

the **SILK ROAD PROJECT** at NAOC

丝绸之路计划

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ASTRONOMIE

ARI ITA LSW Univ. Heidelberg



Direct N-Body Gravothermal Systems

Rainer Spurzem, and Silk Road Team

(mainly: M. Arca Sedda, GSSI; A. Kamlah, F. Flammini Dotti, J. Hurley, Shuo Li, P. Berczik)

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June 2024

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<https://silkroad.zah.uni-heidelberg.de>



Volkswagen **Stiftung**

- 1) Introduction – History and Theory**
- 2) Code(s)* and Supercomputers
- 3) Star Clusters/Black Holes/Grav. Waves
- 4) Wrap-Up and References

*mainly Nbody6++GPU

*Kavli Institute for Astronomy
and Astrophysics, Peking Univ.*



北京大學
PEKING UNIVERSITY



Founded 2008



Astronomisches
Rechen-Institut (ARI)
Univ. of Heidelberg,
Germany

Founded May 10, 1700

Star2000 Conference Heidelberg: Dynamics of Star Clusters and the Milky Way, ASP Conference Series, Vol. 228.
Edited by S. Deiters, B. Fuchs, R. Spurzem, A. Just, and R. Wielen.
San Francisco: Astronomical Society of the Pacific.

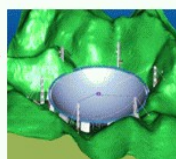




中国科学院国家天文台

NATIONAL ASTRONOMICAL OBSERVATORIES, CHINESE ACADEMY OF SCIENCES

NAOC/
CAS



**Silk Road Project =
Computational Science Project...**

Top: NAOC Headquarter Beijing
Bottom: LAMOST Site



the SILK ROAD PROJECT

丝绸之路计划

IN ANCIENT TIMES ... THERE WAS A BRIDGE
BETWEEN CULTURES AND CONTINENTS ...
... TODAY THERE IS A PROJECT
OF ASTROPHYSICS IN CHINA ON THE MOVE ...
... TO BUILD AN INTERNATIONAL BRIDGE
FOR COMPUTATIONAL SCIENCE

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A A A Login Register     

[Kazakhstan - China - Korea \(KCK\) becomes Silk Road Conference](#)

Kazakhstan-China-Korea meeting becomes Silk Road Conference



13th Silk Road Conference: Dali, China, June 23-27, 2025 (just after the IAU Symposium **Compact Objects and Binaries in Dense Star Clusters** to be held in Seoul, Korea, June 16-20, 2025)

14th Silk Road Conference: Samarkand, Uzbekistan, June 2026 (to be confirmed)

Exploring the Frontiers of Dynamical Astronomy with High-Performance Computing, Artificial Intelligence, and Leading-Edge Observational Techniques.

We continue the numbering of these conferences, which started as Korea-China meeting in 2009; later it became Kazakhstan-China-Korea meeting, and due to increasing interest and participation of colleagues from all over central and east Asia we decided now to rename it to Silk Road Conference.

Last Conference: [12th Kazakhstan-China-Korea meeting in Astana, Kazakhstan, May 20-24, 2024](#)

IAU Symp. No. 312: Star Clusters and Black Holes in Galaxies across Cosmic Time, August 2014, National Library Beijing

Proposed IAU Symposium (H.M. Lee/R. Spurzem), Seoul, June 2025:
Compact Objects and Binaries in Dense Star Clusters



On the Evolution of Stellar Systems

V. A. Ambartsumian

(George Darwin Lecture, delivered on 1960 May 13)

IN THIS lecture we shall consider some aspects of the problem of the evolution of stellar systems. We shall concentrate chiefly on *galaxies*. However, at the same time we shall treat here some questions connected with *star clusters* as component members of galaxies.



Concepts discussed:

Total Energy of grav. star clusters NOT additive

No thermodynamical equilibrium

Statistical Theory of Gases to be used with care

(large mean free path)

Locally truncated Maxwellian distribution.

Physical and Numerical Methods: Modelling the Dynamics

$$\vec{a}_0 = \sum_j Gm_j \frac{\vec{R}_j}{R_j^3} ; \quad \vec{\dot{a}}_0 = \sum_j Gm_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j) \vec{R}_j}{R_j^5} \right]$$

- $N = \infty$

negative specific Heat

gravothermal Collapse

gravothermal Oscillations

- $N = 3$ ($N = 2, \dots, \approx 100$)

History

Exponential Instability

Chaos and Resonance

Regularisation

- $N = 10^6$ ($N = 10^4, 10^5$)

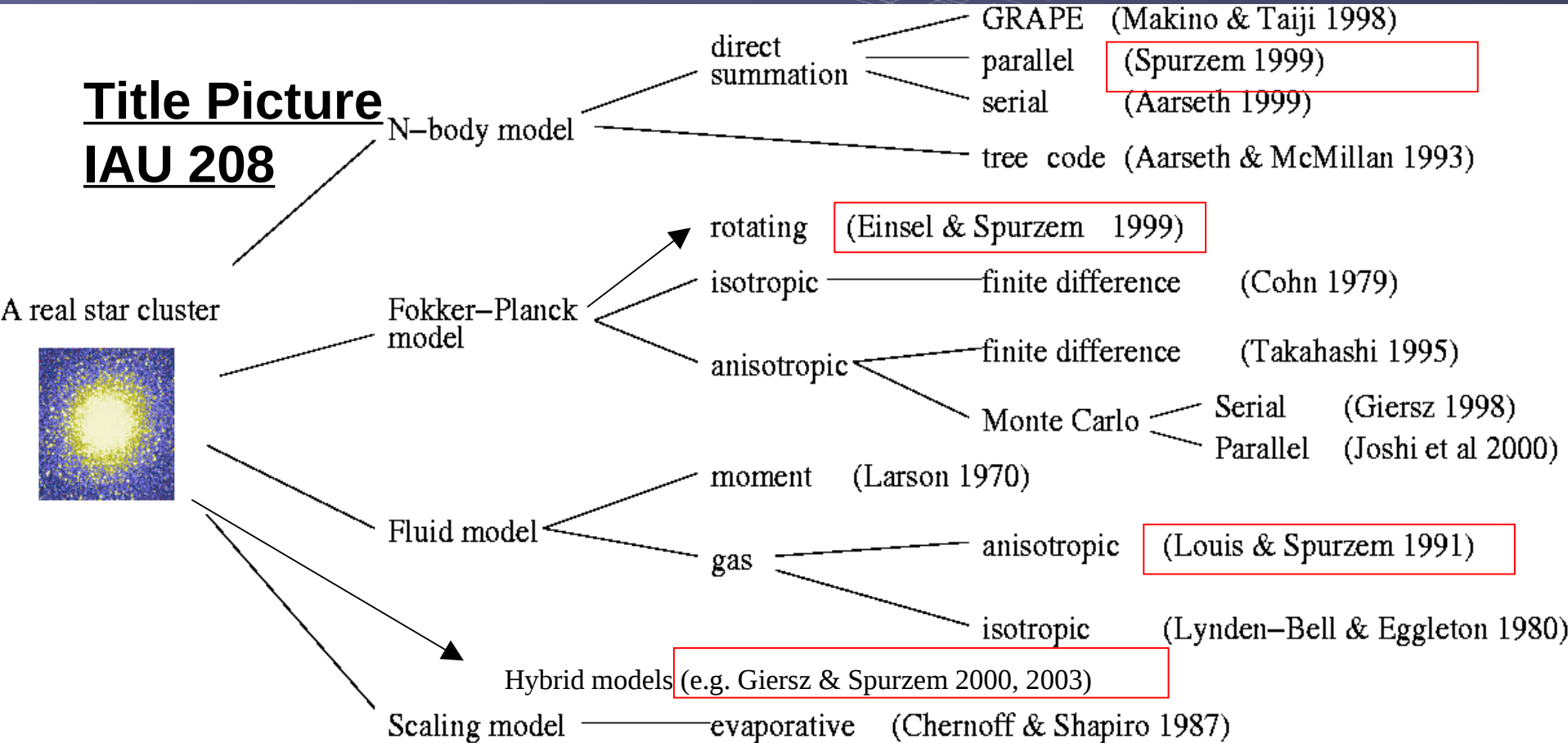
Post-Kollaps-Evolution

Binaries

Globular Clusters

Numerical Stellar Dynamics

Some methods for studying the evolution of globular clusters (by D.C.Heggie)



(citations are not complete for 2006)

1980

On the consequences of the gravothermal catastrophe

D. Lynden-Bell and P. P. Eggleton *Institute of Astronomy,
The Observatories, Cambridge CB3 0HA*

Received 1979 October 9; in original form 1979 July 11

between encounters. In such a stellar system a typical star at any radius moves in and out by about a local Jeans length $\lambda = 1/k_J$ where $k_J^2 v^2 = 4\pi G\rho$. This gives a typical radial distance between encounters. However, the time between those encounters is not λ/v but rather a relaxation time T_r . Hence the energy flux is given by equation (3.5) with $\lambda = k_J^{-1}$ and with τ replaced by T_r

$$\frac{L}{4\pi r^2} = -\frac{C\rho}{k_J^2 T_r} \frac{\partial}{\partial r} \left(\frac{3}{2} v^2 \right)$$

$$= -3GmC(\log N) \frac{\rho}{v} \frac{\partial v^2}{\partial r}$$

- Conductivity $\propto \rho/v$

$$(3.7)$$

- Based on Jeans Length

- Anisotropic generalization

(Louis & Spurzem 1991)

$$(3.8)$$

where C is a dimensionless constant of order unity. As Lecar pointed out, this implies a conductivity K proportional to $\rho T^{-1/2}$ or an opacity κ proportional to $T^{3.5} \rho^{-2}$. Notice that the details of the argument are unimportant, for the use of dimensions, coupled with the fact that the heat flux must be proportional to the relaxation rate (T_r^{-1}) gives the same dependence of conductivity on ρ, v . The detailed computation of Hachisu *et al.* (1978) was

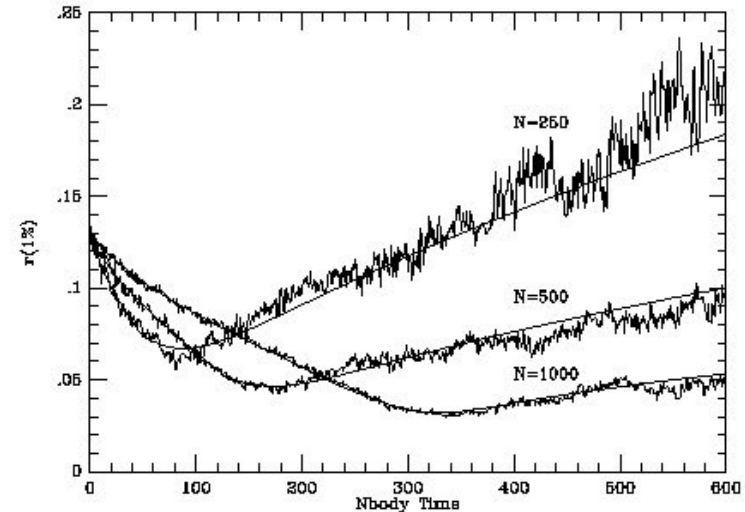
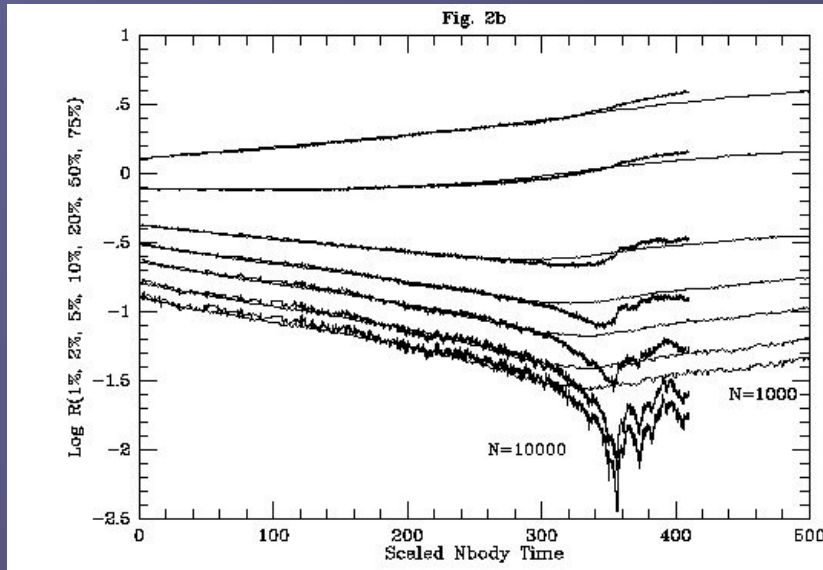
Star Clusters: Modelling the Dynamics

(compare gaseous model with direct N-body integration)

(Spurzem & Aarseth 1996)

(Giersz & Spurzem 1994)

(now in Binney/Tremaine)



N-Body / N-Body

N-Body / Fokker-Planck

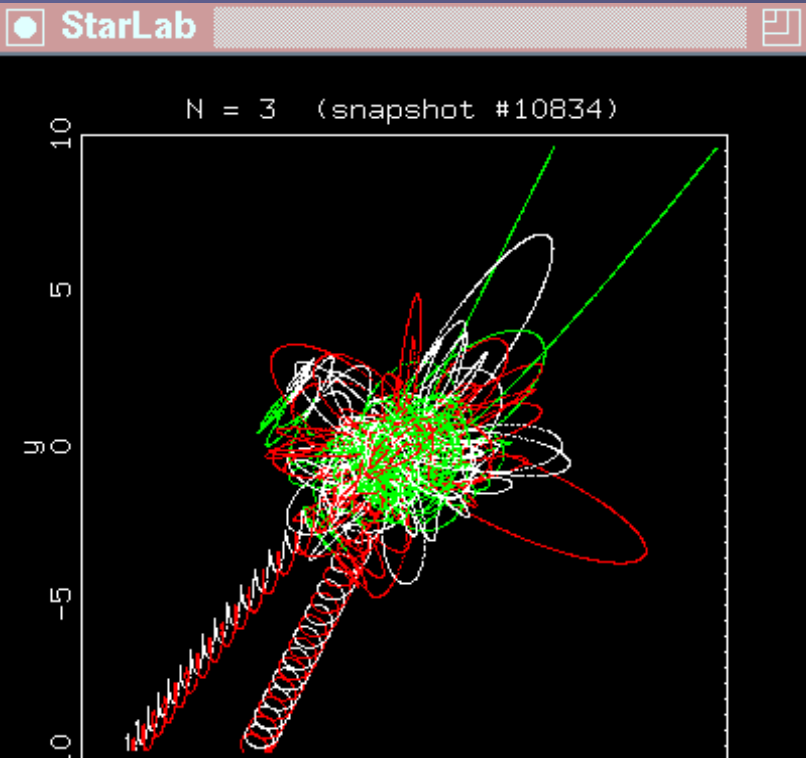
→ Blackboard Talk Week 4 gives more (Tue June 25):

- Fluid and orbit averaged Fokker-Planck models
- central supermassive black hole, rotation
- Anisotropic heat transport, aniso TOV generalization
- Ultracold plasma and self-interacting dark matter

3-body Encounters Starlab Simulation (S.L.W. McMillan)

<http://www.physics.drexel.edu/~steve/>

-> Three-Body-Problem



Gravothermal Oscillations - Attractor in Phase Space Spurzem 1994, Giersz & Spurzem 1994 Amaro-Seoane, Freitag & Sp. 2004

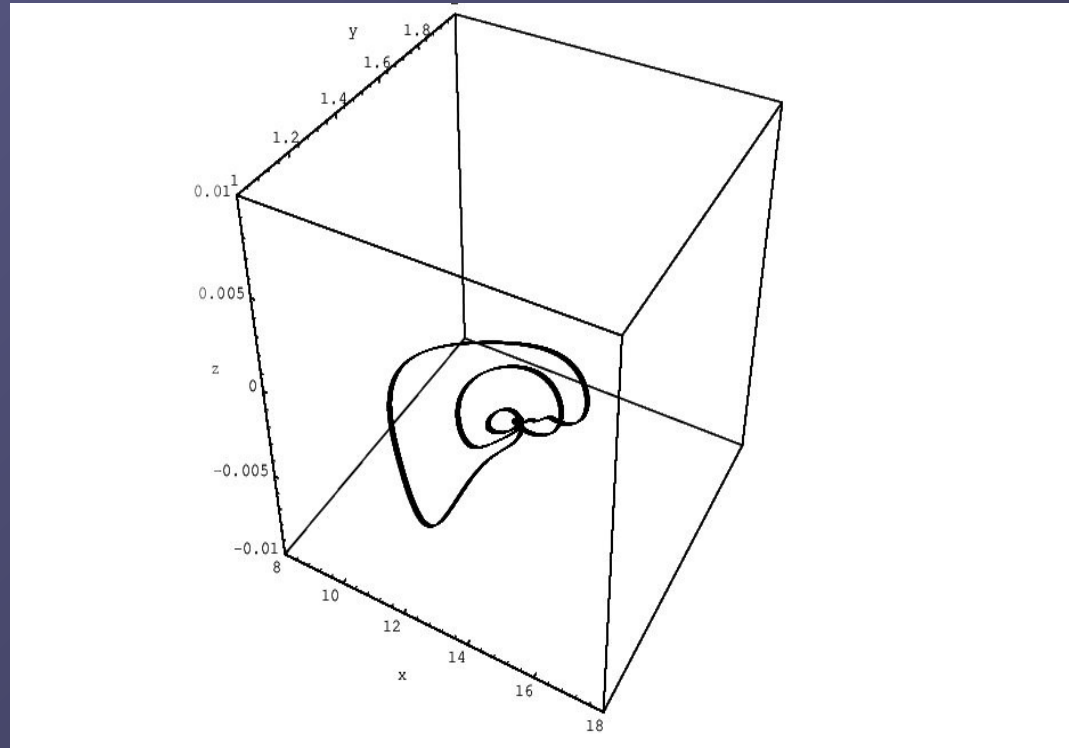


Fig. 3:

6-dimensional attractor for $N = 100.000$ system, $x = \sigma'_c$, $z = \xi$.

Gravothermal instability of anisotropic self-gravitating gas spheres: singular equilibrium solution

R. Spurzem^{1,2,3}

¹Institut für Theoretische Physik und Sternwarte, University of Kiel, Olshausenstraße 40, D-2300 Kiel, Germany

²Institut für Astronomie und Astrophysik, University of Würzburg, Am Hubland, D-8700 Würzburg, Germany

³Universitätssternwarte Göttingen, Geismarlandstraße 11, D-3400 Göttingen, Germany

**Follow-Up of Angeletti &
Giannone and Larson**

Numerical Stellar Dynamics B: Fokker-Planck

$$\frac{\partial f}{\partial t} + \frac{\partial \phi}{\partial t} \frac{\partial f}{\partial E} = \left(\frac{\partial f}{\partial t} \right)_{\text{enc}},$$

with the potential ϕ advanced according to the Poisson equation

$$\nabla^2 \phi = 4\pi G n,$$

and the collisional term on the right-hand side of equation (1) is expressed under the Fokker-Planck assumption of small scatterings:

$$\begin{aligned} \left(\frac{\partial f}{\partial t} \right)_{\text{enc}} = & \frac{1}{V} \left[- \frac{\partial}{\partial E} \langle \Delta E \rangle fV - \frac{\partial}{\partial J_z} \langle \Delta J_z \rangle fV \right. \\ & + \frac{1}{2} \frac{\partial^2}{\partial E^2} \langle (\Delta E)^2 \rangle fV + \frac{\partial^2}{\partial E \partial J_z} \langle \Delta E \Delta J_z \rangle fV \\ & \left. + \frac{1}{2} \frac{\partial^2}{\partial J_z^2} \langle (\Delta J_z)^2 \rangle fV \right], \end{aligned}$$

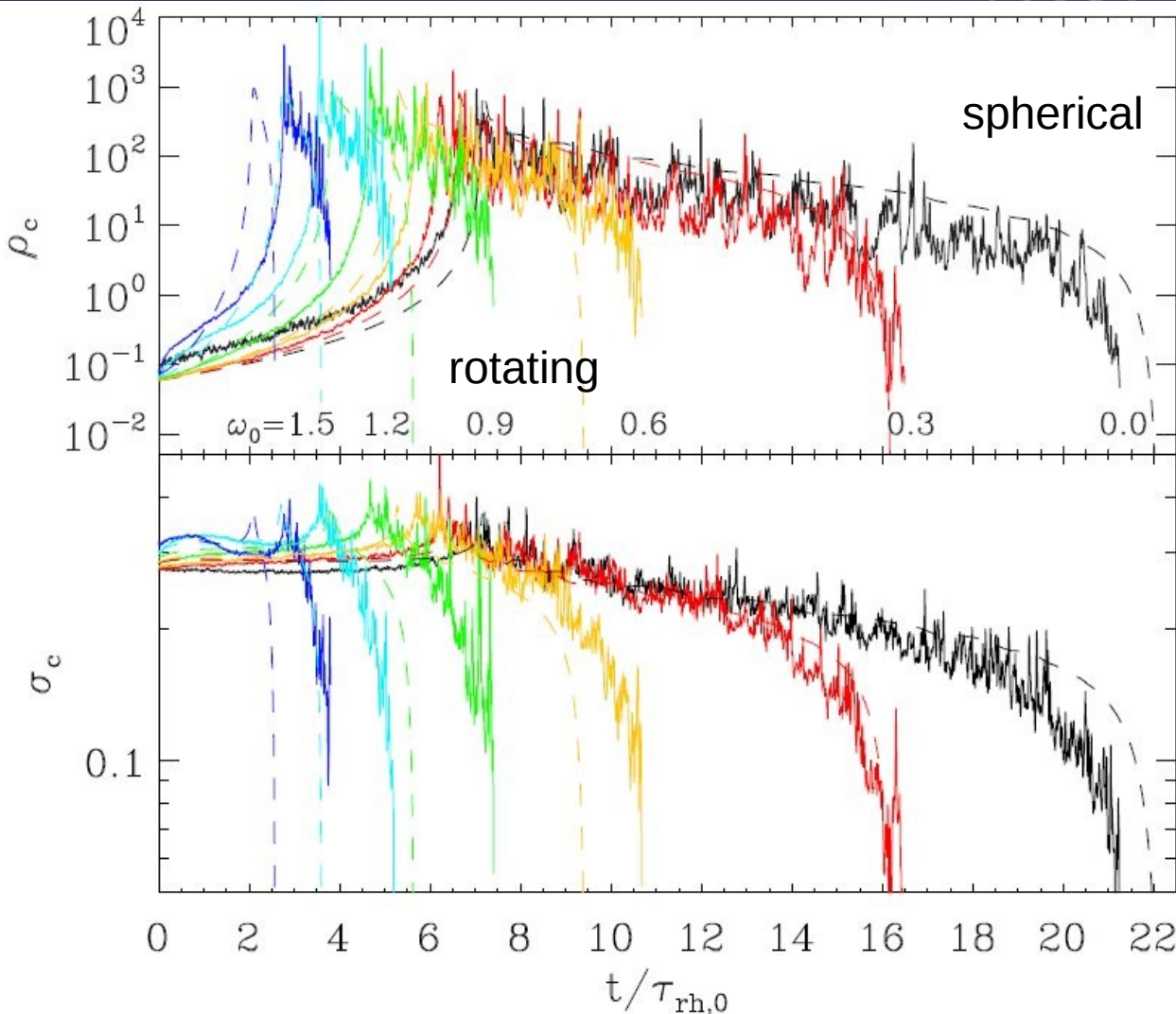
Orbit averaged Fokker-Planck Equation

(here in the 2D
form for axisymm.
systems,
Einsel & Spurzem
1999)
(unpubl.Ph.D. thesis
by J. Goodman 1983)

Rotating Star Clusters

(compare Fokker-Planck model with direct N-body integration)

Dissolution of Star Cluster in Tidal Field



Kim, Yoon,
Lee, Spurzem,
2008, MNRAS

Hong, Kim,
Lee, Spurzem,
2013, MNRAS

Three Phases in
Cluster Dissolution:

- 1) Core Collapse
(Encounters)
- 2) Post-Collapse
Steady Evaporation
(Encount)
- 3) Dynamic
final dissolution

- 1) Introduction – History and Theory
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*mainly Nbody6++GPU

Code(s)

(mainly Nbody6++GPU)

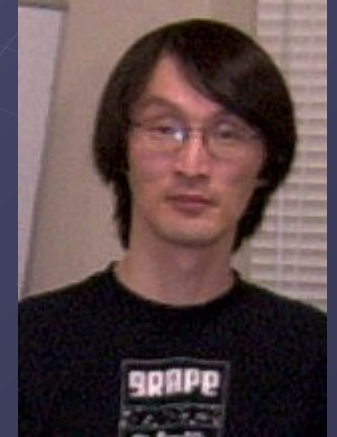
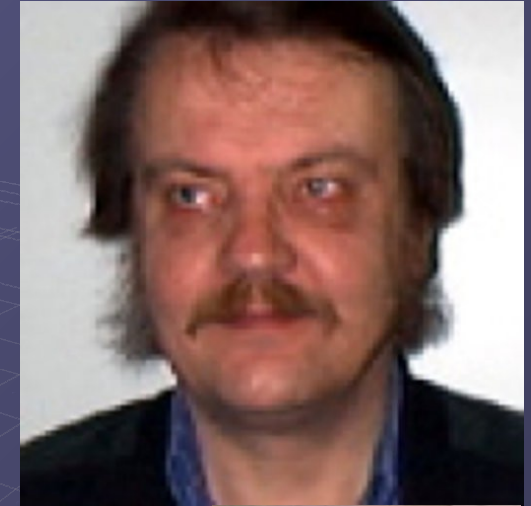
<https://github.com/nbody6ppgpu>

<https://github.com/nbody6ppgpu/Nbody6PPGPU-beijing>

Last but not least: Nbody-X History

After Holmberg and von Hoerner:

Sverre Aarseth, Roland Wielen, Seppo Mikkola



Jarrod Hurley, Steve McMillan, Jun Makino

Later see: Keigo Nitadori, Long Wang, Peter Berczik...

and more: Sambaran Banerjee, Albrecht Kamlah,

Manuel Arca Sedda, ...

(Credit: X-ray: NASA/CfA/J. Grindlay et al.,
Optical: NASA/STScI/R. Gilliland et al.)

X-ray binaries
with neutron stars
and black holes

Globular Cluster 47 Tuc
~ one million stars

$$\vec{a}_0 = \sum_j Gm_j \frac{\vec{R}_j}{R_j^3} ; \quad \vec{a}_0 = \sum_j Gm_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$

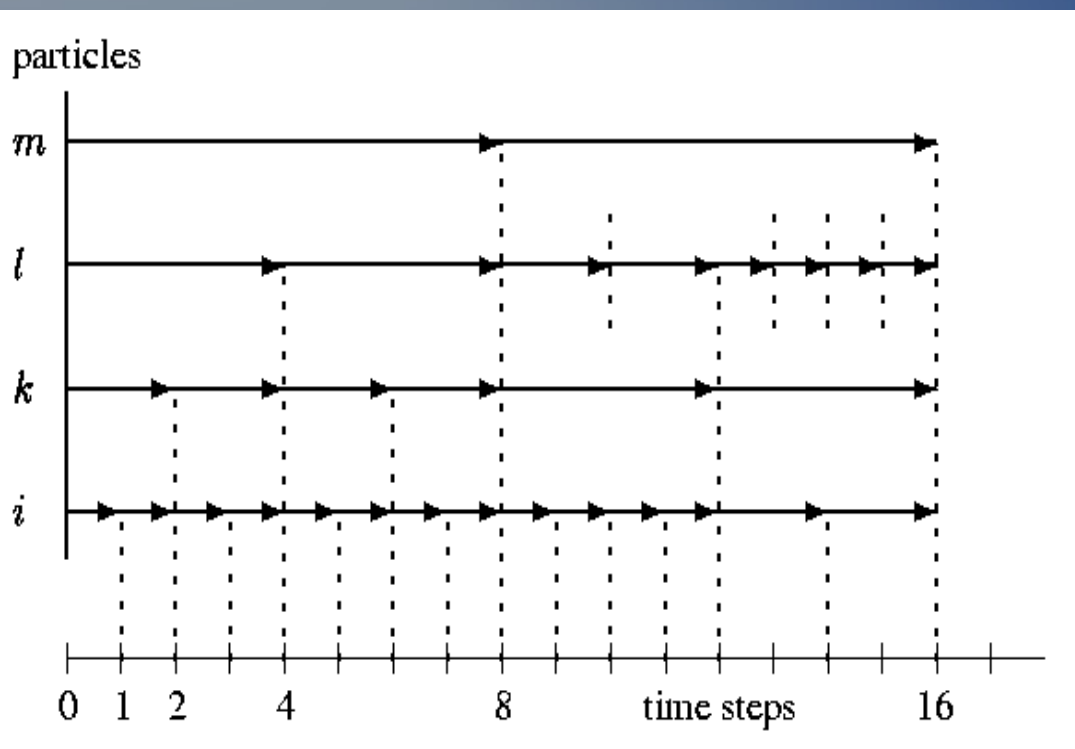
Code(s) – mainly Nbody6++GPU

S.J.Aarseth, S. Mikkola

(ca. 20.000 lines):

- Hierarchical Block Time Steps
- Ahmad-Cohen Scheme
- Regularisations
- 4th order Hermite scheme

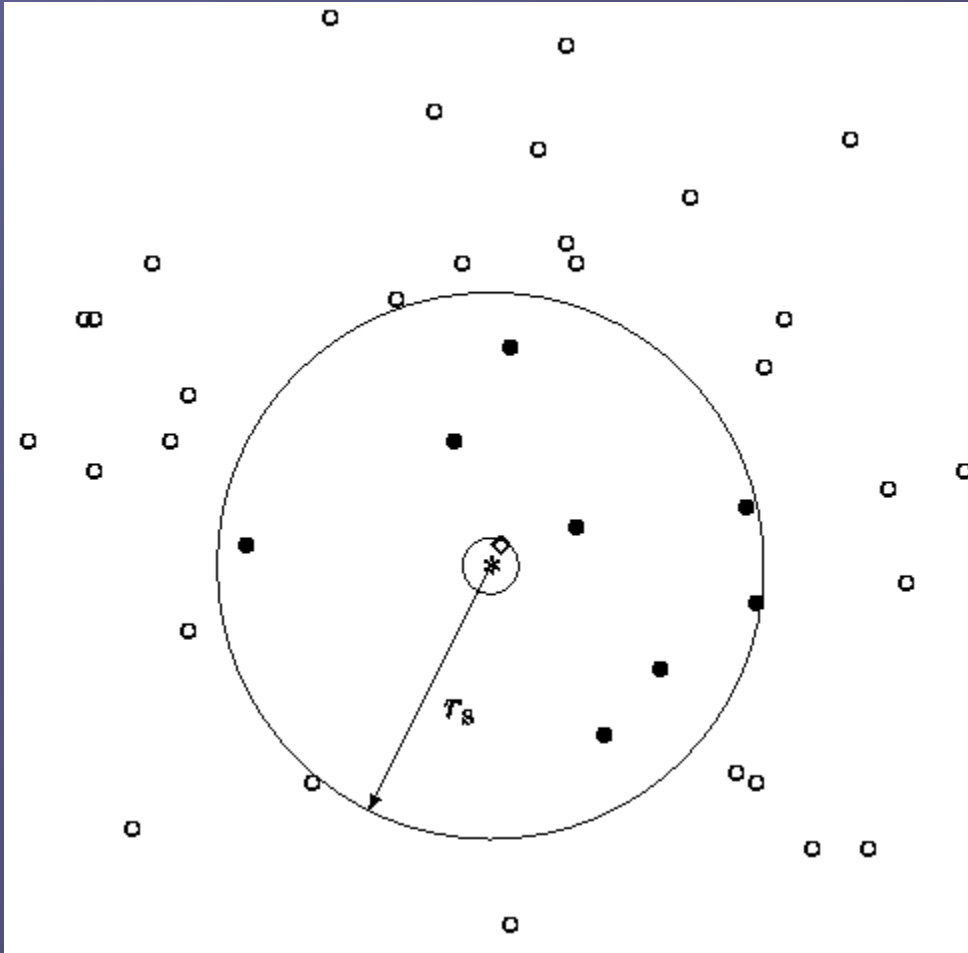
- NBODY6 (Aarseth 1999)
- NBODY6++ (Spurzem 1999) MPI
- NBODY6++GPU (Wang, Spurzem, Aarseth et al. 2015, Kamlah, Sp, et al. 2022A, Spurzem & Kamlah LRCA 2023)



Hierarchical Block Time Steps
“Aarseth step” →

$$\Delta t = \sqrt{\eta \frac{|\vec{a}| |\vec{a}^{(2)}| + |\vec{a}|^2}{|\vec{a}| |\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}}$$

Code(s) – mainly Nbody6++GPU



Ahmad-Cohen
Neighbour Scheme

(Double Volume for
Incoming Particles)

Special Care for fast
Particles

Complexity

$$\alpha N^2 + \beta N \cdot N_n$$

α small, $N_n \sim 50-200$

Physical and Numerical Methods: Direct Simulations
Direct: high accuracy / active-inactive particles

The Hermite Scheme: 4th Order on two time points

$$\vec{a}_0 = \sum_j Gm_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \ddot{\vec{a}}_0 = \sum_j Gm_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right] ,$$

$$\vec{x}_p(t) = \frac{1}{6}(t - t_0)^3 \ddot{\vec{a}}_0 + \frac{1}{2}(t - t_0)^2 \vec{a}_0 + (t - t_0)\vec{v} + \vec{x} ,$$
$$\vec{v}_p(t) = \frac{1}{2}(t - t_0)^2 \ddot{\vec{a}}_0 + (t - t_0)\vec{a}_0 + \vec{v} ,$$

Repeat Step 1 at t_1 using predicted $x, v \rightarrow a_1, \ddot{a}_1$

Physical and Numerical Methods: Direct Simulations

$$\frac{1}{2}\vec{a}^{(2)} = -3\frac{\vec{a}_0 - \vec{a}_1}{(t - t_0)^2} - \frac{2\vec{a}_0 + \vec{a}_1}{(t - t_0)}$$
$$\frac{1}{6}\vec{a}^{(3)} = 2\frac{\vec{a}_0 - \vec{a}_1}{(t - t_0)^3} - \frac{\vec{a}_0 + \vec{a}_1}{(t - t_0)^2},$$

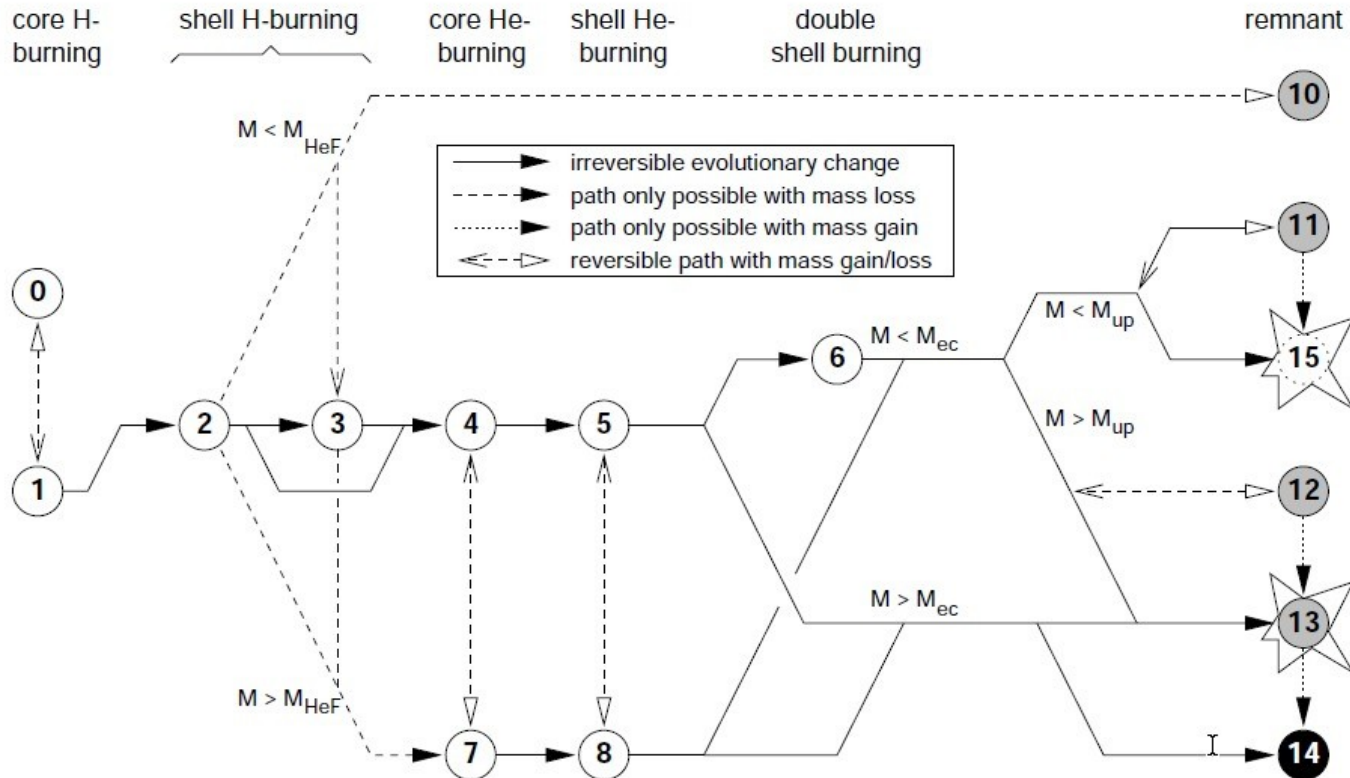
The Hermite Step
Get Higher Derivatives

$$\vec{x}(t) = \vec{x}_p(t) + \frac{1}{24}(t - t_0)^4\vec{a}_0^{(2)} + \frac{1}{120}(t - t_0)^5\vec{a}_0^{(3)},$$
$$\vec{v}(t) = \vec{v}_p(t) + \frac{1}{6}(t - t_0)^3\vec{a}_0^{(2)} + \frac{1}{24}(t - t_0)^4\vec{a}_0^{(3)}.$$

The Corrector Step – this is not time symmetric!

Most NBODY Codes ...

Hurley, 2001
Ph.D. thesis
and many papers
Following...



0 = main sequence $M < 0.7 M_{\odot}$

1 = main sequence $M > 0.7 M_{\odot}$

2 = Hertzsprung gap / subgiant

3 = first-ascent red giant

4 = horizontal branch / helium-burning giant

5 = early asymptotic giant / red supergiant

6 = thermally pulsating asymptotic giant

7 = naked helium main sequence

8 = naked helium (sub) giant

10 = helium white dwarf

11 = carbon/oxygen white dwarf

12 = oxygen/neon white dwarf

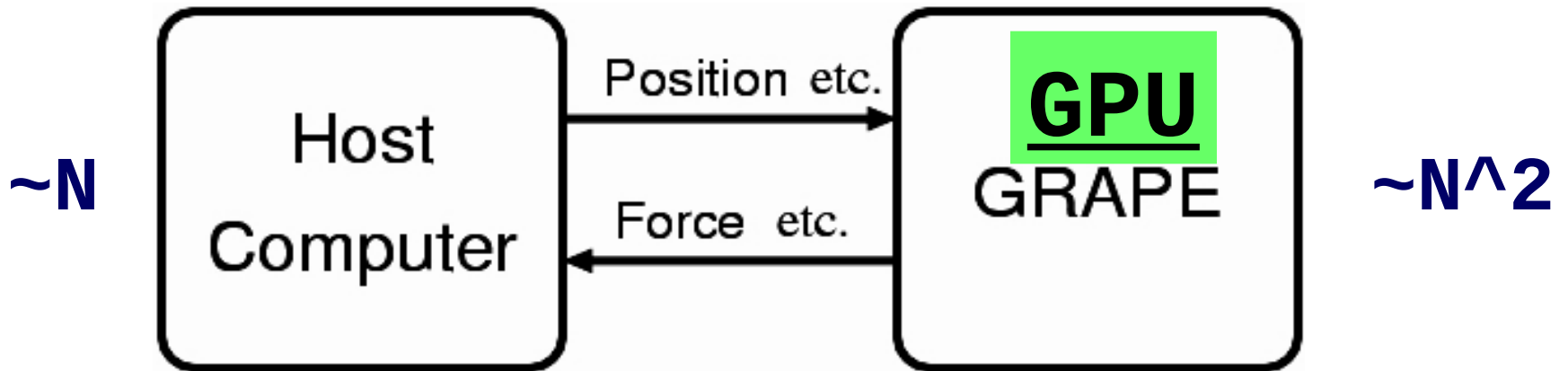
13 = neutron star

14 = black hole

15 = no stellar remnant

Code(s) - mainly Nbody6++GPU

(note also ϕ GRAPE/ ϕ GPU, Berczik et al. 2013)



$$\vec{a}_i = \sum_{j=1; j \neq i}^N \vec{f}_{ij} \quad \vec{f}_{ij} = - \frac{G \cdot m_j}{(r_{ij}^2 + \varepsilon^2)^{3/2}} \vec{r}_{ij}$$

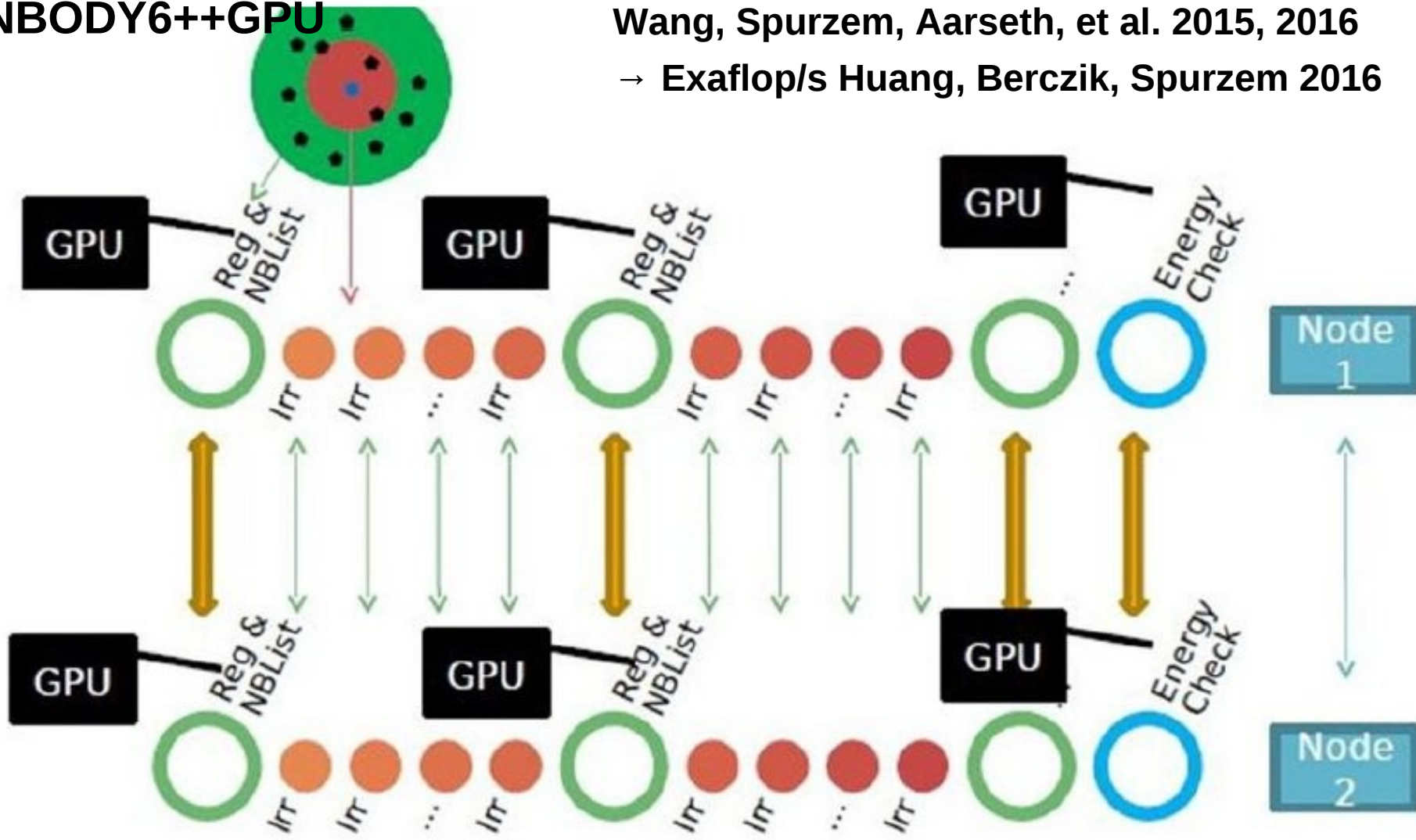
(Here: $\varepsilon=0$!)

Our CPU/GPU N-body (AC) code

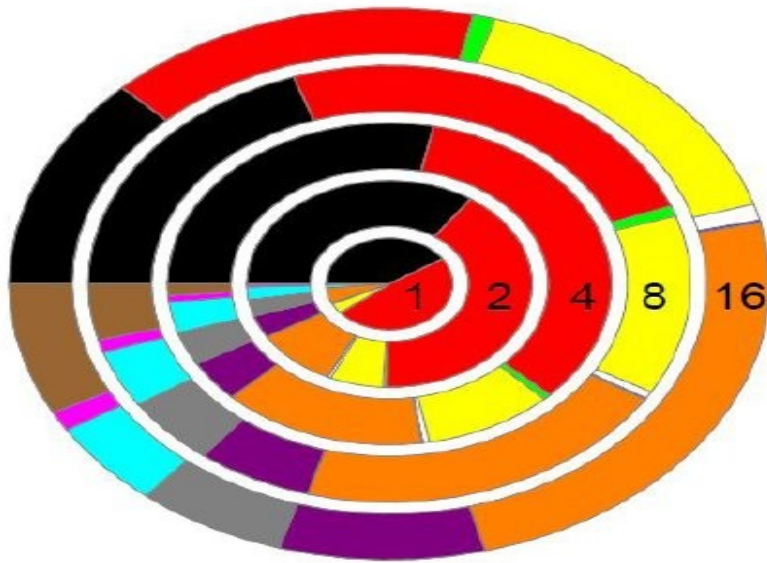
NBODY6++GPU

Wang, Spurzem, Aarseth, et al. 2015, 2016

→ Exaflop/s Huang, Berczik, Spurzem 2016



<https://github.com/lwang-astro/betanb6pp>



Huang, Spurzem. Berczik, 2016, RAA

Wang, Spurzem, Aarseth, et al. 2015, MNRAS



A timing model of Nbody6++GPU

Table 1 Main components of NBODY6++

Description	Timing variable	Expected scaling		Fitting value [sec]
		N	N_p	
Regular force computation	T_{reg}	$\mathcal{O}(N_{\text{reg}} \cdot N)$	$\mathcal{O}(N_p^{-1})$	$(2.2 \cdot 10^{-9} \cdot N^{2.11} + 10.43) \cdot N_p^{-1}$
Irregular force computation	T_{irr}	$\mathcal{O}(N_{\text{irr}} \cdot \langle N_{nb} \rangle)$	$\mathcal{O}(N_p^{-1})$	$(3.9 \cdot 10^{-7} \cdot N^{1.76} - 16.47) \cdot N_p^{-1}$
Prediction	T_{pre}	$\mathcal{O}(N^{kn_p})$	$\mathcal{O}(N_p^{-kp_p})$	$(1.2 \cdot 10^{-6} \cdot N^{1.51} - 3.58) \cdot N_p^{-0.5}$
Data moving	T_{mov}	$\mathcal{O}(N^{kn_{m1}})$	$\mathcal{O}(1)$	$2.5 \cdot 10^{-6} \cdot N^{1.29} - 0.28$
MPI communication (regular)	T_{mcr}	$\mathcal{O}(N^{kn_{cr}})$	$\mathcal{O}(kp_{cr} \cdot \frac{N_p-1}{N_p})$	$(3.3 \cdot 10^{-6} \cdot N^{1.18} + 0.12)(1.5 \cdot \frac{N_p-1}{N_p})$
MPI communication (irregular)	T_{mci}	$\mathcal{O}(N^{kn_{ci}})$	$\mathcal{O}(kp_{ci} \cdot \frac{N_p-1}{N_p})$	$(3.6 \cdot 10^{-7} \cdot N^{1.40} + 0.56)(1.5 \cdot \frac{N_p-1}{N_p})$
Synchronization	T_{syn}	$\mathcal{O}(N^{kn_s})$	$\mathcal{O}(N_p^{kp_s})$	$(4.1 \cdot 10^{-8} \cdot N^{1.34} + 0.07) \cdot N_p$
Sequential parts on host	T_{host}	$\mathcal{O}(N^{kn_h})$	$\mathcal{O}(1)$	$4.4 \cdot 10^{-7} \cdot N^{1.49} + 1.23$

NBODY6++GPU with up to 16M particles (Benchmarks on raven at MPCDF)

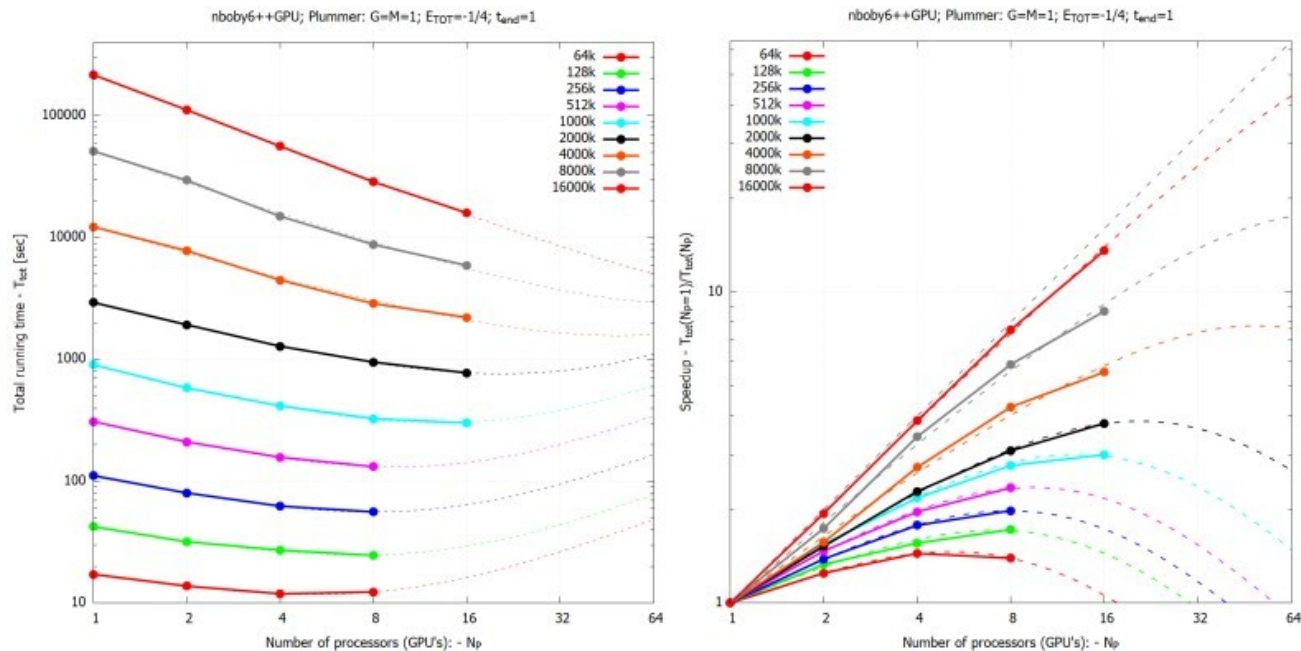
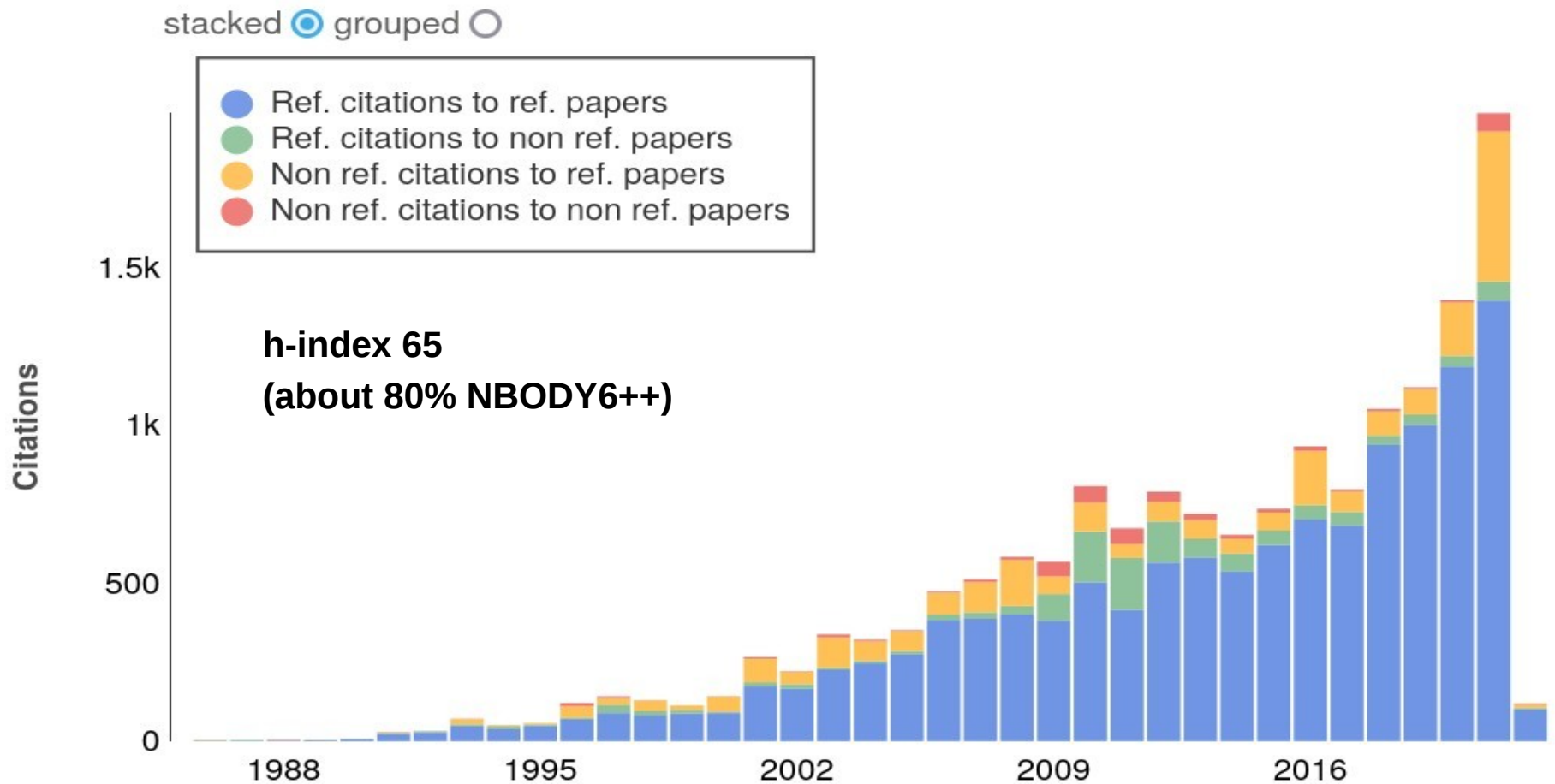


Figure 6: Benchmark results and extrapolated scaling for NBODY6++GPU, initial Plummer model, on the raven cluster at MPCDF, see main text. **Left:** Total time for one NBODY model unit in secs; **Right:** Speed-Up compared to using one GPU only. In both cases different curves for particle numbers from 64k to 16m. Ideal Speedup is the diagonal dashed lines, other dashed lines extrapolations from the timing model.

To demonstrate how successful the direct NBODY codes are in our field we have collected the following three figures from the ADS Bumblebee (full text search) facility. The search string

full:NBODY5 OR full:NBODY6 OR full:"NBODY6++" OR full:NBODY7 OR full:NBODY4

has been used to catch all publications using or citing the different variants of the code.



NBODY1 – NBODY7: “The Growth of an Industry” (Aarseth 1999)

	ITS	ACS	KS	HITS	PN	AR	CC	MPI	GPU
NBODY1	✓								
NBODY2		✓		✓					
NBODY3	✓		✓						
NBODY4			✓	✓					
NBODY5	✓	✓	✓						
NBODY6		✓	✓	✓					
NBODY6GPU		✓	✓	✓				✓	
NBODY6++		✓	✓	✓			✓		
NBODY6++GPU		✓	✓	✓	✓		✓	✓	✓
NBODY7		✓	✓	✓	✓	✓			✓

ITS: Individual time-steps [107] Aarseth 1985

ACS: Ahmad-Cohen neighbour scheme [109] Ahmad, Cohen 1973

KS: KS-regularization of few-body subsystems [104] Kustaanheimo, Stiefel 1965

HITS: Hermite scheme integration method combined with hierarchical block time-steps [111]

PN: Post-Newtonian [150,125,151] Kupi, Amaro-Seoane, Sp. 2006, \ Makino, Aarseth 1992

AR: Algorithmic regularization [125] Mikkola, Merritt 2008, Aarseth 2012, Banerjee et al. 2020

CC: Classical chain regularization [114] Mikkola, Aarseth 1998

MPI: Message Passing Interface, multi-node multi-CPU parallelization [139]

GPU: use of GPU acceleration [138] (if also MPI: multi-node many GPU [145]) Spurzem 1999

Berczik, Spurzem, et al., LNCS 2013; Table from: Spurzem, Kamlah 2023, LRCA

NBODY6++GPU: <https://github.com/nbody6ppgpu/>

Part of <https://www.punch4nfdi.de/> PUNCH4NFDI Consortium w. Jülich

PeTar: a high-performance N -body code for modeling massive collisional stellar systems

Long Wang,^{1,2}★ Masaki Iwasawa,^{2,3} Keigo Nitadori² and Junichiro Makino^{2,4}

¹Department of Astronomy, School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-0033, Japan

²RIKEN Center for Computational Science, 7-1-26 Minatojima-minami-machi, Chuo-ku, Kobe, Hyogo 650-0047, Japan

³National Institute of Technology, Matsue College, 14-4, Nishi-ikuma-cho, Matsue, Shimane 690-8518, Japan

⁴Graduate School of Science, Kobe University, 1-1 Rokko, Nada-ku, Kobe, Hyogo 657-8501, Japan

Slide 50

Accepted XXX. Received YYY; in original form ZZZ



New competition 1:
PeTar: MNRAS 2020

ABSTRACT

The numerical simulations of massive collisional stellar systems, such as globular clusters (GCs), are very time-consuming. Until now, only a few realistic million-body simulations of GCs with a small fraction of binaries (5%) have been performed by using the NBODY6++GPU code. Such models took half a year computational time on a GPU based super-

FROST: a momentum-conserving CUDA implementation of a hierarchical fourth-order forward symplectic integrator

Antti Rantala¹★, Thorsten Naab¹, Volker Springel¹

¹Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85748, Garching, Germany

New competition 2:
FROST: MNRAS 2021

Slide 50

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

We present a novel hierarchical formulation of the fourth-order forward symplectic integrator and its numerical implementation in the GPU-accelerated direct-summation N -body code FROST. The new integrator is especially suitable for simulations with a large dynamical range due to its hierarchical nature. The strictly positive integrator sub-steps in a fourth-

Supercomputers

A special-purpose computer for gravitational many-body problems

Nature 1990

Daiichiro Sugimoto*, Yoshihiro Chikada†, Junichiro Makino*, Tomoyoshi Ito*, Toshikazu Ebisuzaki* & Masayuki Umemura‡

* Department of Earth Science and Astronomy, College of Arts and Sciences, University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo 153, Japan

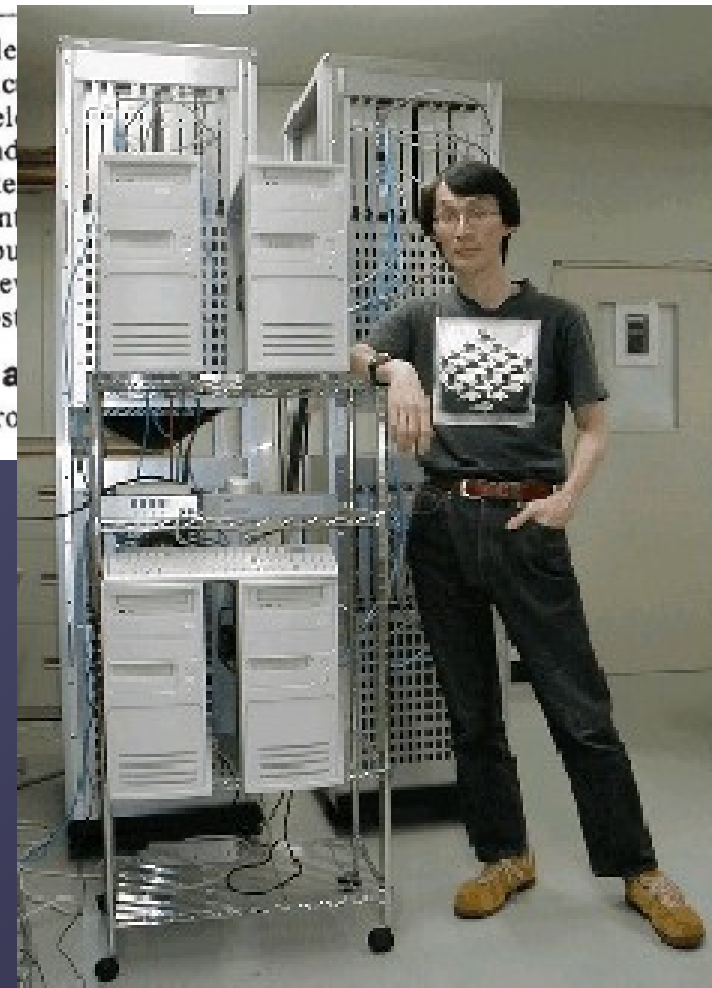
† Nobeyama Radio Observatory, Minamimaki-mura, Minamisaku-gun, Nagano 384-13, Japan

‡ National Astronomical Observatory, Mitaka, Tokyo 181, Japan

A processor has been constructed using a 'pipeline' architecture to simulate many-body systems with long-range forces. It has a speed equivalent to 120 megaflops, and the architecture can be readily parallelized to make teraflop machines a feasible possibility. The machine can be adapted to study molecular dynamics, plasma dynamics and astrophysical hydrodynamics with only minor modifications.

(very-large-scale computer is constructed. The problem is to develop construction and of physical systems to these apparent in the computer GRAPE-1 achieved XMP/1 at a cost

The pipeline architecture
The N-body problem

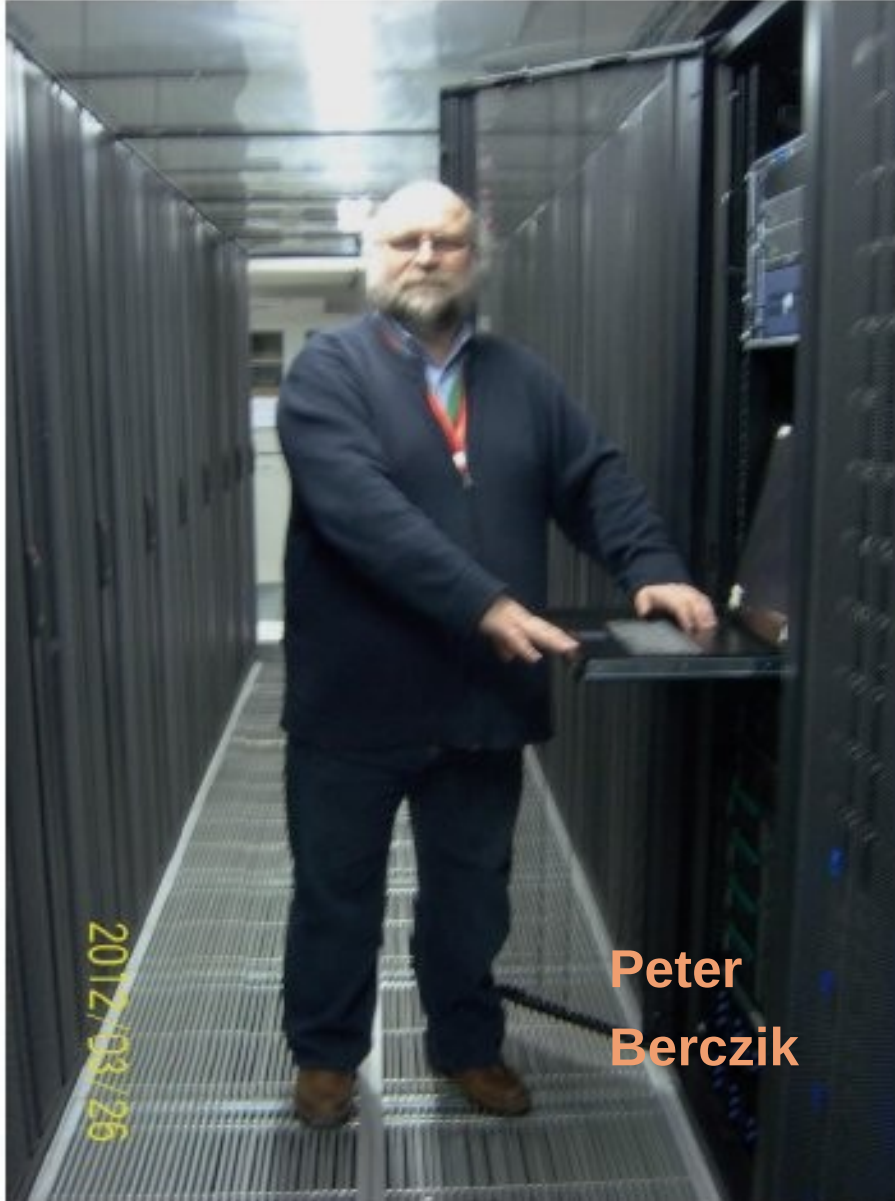


Jun Makino
with GRAPE6 →
(2002)

GRAPE-6 Gravity/Coulomb Part

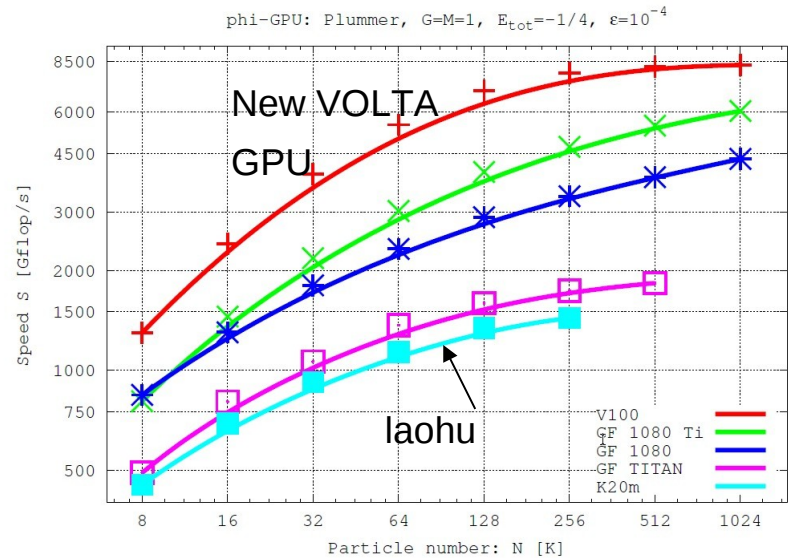
- G6 Chip: 0.25 μ 2MGate ASIC, 6 Pipelines
- at 90MHz, 31Gflops/chip
- 48Tflops full system (March 2002)
- Plan up to 72Tflops full system (in 2002)
- Installed in Cambridge, Marseille, Drexel, Amsterdam, New York (AMNH), Mitaka (NAO), Tokyo, etc..

2010 Beijing: NAOC laohu cluster 2015: 64 Kepler K20



Laohu: 2009/2015
(Kepler GPU)
100 Tflop/s 150k cores

New GPUs 5-6 times
faster... (see below)



JUWELS Booster 936 nodes (AMD CPU, 4x Ampere GPU)
~450.000 AMD cores, 25 million NVIDIA Ampere GPU cores
~ 70 Pflop/s SP ~ 44 Pflop/s DP
No. 12 in top500 list, No. 25 in green500 list

Jülich Wizard for European Leadership Science



LUMI

Supercomputer, Kajaani, Finland

Using only
Hydroelectric
Power and its
Heat used for heating
buildings.

No. 3 in top500

No. 7 in green500

2.2 million cores

10.000 AMD GPUs



EuroHPC and LUMI consortium:

Finland, Belgium, Czech Republic, Denmark, Estonia, Iceland, Norway, Poland, Sweden, and Switzerland.

- 1) Introduction – History and Theory
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DRAGON I Simulation

<http://silkroad.zah.uni-heidelberg.de/dragon/>

<https://github.com/nbody6ppgpu>

Also in: <https://www.punch4nfdi.de/>



One million stars direct simulation,

biggest and most realistic direct N-Body simulation of globular star clusters.

With stellar mass function, single and binary stellar evolution, regularization of close encounters, tidal field (NBODY6++GPU).

(NAOC/Silk Road/MPA collaboration).

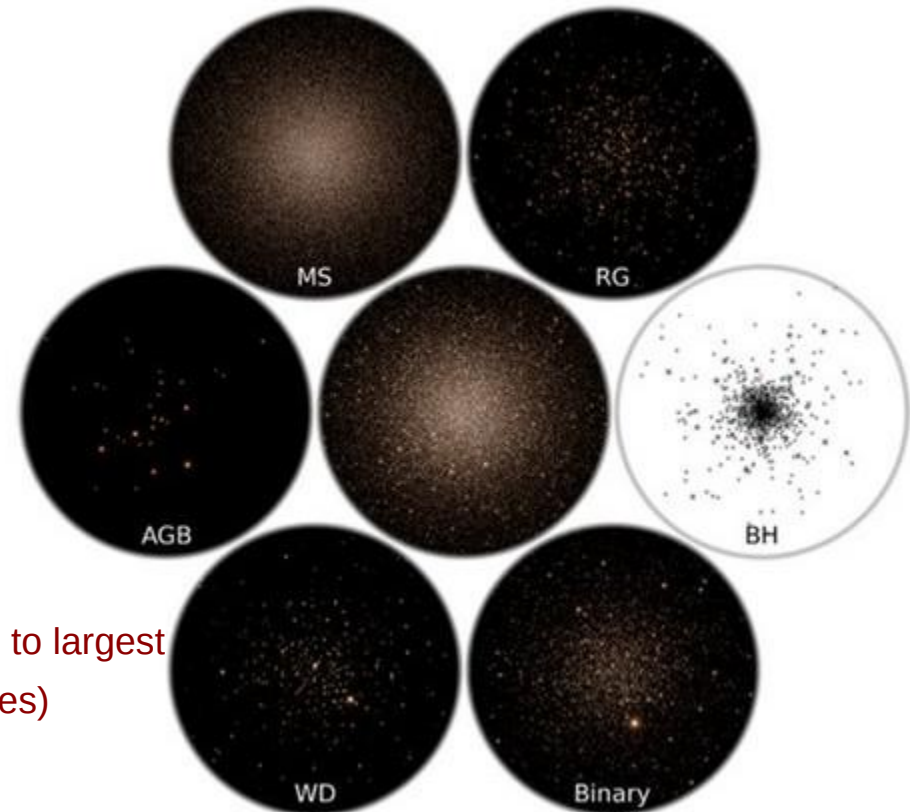
Wang, Spurzem, Aarseth, Naab et al.

MNRAS, 2015

Wang, Spurzem, Aarseth Naab, et al.

MNRAS 2016

Number of Floating Point Operations (~1M bodies) similar to largest
Cosmological simulations (Millennium, Illustris, ~10¹⁰ bodies)



What about black holes?
Black Hole Binaries?
DRAGON II Models

GW190521:
mass-gap remnant with high $m \sim 142 M_{\text{SUN}}$

O3a observation run

GW190412:
smallest mass ratio

LIGO-Virgo Black Holes

EM Black Holes

GW190814:
massive NS?

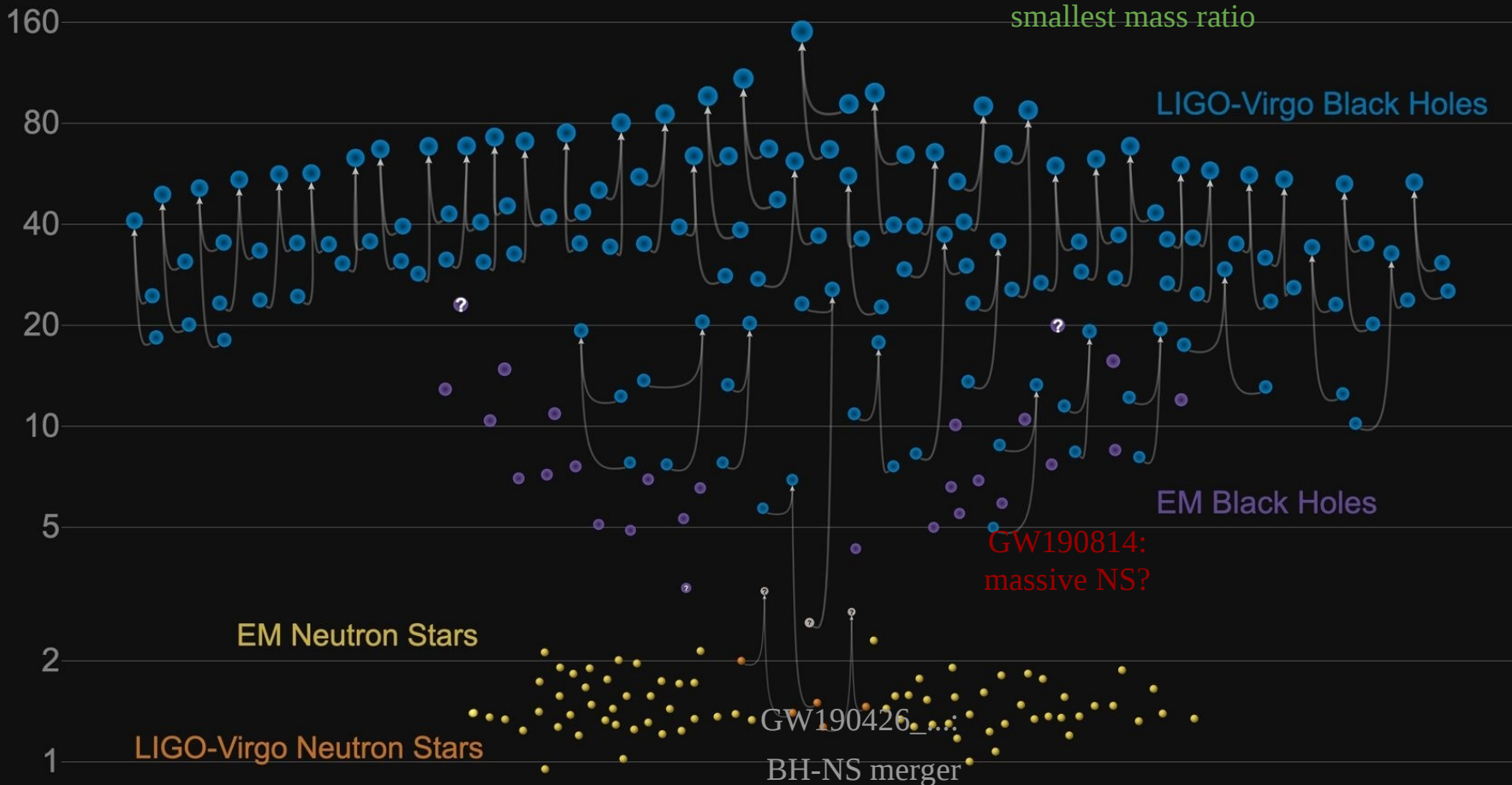
EM Neutron Stars

LIGO-Virgo Neutron Stars

GW190426...
BH-NS merger

GWTC-2 plot v1.0

LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern



Binary Evolution Relativistic – current

(taken from Rizzuto et al. 2021, 2022, Arca Sedda et al. 2021, 2022, and DRAGONII Papers I,II,III, In preparation, using citations given here and cited in papers)

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64 G^3 m_1 m_2 (m_1 + m_2)}{5 c^5 a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right),$$

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304 G^3 m_1 m_2 (m_1 + m_2)}{15 c^5 a^4 (1 - e^2)^{5/2}} \left(1 + \frac{121}{304} e^2 \right).$$

$$\vec{v}_{\text{GW}} = v_m \hat{e}_{\perp,1} + v_{\perp} (\cos \xi \hat{e}_{\perp,1} + \sin \xi \hat{e}_{\perp,2}) + v_{\parallel} \hat{e}_{\parallel},$$

$$v_m = A \eta^2 \sqrt{1 - 4\eta} (1 + B\eta),$$

$$v_{\perp} = \frac{H \eta^2}{1 + q_{\text{BBH}}} \left(S_{2,\parallel} - q_{\text{BBH}} S_{1,\parallel} \right),$$

$$v_{\parallel} = \frac{16 \eta^2}{1 + q_{\text{BBH}}} \left[V_{11} + V_A \Xi_{\parallel} + V_B \Xi_{\parallel}^2 + V_C \Xi_{\parallel}^3 \right] \times \left| \vec{S}_{2,\perp} - q_{\text{BBH}} \vec{S}_{1,\perp} \right| \cos(\phi_{\Delta} - \phi_1).$$

Orbit Averaged

Post-Newtonian

(Peters & Mathews 1963,

Peters 1964)

semi-major axis a,

Eccentricity a

(Rizzuto et al. 2021, 2022)

Implementation of

relativistic kick after

gravitational wave

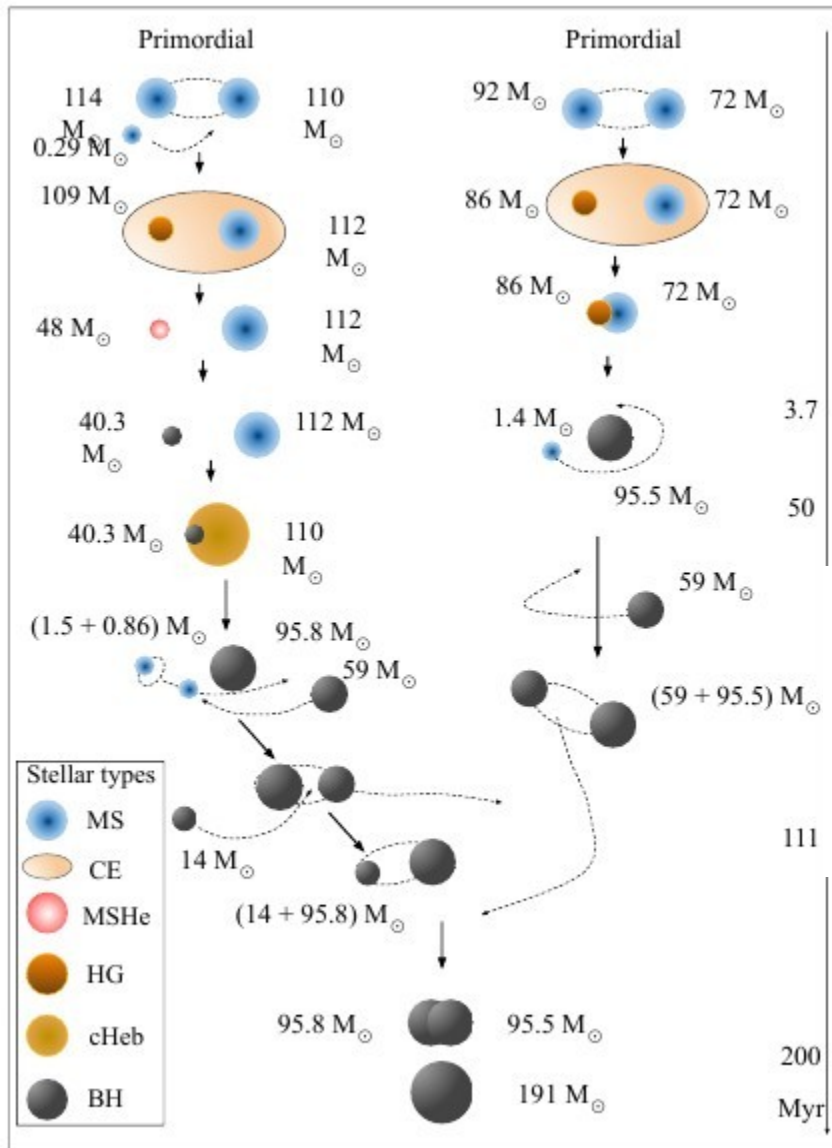
Induced coalescence

(Arca Sedda et al. 2023abc,

following papers cited therein)

DRAGON-II Simulations – Paper II

using NBODY6++GPU



*Arca Sedda et al. 2023abc:
MNRAS:*

*19 models, up to 1 million stars, up
to 33% initial hard binaries*

Including GR kicks for mergers!

Figure 2. Formation of an IMBH in simulation with $N = 120k$, $R_{\text{HM}} = 1.75$ pc, and $f_b = 0.2$, realization ID 0. Two massive primordial binaries undergo common envelope that eventually lead to the formation of two nearly equal mass BHs ($m_{\text{BH}} \sim 95 M_{\odot}$) that eventually find each other via a complex series of binary-binary interactions. The binary eventually merge and builds-up an IMBH with mass $m_{\text{IMBH}} \simeq 191 M_{\odot}$. The color-coded legend is ent colors correspond to different evolutionary stages: main sequence (MS), common envelope (CE), naked main sequence He star (MSHe), Hertzsprung gap (HG), core He burning (cHeB), and black hole (BH).

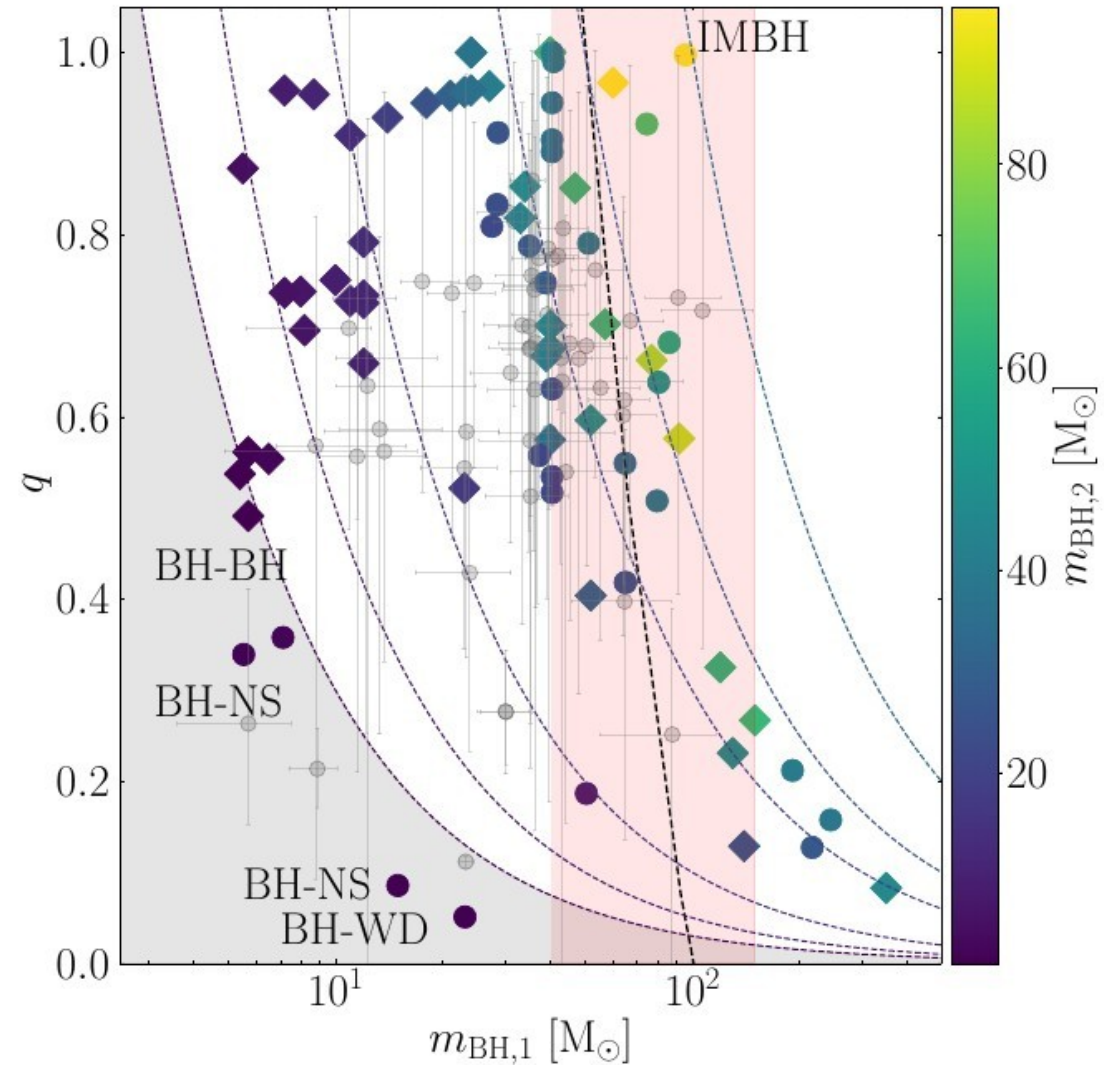
DRAGON-II Simulations – Paper III using NBODY6++GPU

*Arca Sedda et al. 2023c:
Submitted to MNRAS:
19 models, up to 1 million
stars, up to 33% initial hard
binaries*

Compact Object Mergers
Compared with LIGO-Virgo
GWTC-3 catalogue (grey
symbols)

Mass ratio q vs. primary
mass m_1 ; colour code:
secondary mass m_2

grey shade: neutron star
involved



Work in Progress: Very Massive Stars, ... Black Holes, Relativistic Dynamics

- DRAGON-II simulations: approximate Post-Newtonian, max. 1 Gyr or shorter, need longer simulations, currently waiting: PopIII clusters (with Ataru Tanikawa)
- Pure Black Hole Clusters: check effect of spins and recoil kicks (Ricarda)
- Growth of Massive Stars: Albrecht Kamlah with Rainer, Marcelo, Francesco, Peter, and many collaborators (e.g. Renyue Cen, Nadine Neumayer)
- Full Post-Newtonian up to PN3.5 with spins in large cluster models

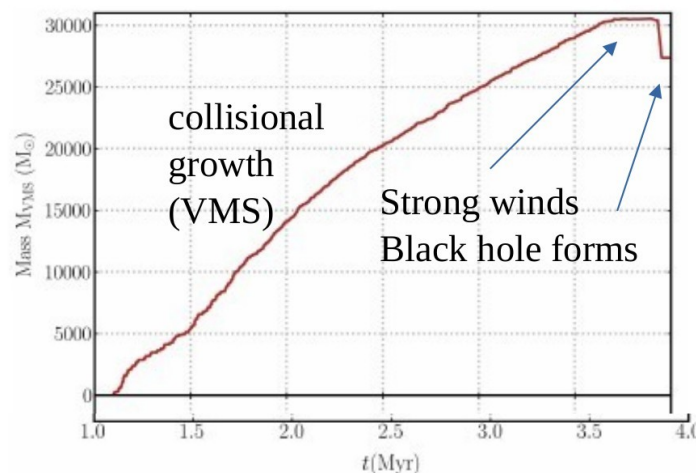
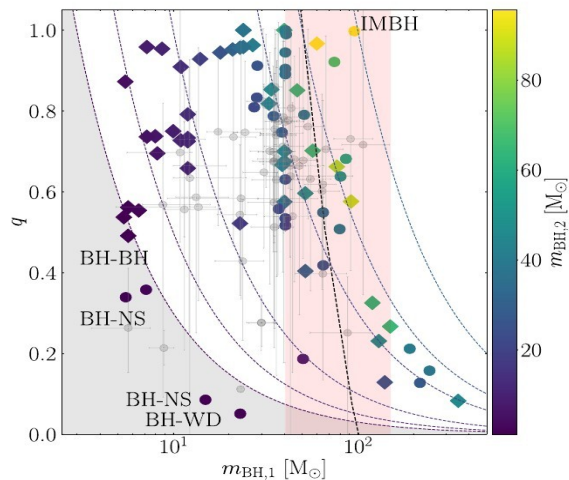


Figure 8. Figure showing temporal evolution of the mass M_{VMS} (M_{\odot}) of the VMS (and IMBH thereafter), the logarithm

Kamlah,
Spurzem,
et al. (incl. Jarrod
Hurley) to be
subm. ASAP

Issues:

- Rejuvenation
- He ageing
- Massive star evolution

- 1) Introduction – History and Theory
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Summary Message

Massive Star Clusters:

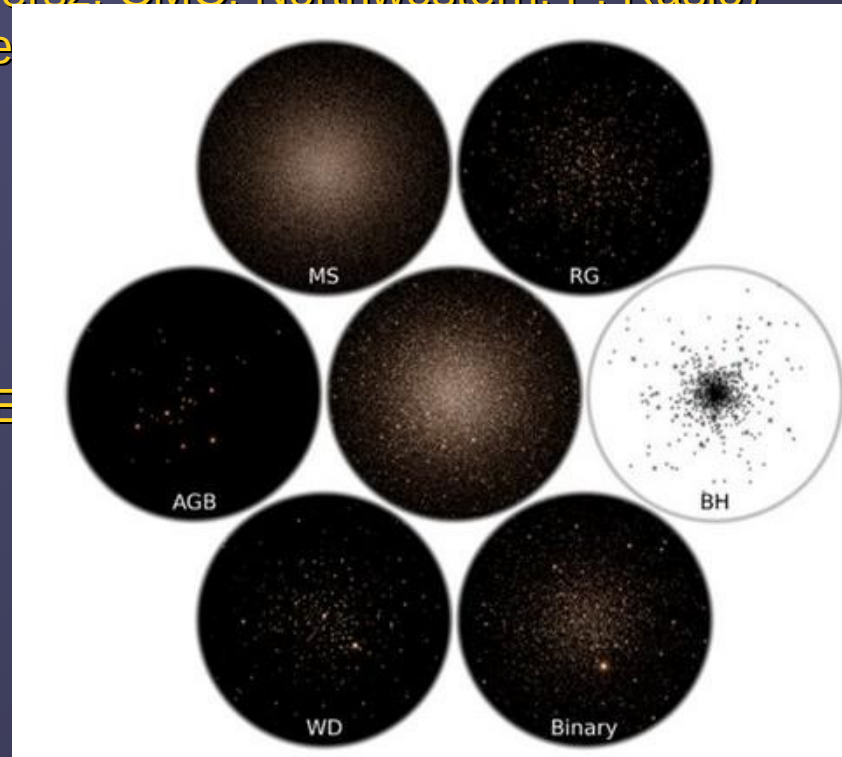
- Direct N-Body Simulations of star clusters give LIGO/Virgo Sources (are consistent with them, it does not mean all sources are from star clusters)
- Necessary Input: single/binary stellar evolution / relativistic (PN) dynamics
- Still very long computing time for few models (in spite of GPU, Lumi, ...)

Monte Carlo Models (MOCCA, Warsaw, M. Giersz, CMC, Northwestern, F. Rasio) needed to get good sweep of parameter space

Nuclear Star Clusters

(not covered in this talk):

- Observable Tidal Disruption Events (TDE)
- Light Curves correspond to dynamics of TDE
- Future Work: relativistic inspirals
- Future Work: pulsars
- Future Work: star disk



Stars, Stellar Dynamics, Black Holes, and Gravitational Waves at UNC

The Cluster Monte Carlo (CMC) Code

January 12, 2022 By [carloodr](#)



After several years of work that started before I was an assistant prof, we have the first public release of the Cluster Monte Carlo code, CMC. The **paper** is available [here](#) while the **code itself** is available [here](#). CMC is a Monte Carlo technique for stellar dynamics, based on an approach developed by Hénon in 1971. Instead of integrating the force of every particle on every other particle, Hénon's approach was to model the effect of many two-body encounters as single "effective" encounters.

University of North Carolina, UNC, Carl Rodriguez et al.
Canadian Inst. For Theoretical Astrophysics, CITA, Claire Ye
Center for Interdisciplinary Exploration and Research in
Astrophysics CIERA, Northwestern Uni, Evanston, IL, Chicago,
Fred Rasio et al.

Contact Info

Carl Rodriguez

Phillips Hall 268

120 E. Cameron Ave | CB3255
| Chapel Hill, NC 27599

MOCCA

The MOCCA code is currently one of the most advanced codes for simulating real size star clusters. It follows the star cluster evolution closely to N-body codes but is much faster!
More about [features](#), [publications](#) and [team](#)

Warsaw PL: MOCCA team:
M. Giersz, A. Hypki, A. Askar
et al.

Team & Collaborators as below and further:

Thorsten Naab (MPA), Mirek Giersz (CAMK),
Ataru Tanikawa (Tokyo U), Nadine Neumayer (MPIA)

DRAGON simulations – globular and nuclear star clusters

- **DRAGON simulation: PhD thesis Long Wang, KIAA/PKU**, awarded for first realistic globular cluster simulation using **NBODY6++GPU** with one million stars and many binaries (**Wang, Spurzem, Aarseth, et al.**, MNRAS 2016).
- **The Dragon-II simulations – Paper III. Compact binary mergers in clusters with up to 1 million stars: mass, spin, eccentricity, merger rate and pair instability supernovae rate** (**Arca Sedda, M., Kamlah, A. W. H., Spurzem, R., et al.**) arXiv e-prints arXiv:2307.04807, **Paper II: MNRAS 525, 429 (2023), Paper I: arXiv e-prints arXiv:2307.04805**

NBODY6++GPU and more, current state:

- **Spurzem, R., Kamlah A.W.H. Direct N-body simulations, in Living Rev. in Comp. Astrophysics 9, id.3 (2023) (NBODY7 see also Banerjee, Sambaran papers)**

Direct Nuclear Star Cluster Models with SMBH and TDE:

- **DRAGON simulation of the Galactic Center**, PhD thesis of Taras Panamarev, ARI/ZAH Univ. of Heidelberg (**Panamarev, Just, Spurzem, Berczik, Wang, Arca Sedda**, MNRAS 2019), simple TDE
- **Revisit the Rate of Tidal Disruption Events: The Role of the Partial Tidal Disruption Event** (**Zhong, S., Li, S., Berczik, P., Spurzem, R.**) 933, 96 (2022), TDE improved 1
- **Marija Minzberg, Philip Cho**: Master Theses Heidelberg 2023, publication in progress, TDE improved 2 and 3.

Some other papers and collaborators may be mentioned:

- Rizzuto, Naab, Spurzem et al. (2021, 2022) precursor of DRAGON II but no GW kicks, no TDE
- Li, Zhong, Berczik, Spurzem, Chen, Liu (MNRAS 2023 and earlier): merging nuclei with TDE