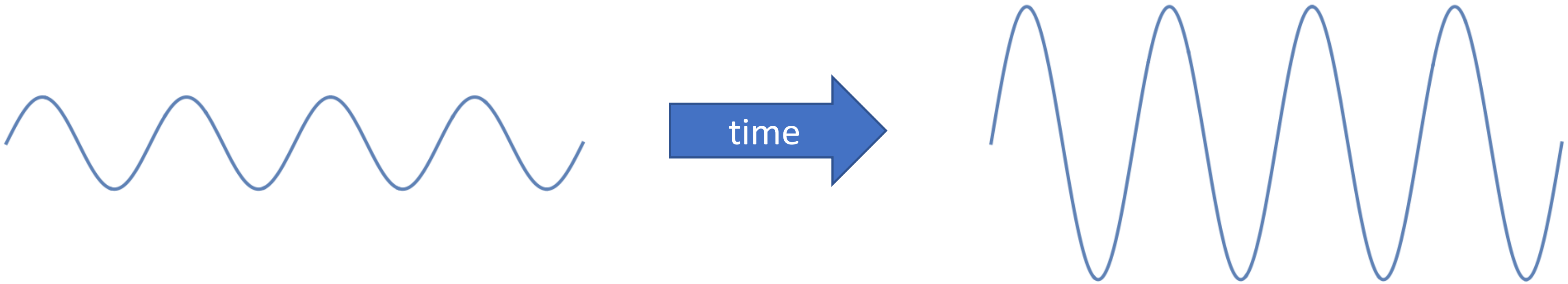




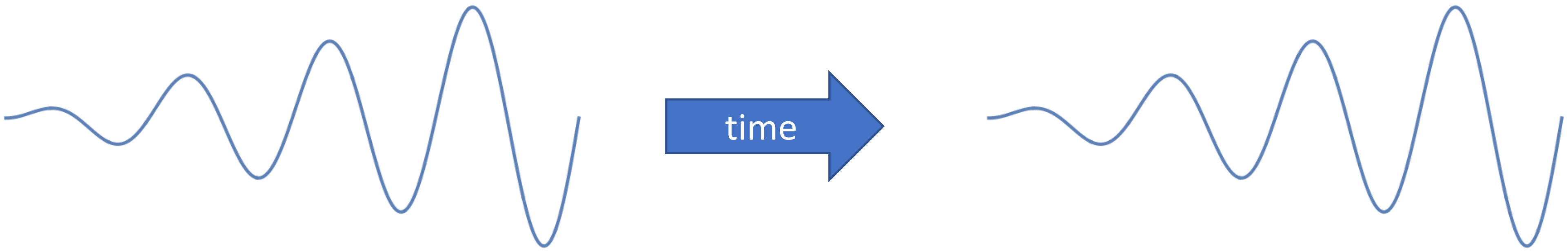
- Wave gains Minkowski momentum, but plasma does not lose momentum
- Should have implications for rotation drive

What is wave damping / amplification?

- Time: Plane Wave Initial Value Problem – Instabilities



- Space: Steady-State Boundary Value Problem – Ray Tracing, Current Drive



Problem defined, now need to solve it…

Types of theory

- Eulerian
  - Consider Lorentz force on **volume** of plasma

- Oscillation Center (OC)
  - Consider single-particle Hamiltonian / Lagrangian with wave
  - Transform to nearby coordinates with slow dynamics (~E^2)
  - Coordinate transform depends on wave amplitude

$$\boldsymbol{\mathcal{X}} = \mathbf{x} + \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{p}} - \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{x}} \cdot \frac{\partial^2 S(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{p}\partial \mathbf{p}}$$
$$\boldsymbol{\mathcal{P}} = \mathbf{p} - \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{x}} + \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{x}} \cdot \frac{\partial^2 S(\mathbf{x},\mathbf{p},t)}{\partial \mathbf{p}\partial \mathbf{x}}$$

[G. W. Kentwell and D. A. Jones, “The time-dependent ponderomotive force,” *Physics Reports*, **145**(6), 319 (1987)]
[R. L. Dewar, “Oscillation center quasilinear theory,” *Physics of Fluids*, **16**(7), 1102, (1973)]

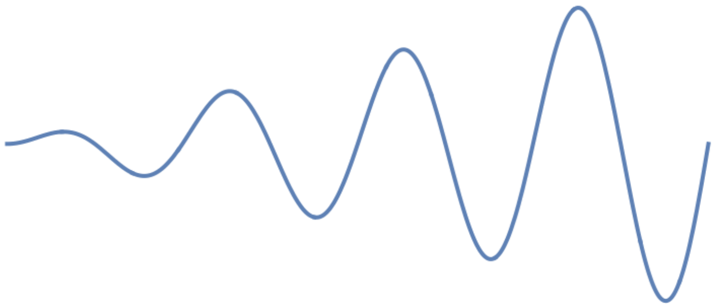
Each theory is naturally suited to one problem type

Type of Theory

	Space	Time
Oscillation Center	Forces only come from resonances and gradients	Nonresonant reactions reside in complicated coordinate transformations
Eulerian	Subtle surface stress terms	Straightforward averages

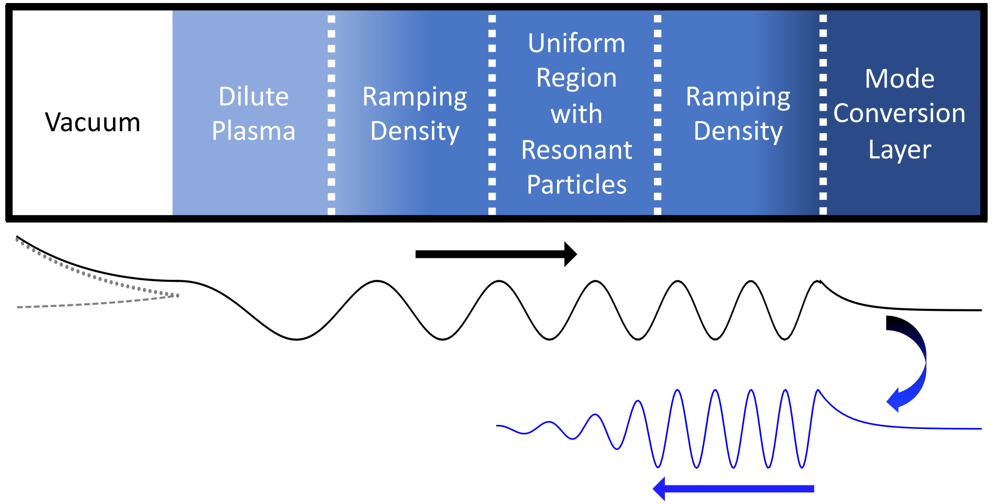
Type of Problem

Start with steady-state boundary value problem



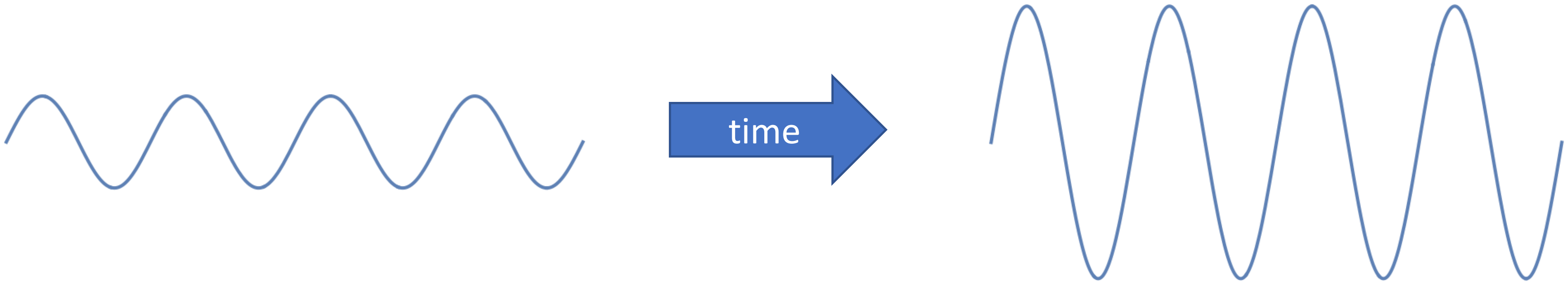
Flow is driven in the steady-state boundary value problem

- Consider wave that decays in **x**
- Oscillation Center
  - Wave damps on resonant particles in **y**
  - Nonresonant particles: no forces along **y**
  - Net effect: resonant charge extracted in **x**
  - **Flow is driven**
- Eulerian
  - Laborious, but can track momentum injection into plasma from edge, giving same result



[P.H. Diamond and Y.B. Kim, Physics of Fluids B, 3(7), 1626 (1991)]
[J.R. Myra, L.A. Berry, D.A. D'Ippolito, D. A., and E.F. Jaeger, Physics of Plasmas, *11*, 1786 (2004)]
[Z. Gao, N.J. Fisch, H. Qin, and J.R. Myra, Physics of Plasmas, 14, 084502 (2007)]
[I. E. Ochs and N. J. Fisch, *Physics of Plasmas*, **29**, 062106 (2022)]

What about the initial value problem?



IVP: Eulerian electrostatic quasilinear theory

- Dispersion for ES wave

$$0 = 1 + \sum_s D_s$$

$$D_s \equiv -\frac{4\pi q_s}{k^2}\frac{\tilde{n}_s}{\tilde{\phi}}$$

- Force exerted by field:

$$F_s^y = q_s \langle E^y n_{s1} \rangle$$

$$F_s^y = 2W_{EM} k^y \left[\underbrace{D_{is}}_{\text{Resonant}} + \underbrace{\omega_i \frac{\partial D_{rs}}{\partial \omega_r}}_{\text{Nonresonant}} \right]$$

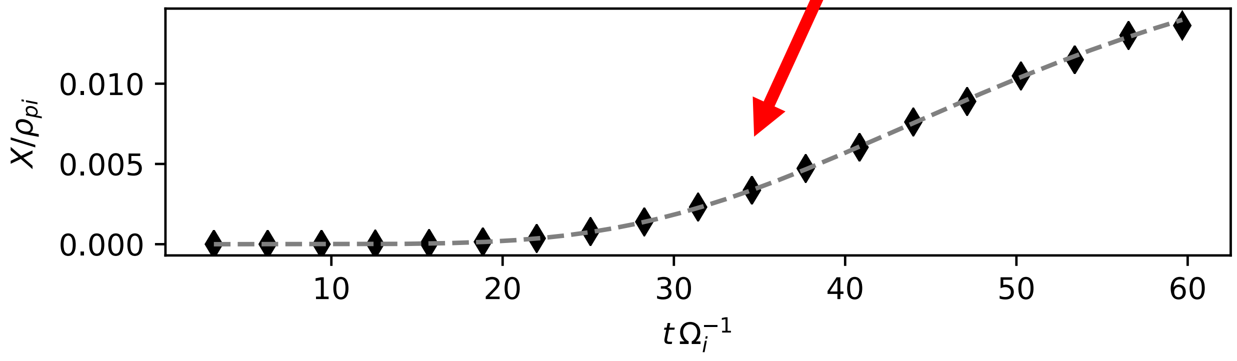
Species sum is imaginary part of dispersion relation = 0

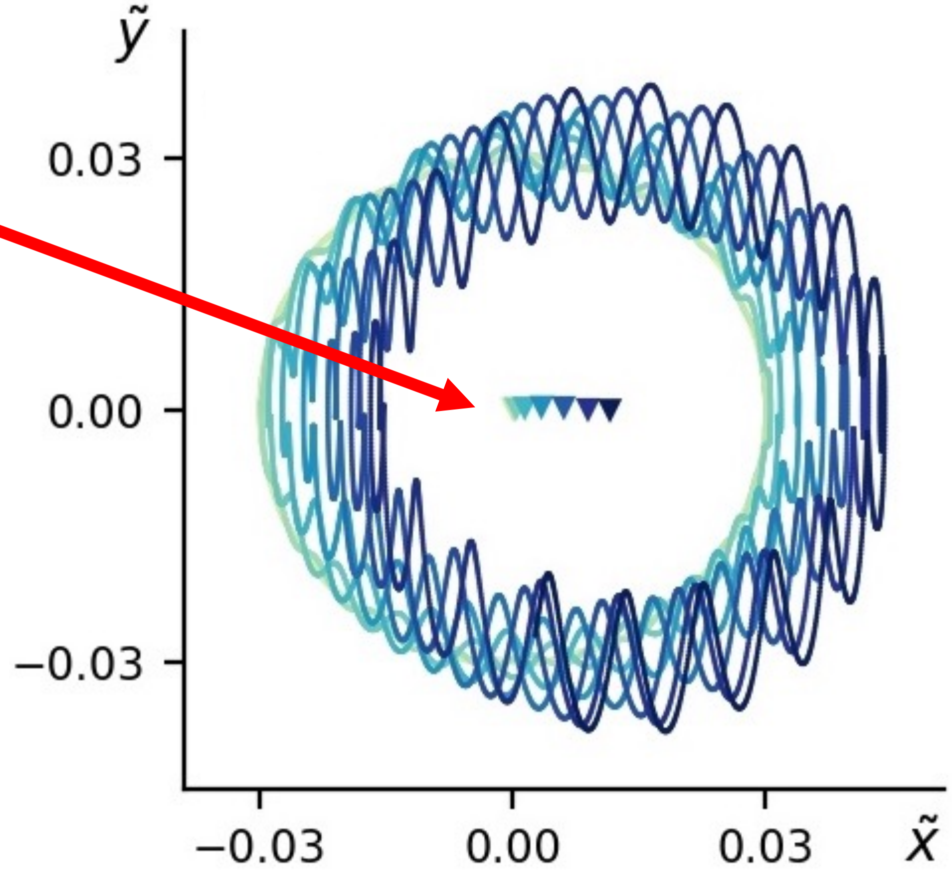
Wave cannot apply force, extract charge, or drive ExB flow

[I.E. Ochs & N.J. Fisch, Physics of Plasmas **27**, 062109 (2020)]

IVP: Simulations confirm that nonresonant charge transport cancels resonant charge transport

$$F_s^y \approx 2W_{EM}k^y \left[\underbrace{D_{is}}_{\text{Resonant}} + \underbrace{\boxed{\omega_i \frac{\partial D_{rs}}{\partial \omega_r}}}_{\text{Nonresonant}}\right]$$





[I.E. Ochs and N.J. Fisch, Physical Review Letters, 127(2), 025003 (2021)]

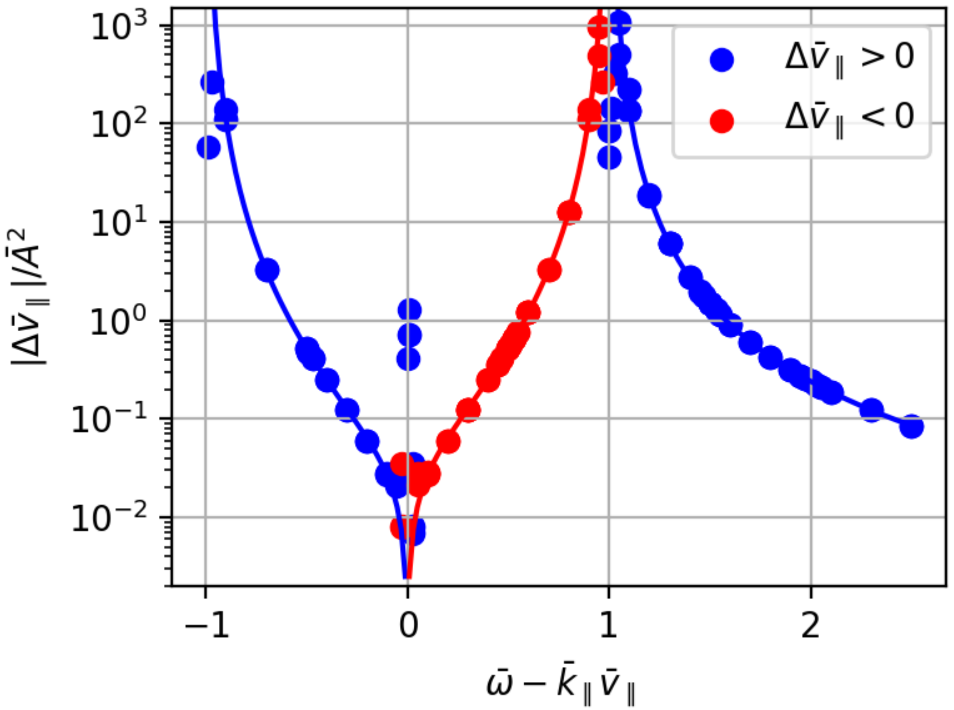
Electromagnetic Waves: Waves with Momentum

- Same QL procedure, fully kinetic

$$\langle \mathbf{F}_s \rangle = \frac{\partial}{\partial t}\mathbf{p}_{Ns} + \mathbf{F}_{Rs}$$

$$\mathbf{p}_{Ns} = \frac{1}{8\pi}\mathrm{Re}\left[\left(\boldsymbol{\chi}_s^H \cdot \mathbf{E}\right) \times \left(\frac{\mathbf{k}_r}{\omega_r} \times \mathbf{E}^*\right) + \frac{\mathbf{k}_r}{2}\frac{\partial}{\partial \omega_r}\left(\mathbf{E}^* \cdot \boldsymbol{\chi}_s^H \cdot \mathbf{E}\right)\right]$$

$$\mathbf{F}_{Rs} = \frac{1}{8\pi}\mathbf{k}_r \left(\mathbf{E}^* \cdot \boldsymbol{\chi}_s^A \cdot \mathbf{E}\right).$$

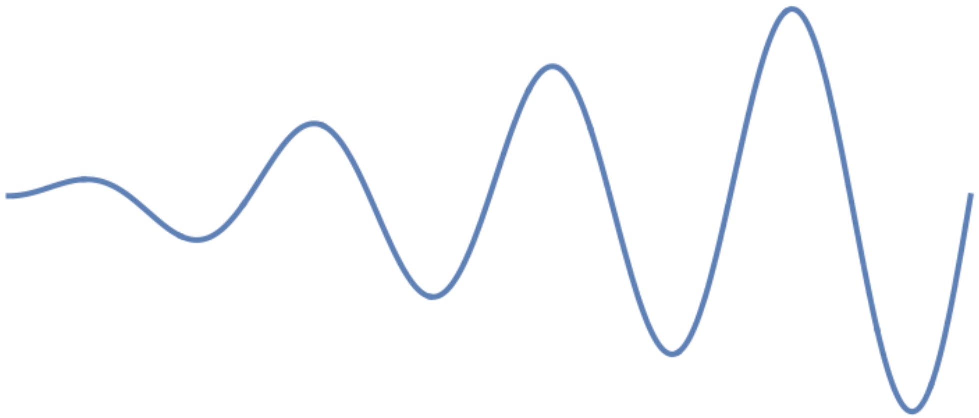


- Plasma absorbs Poynting flux: $\sum_s \langle \mathbf{F}_s \rangle = -\frac{\partial}{\partial t}\left(\frac{\langle \mathbf{E} \times \mathbf{B} \rangle}{4\pi c}\right) = -\frac{\partial \mathbf{p}_{EM}}{\partial t}$

[I. E. Ochs and N. J. Fisch, *Physics of Plasmas*, 30(2), 022102 (2023)]

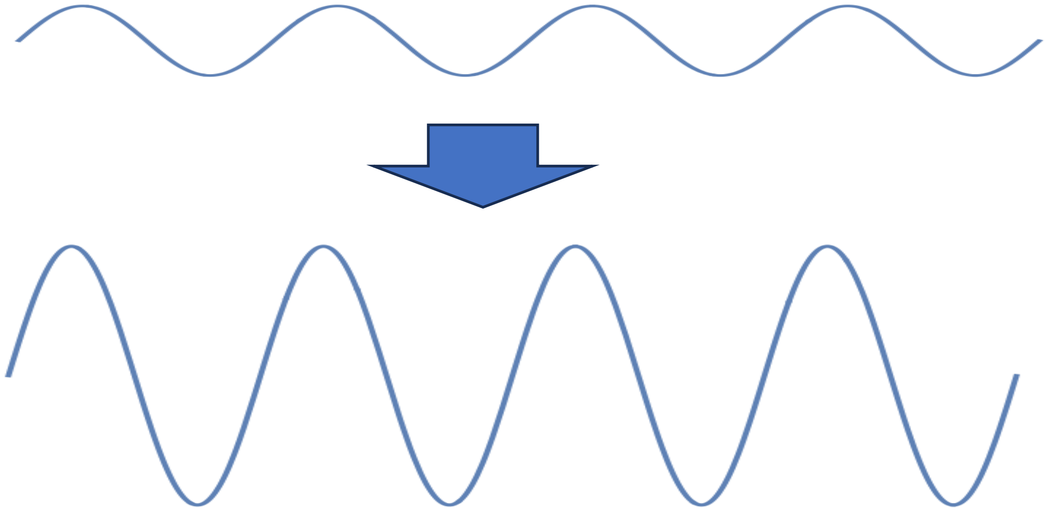
How much flow do waves drive?

Steady-State BVP



Drives flow according to wave’s Minkowski momentum

Plane Wave IVP



Drives flow according to wave’s electromagnetic momentum

What is Minkowski momentum physically?

- Result of electromagnetic Eulerian theory:

$$\mathbf{p}_M = \mathbf{p}_{EM} + \sum_s \mathbf{p}_{Ns}$$

- What does this mean?
    - Start with still plasma and grow wave from 0 amplitude
    - Final sum of electromagnetic and nonresonant particle momenta is Minkowski momentum
    - Minkowski momentum is the momentum required to “put the wave into the plasma”

Summary

- Deep relationships between **charge transport, momentum, and current** pervade many plasma systems, so self consistency is important!
- We have found a **self-consistent quasilinear theory** for perpendicular Landau damping and the alpha channeling instability
- Result: **time-evolving and space-evolving waves behave very differently**
  - Space: wave drives flow if the wave Minkowski momentum changes
  - Time: wave drives flow if the wave electromagnetic momentum changes
- Roughly: For the same resonant process (eg alpha channeling), a growing instability will not drive rotation, but a steady-state process will