Understanding electron heating of kinetic turbulence through phase-space dynamics

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with

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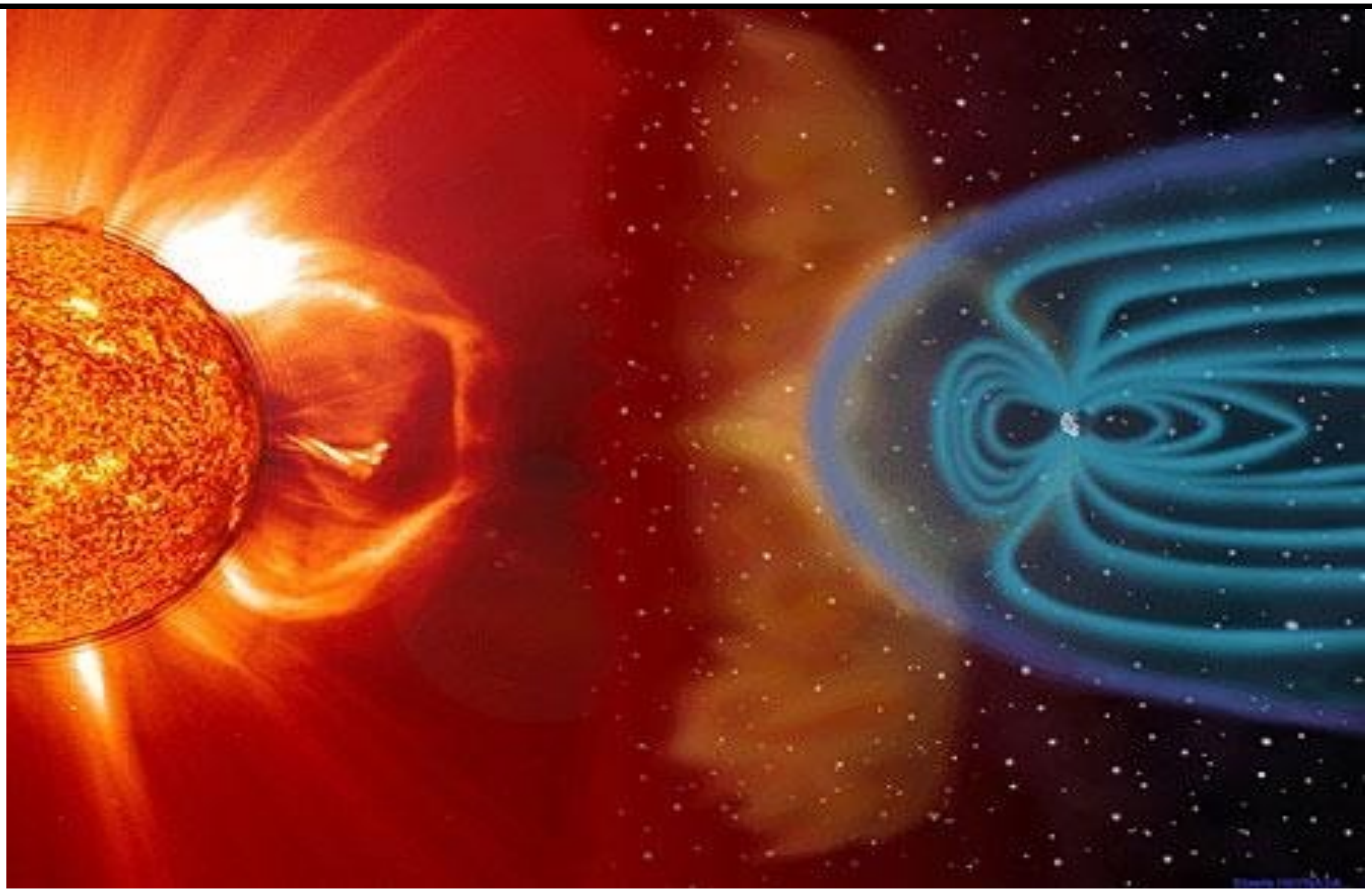


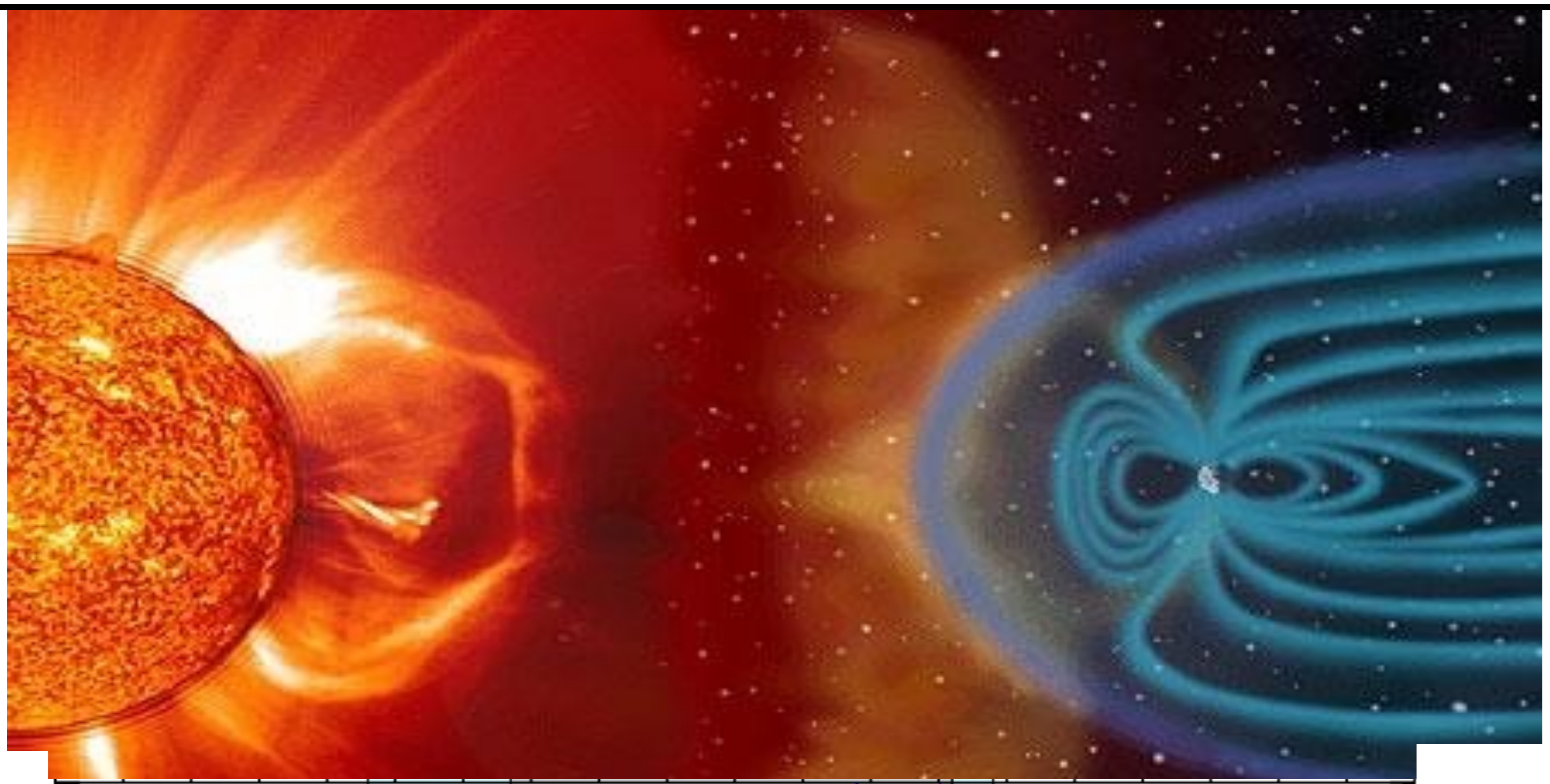
The dissipation of turbulence causes the heating of plasma in space and astrophysical systems.

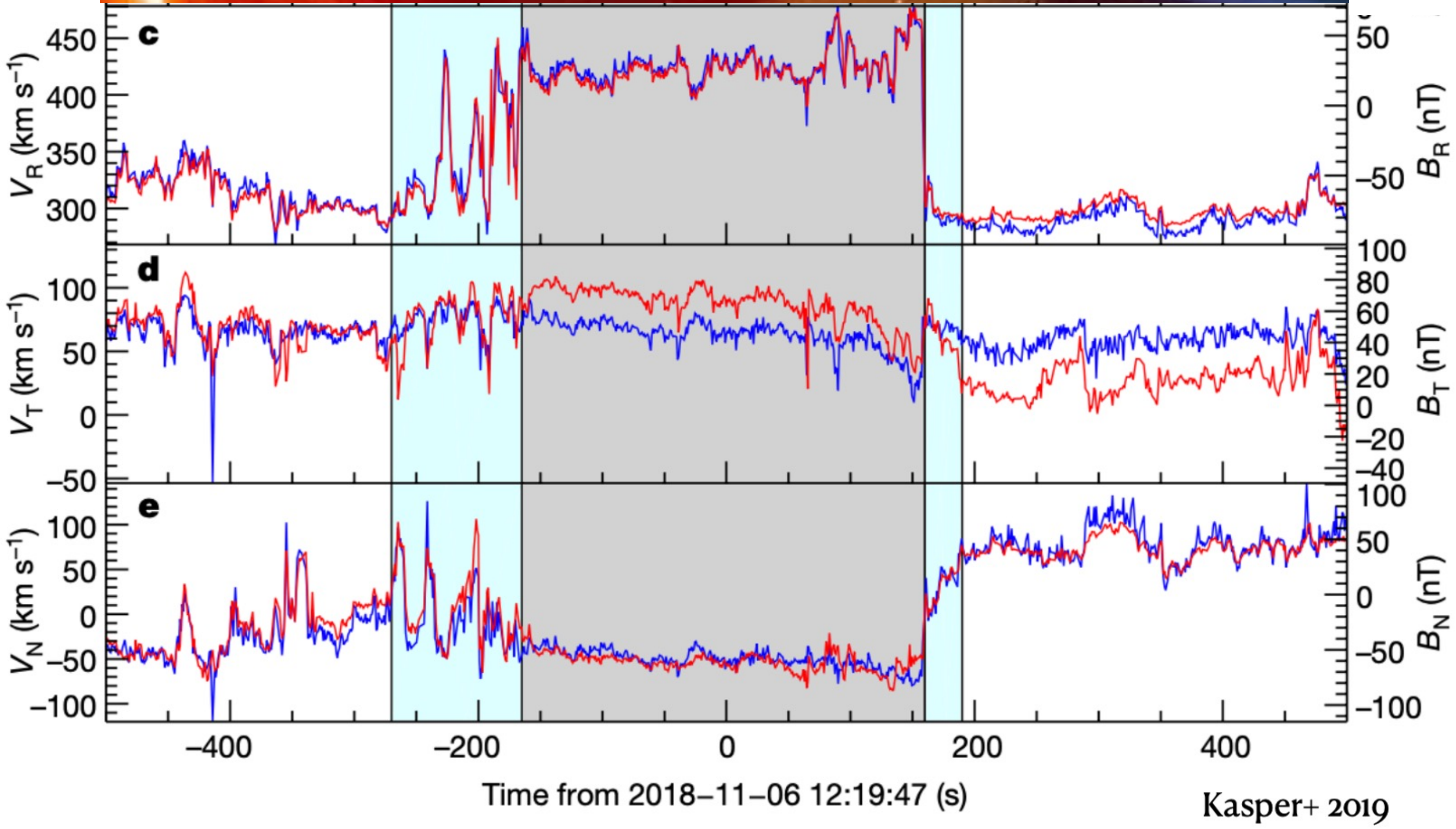
A predictive theory of turbulence (plasma couples with EM fields) is crucial:

→ nature of the turbulent fluctuations (e.g., spectral properties,)

→heating mechanisms for particles (dissipative mechanisms for electromatic fluctuations)

Turbulence composed of waves

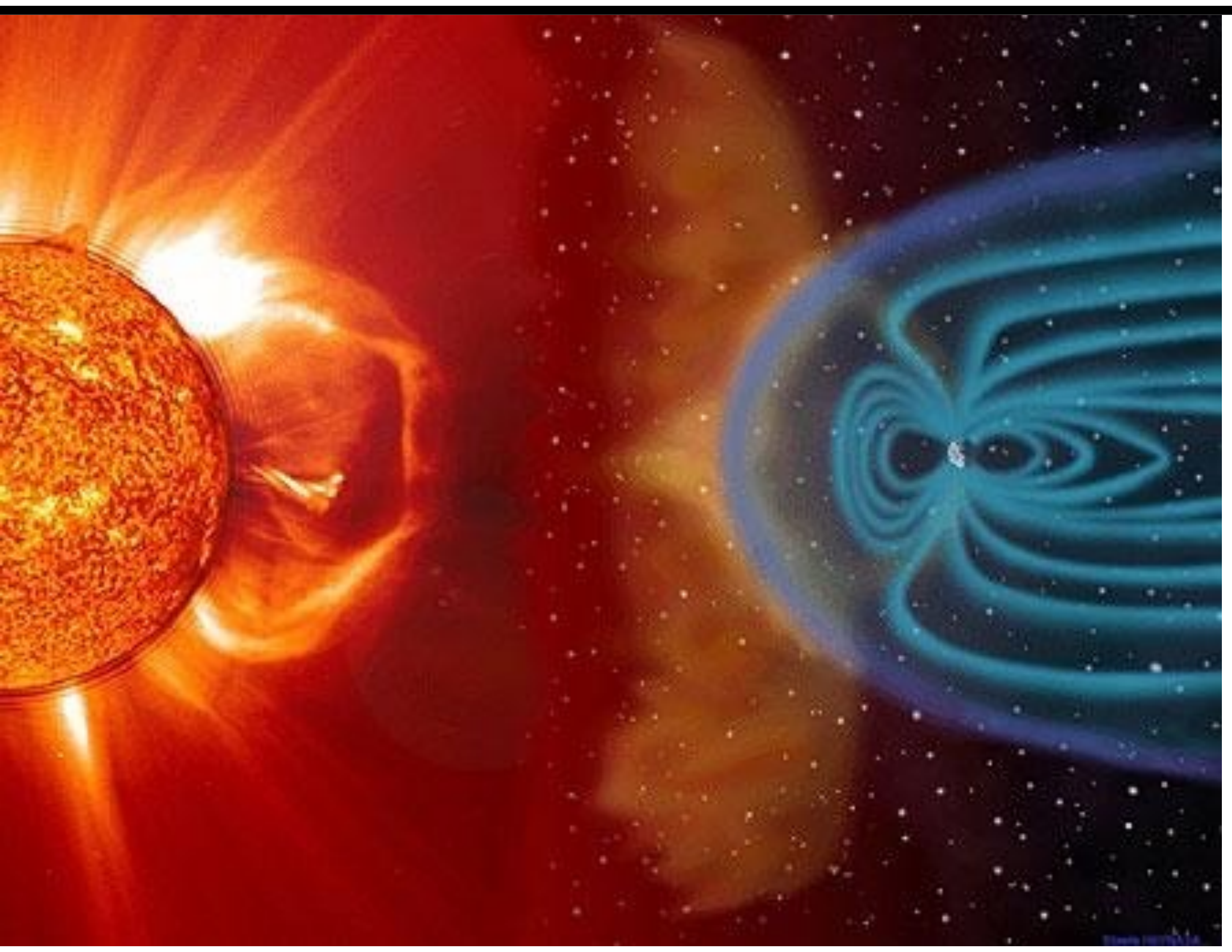


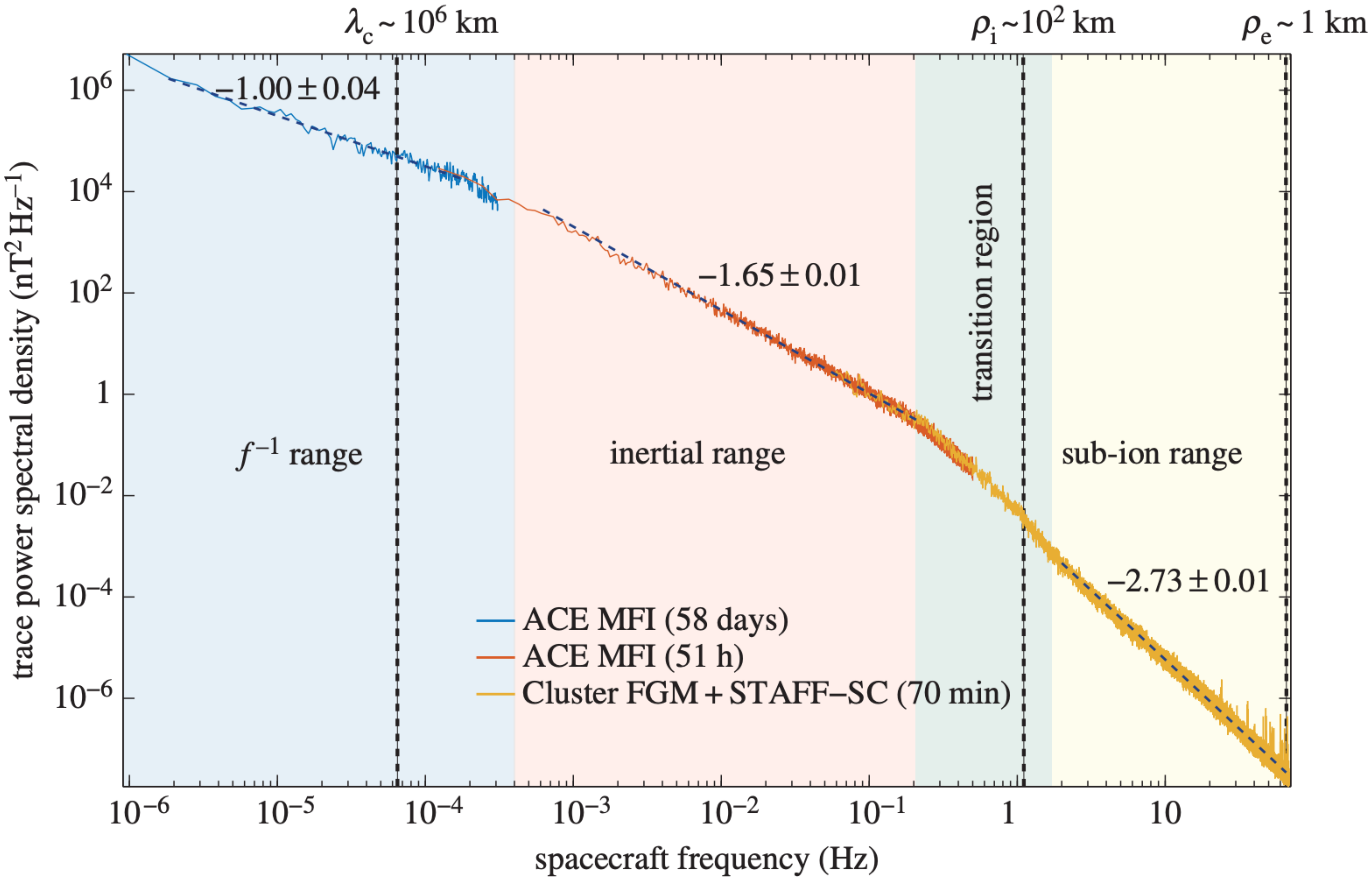


Kasper+ 2019

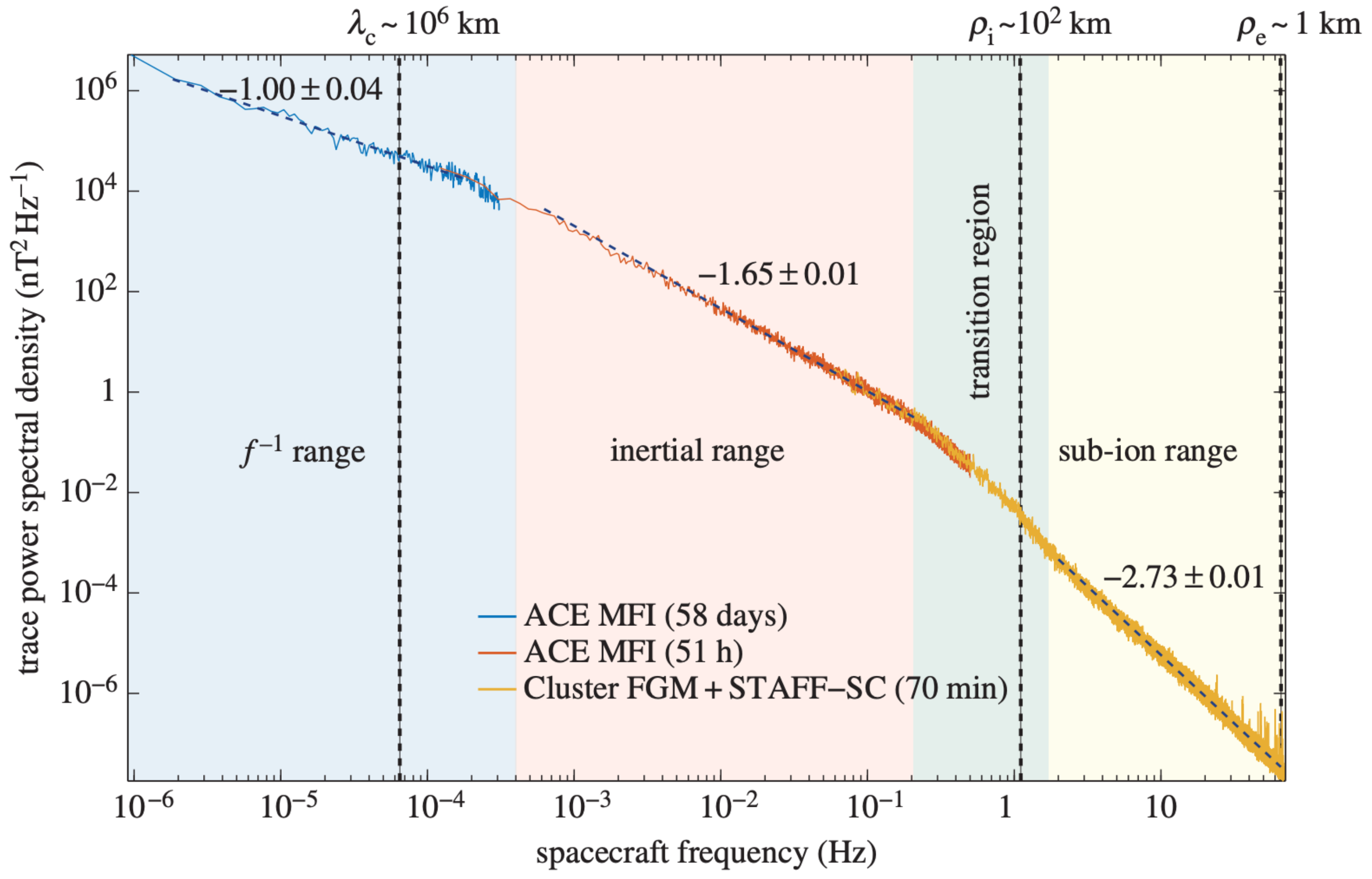
PSP Solar wind.
[Kasper+(2019)]

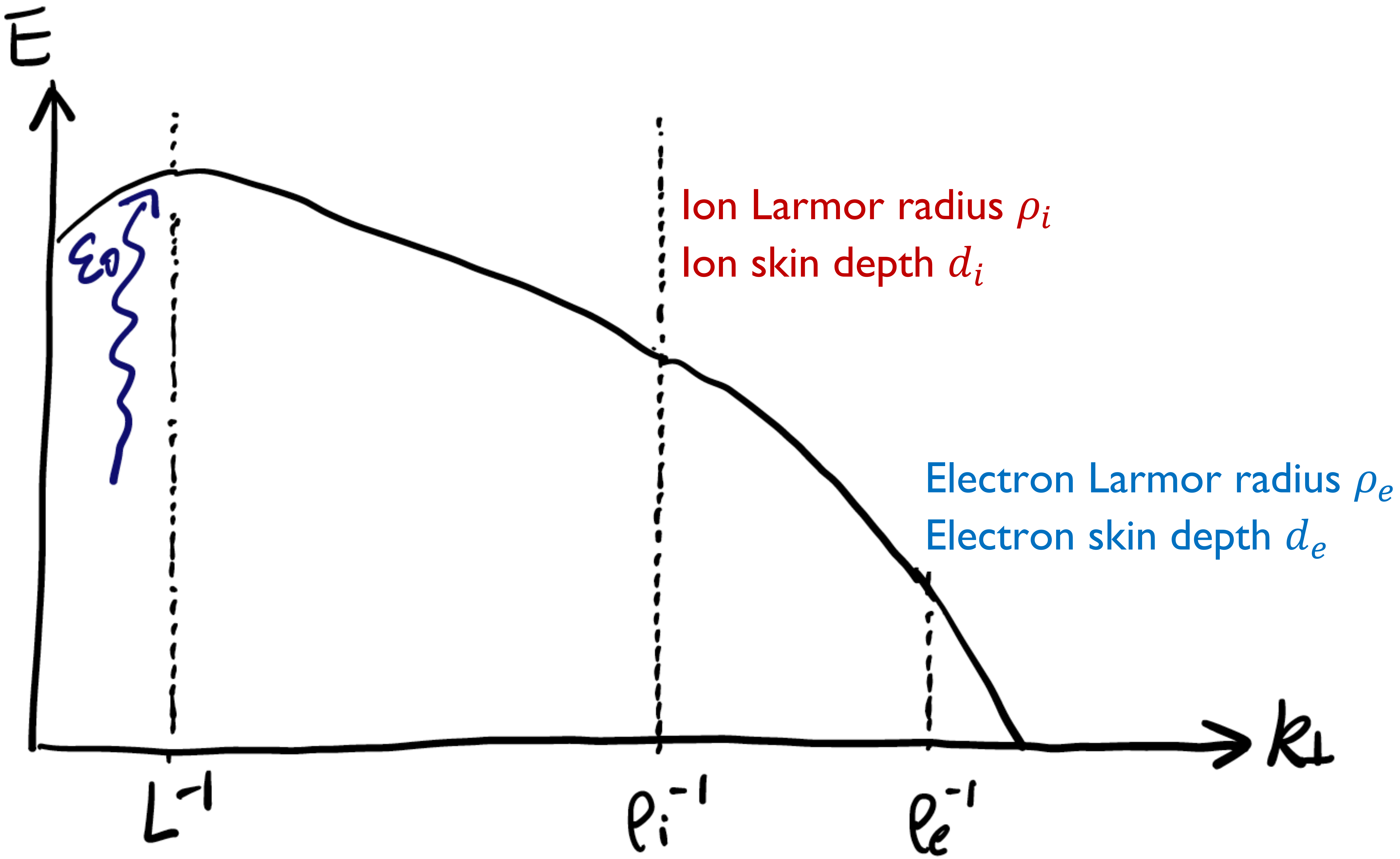
Turbulent spectrum in the Solar wind



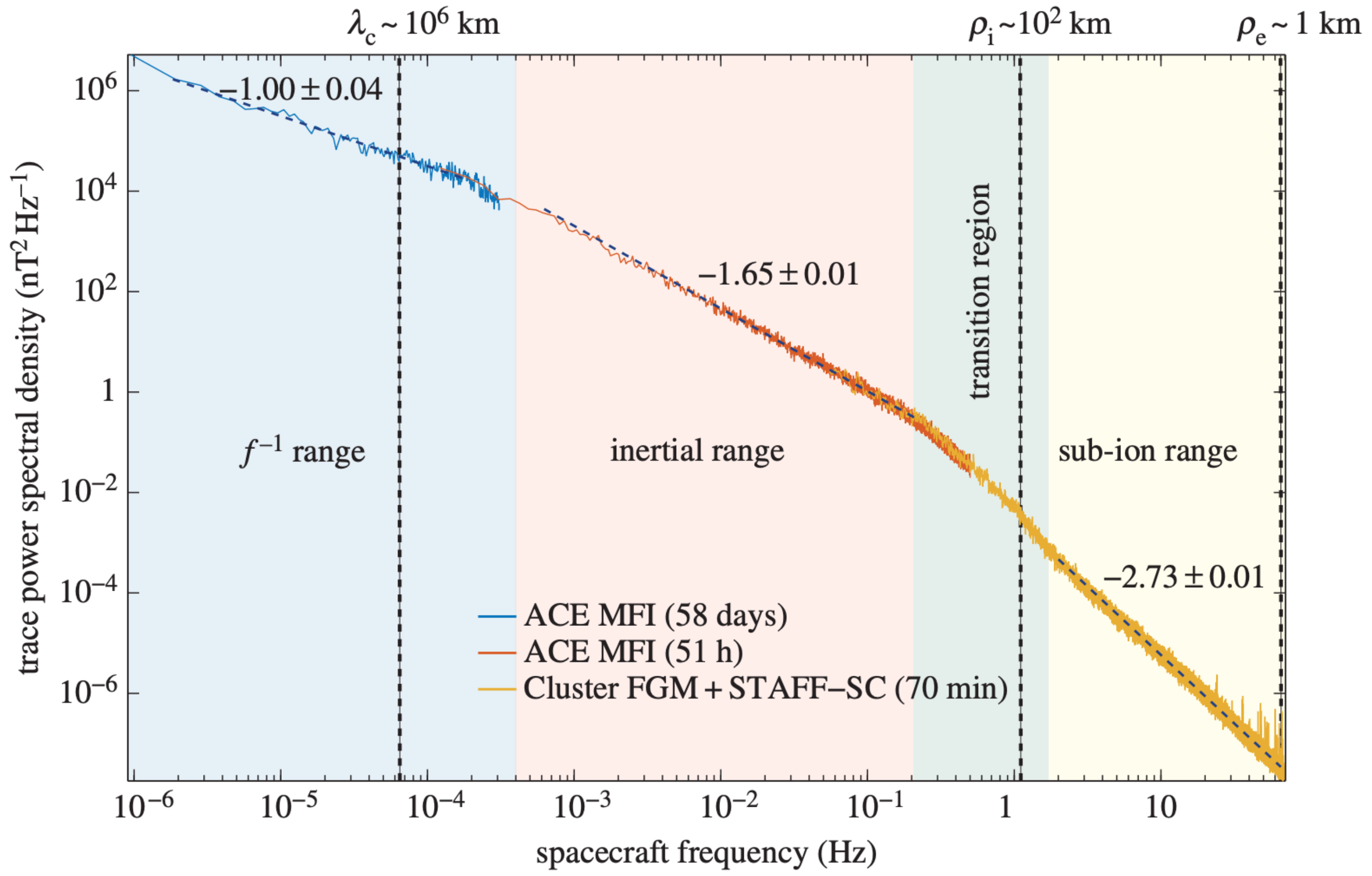


Turbulent spectrum in a theorist's mind

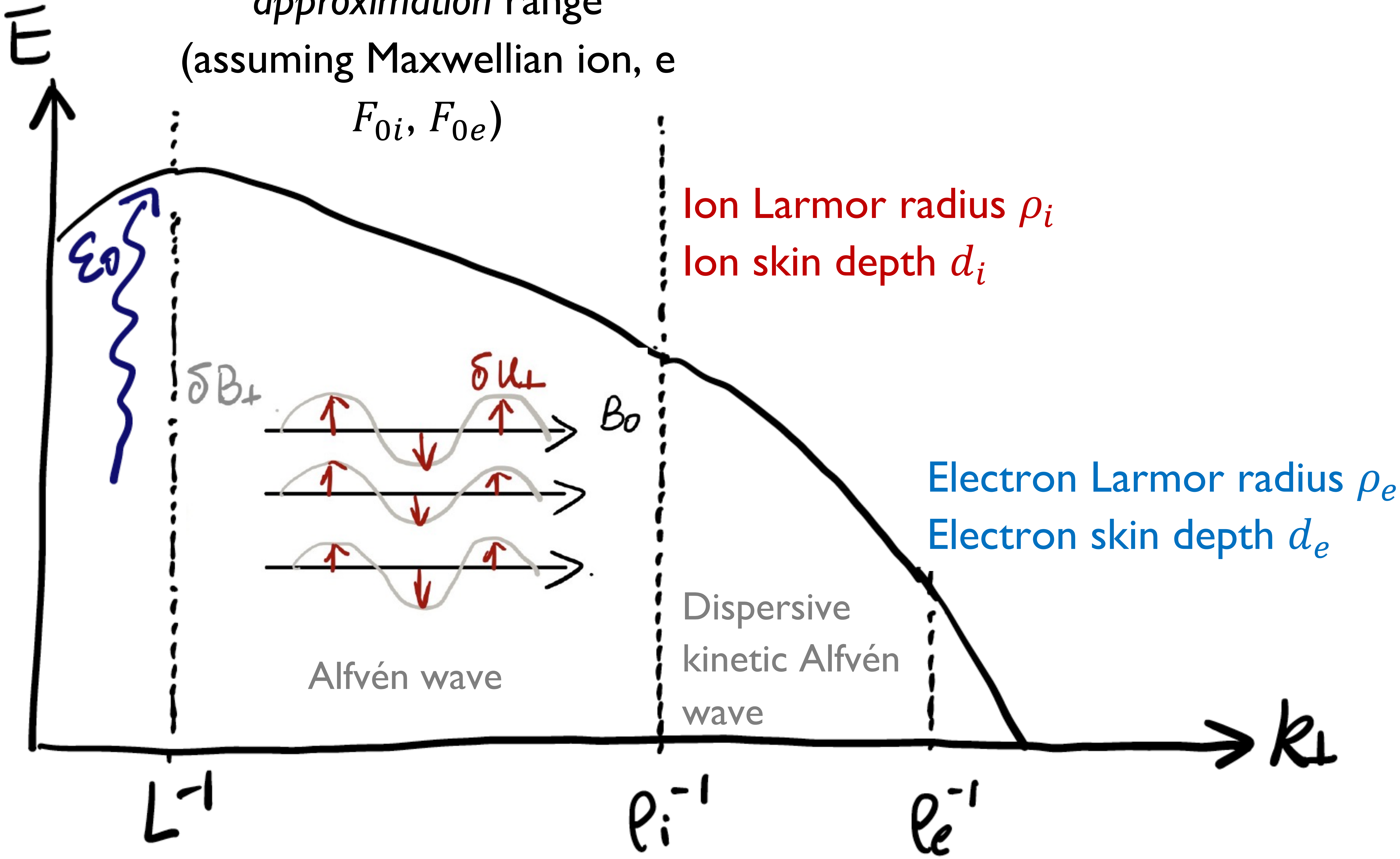




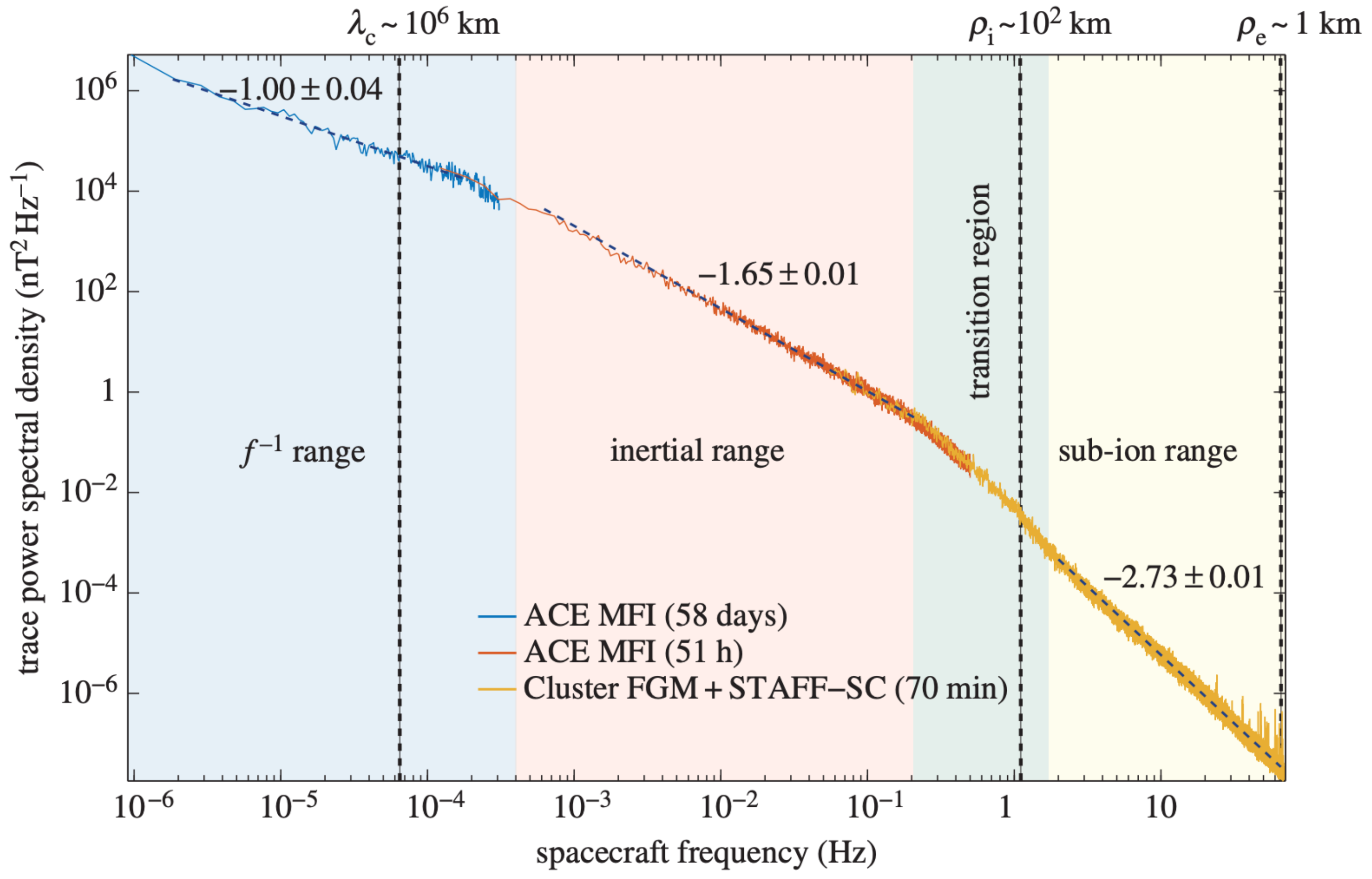
Inertial range (MHD approximation)



Magnetohydrodynamics (MHD)
approximation range
(assuming Maxwellian ion, e
F_{0i}, F_{0e})



Kinetic range (electron and ion kinetic/nonequilibrium physics matters)



Magnetohydrodynamics (MHD) *approximation* range (assuming Maxwellian ion, e F_{0i}, F_{0e})

Kinetic range

not necessarily small

$F_i = F_{0i} + \delta f_i$

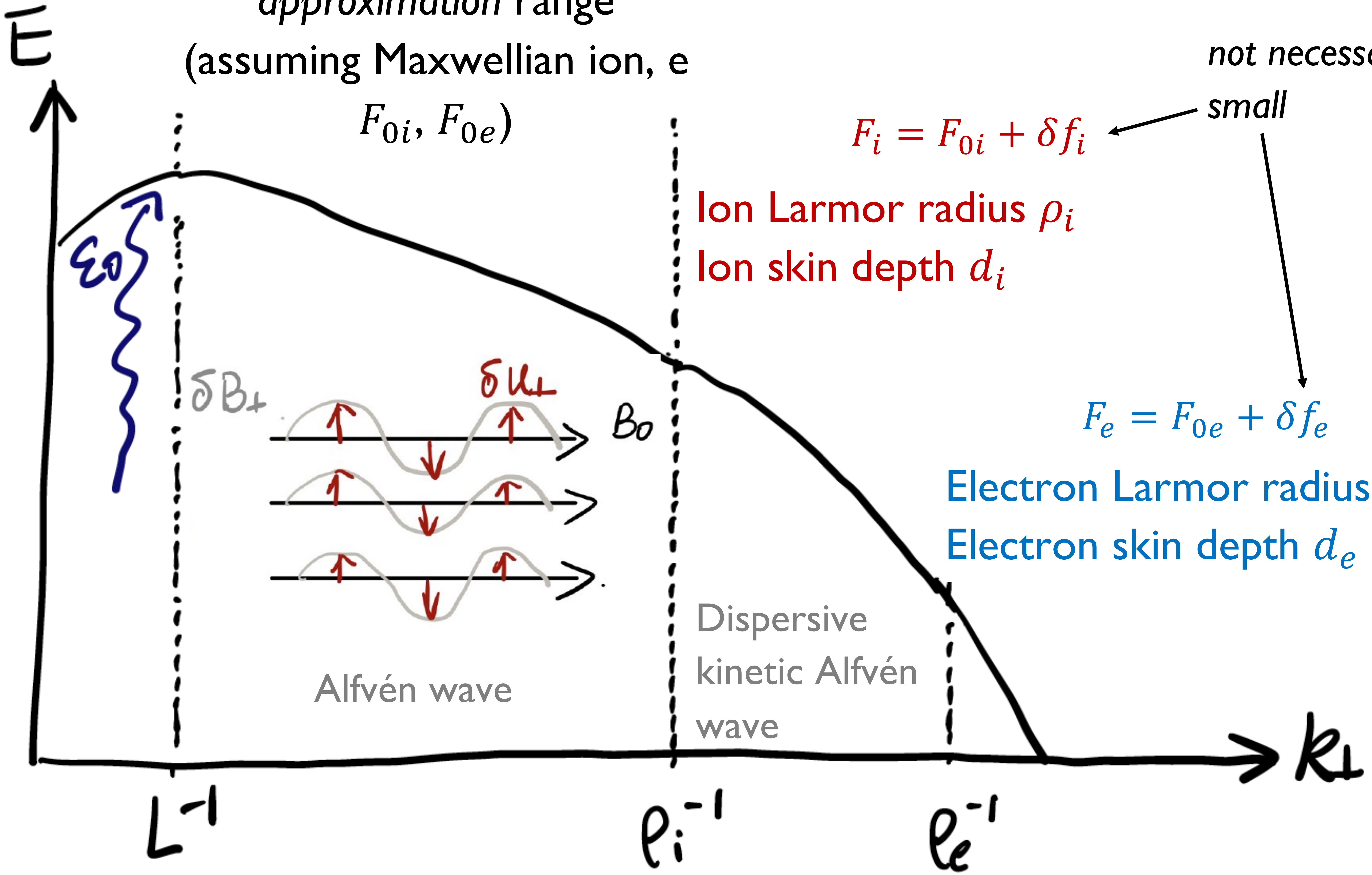
Ion Larmor radius ρ_i

Ion skin depth d_i

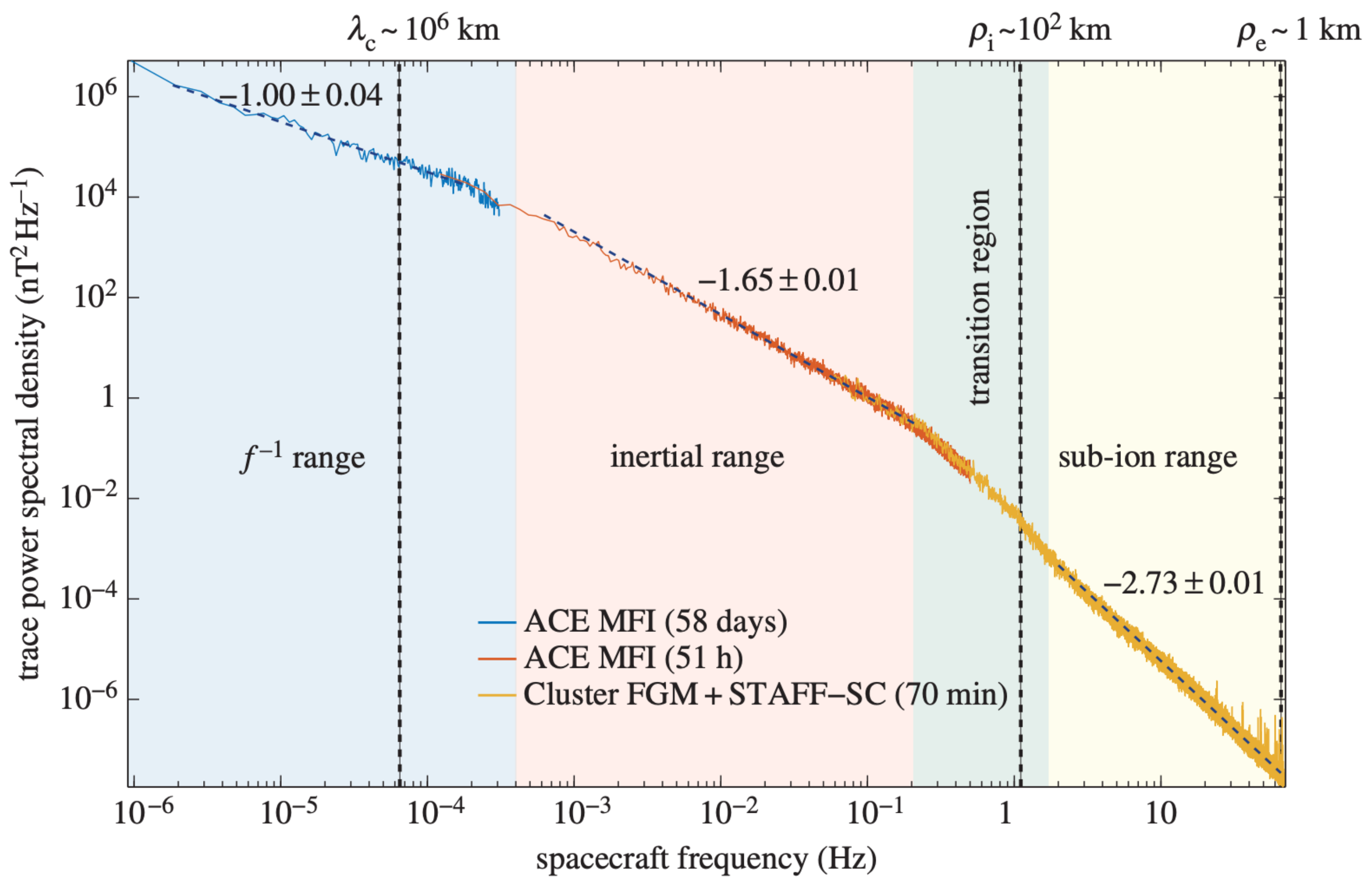
$F_e = F_{0e} + \delta f_e$

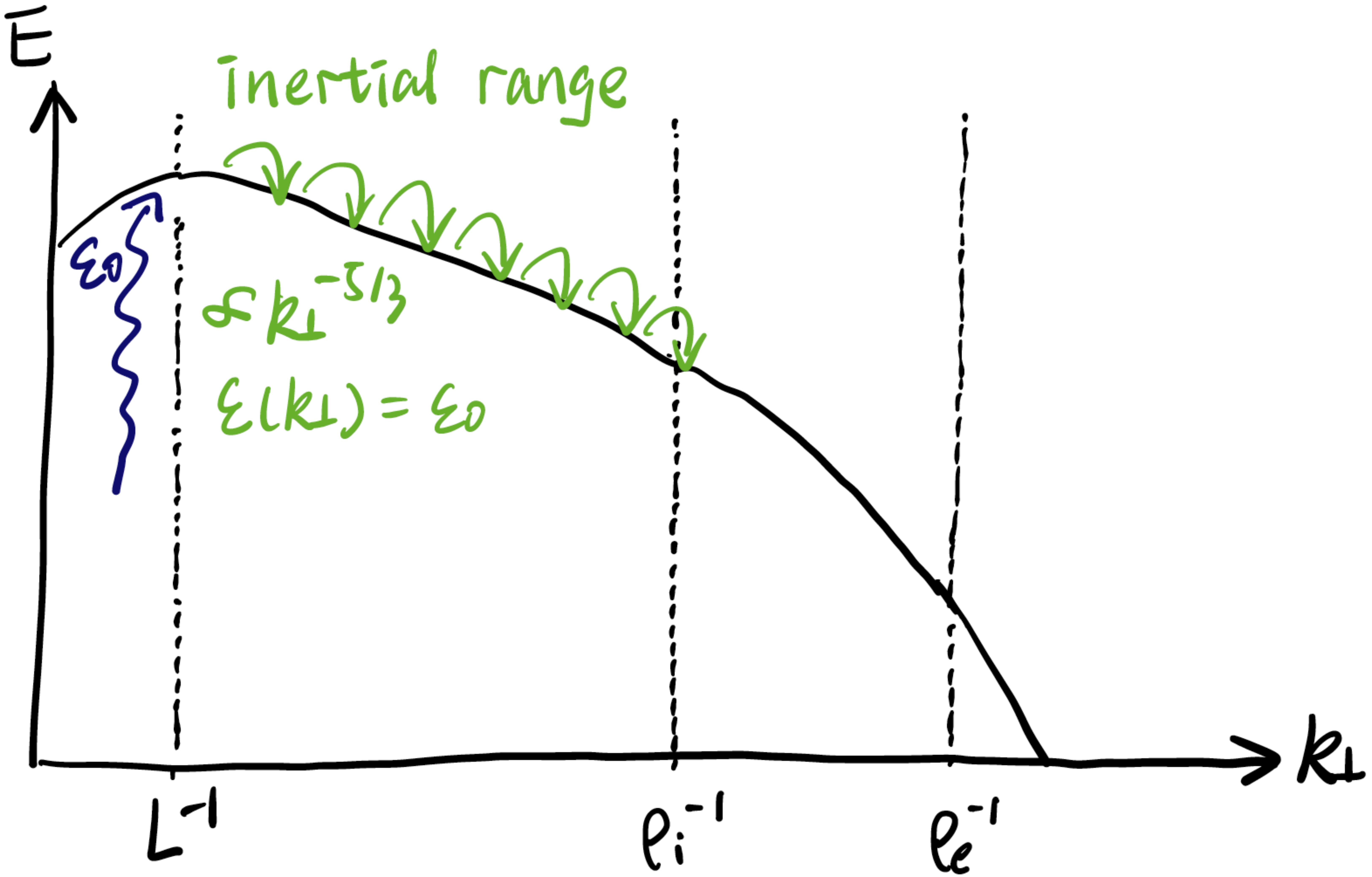
Electron Larmor radius ρ_e

Electron skin depth d_e

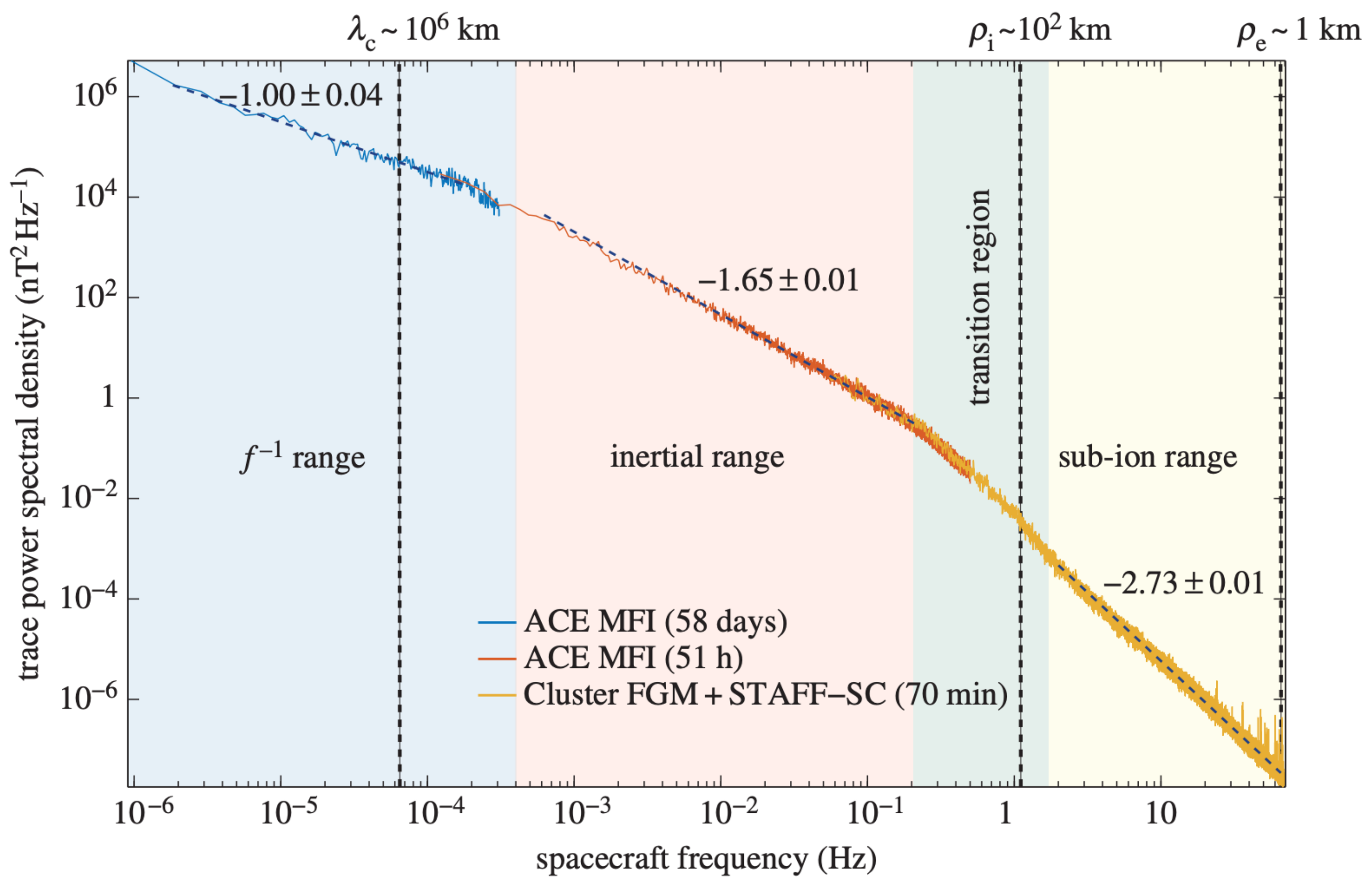


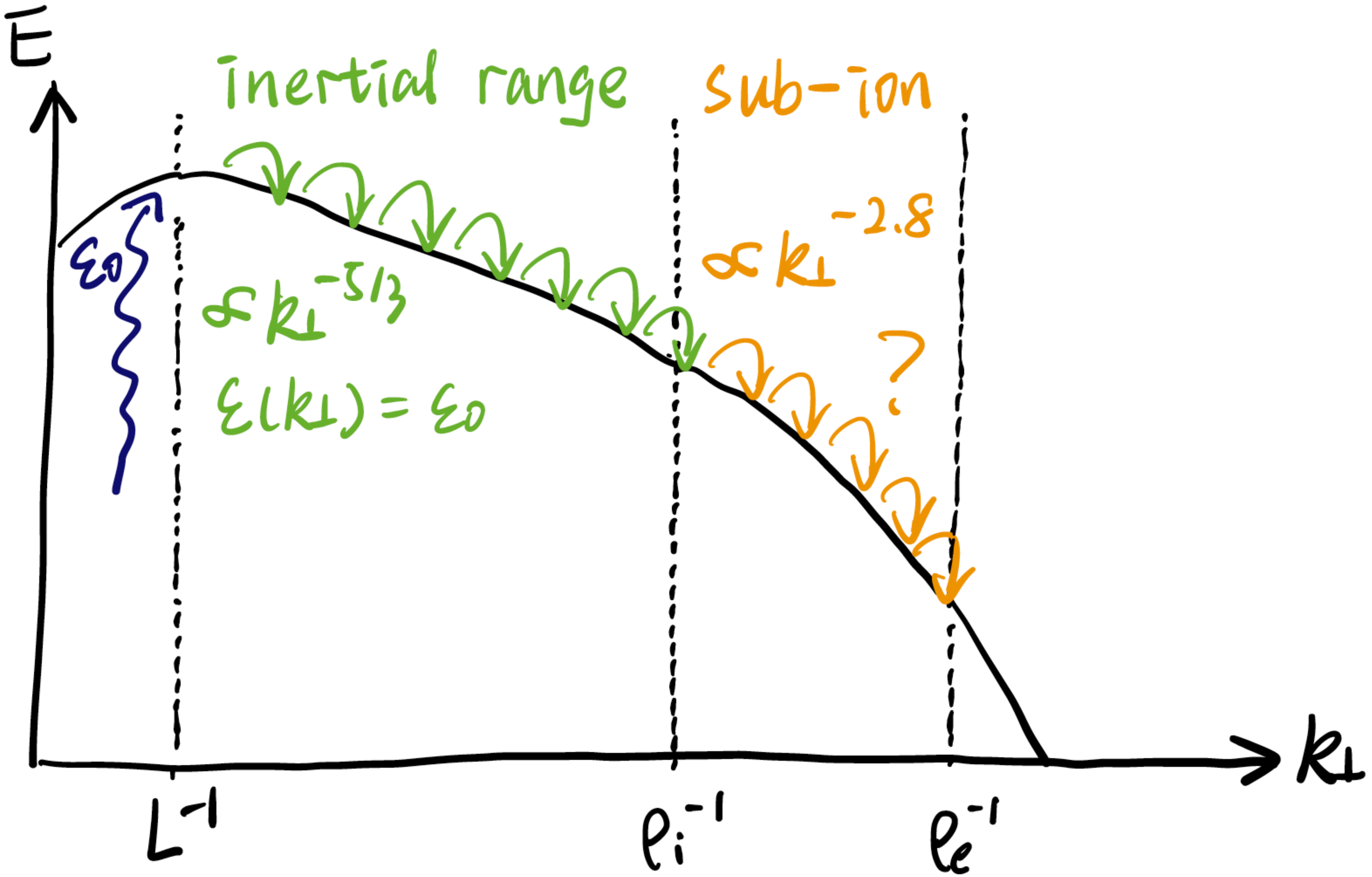
Kolmogorov-like cascade; constant energy flux



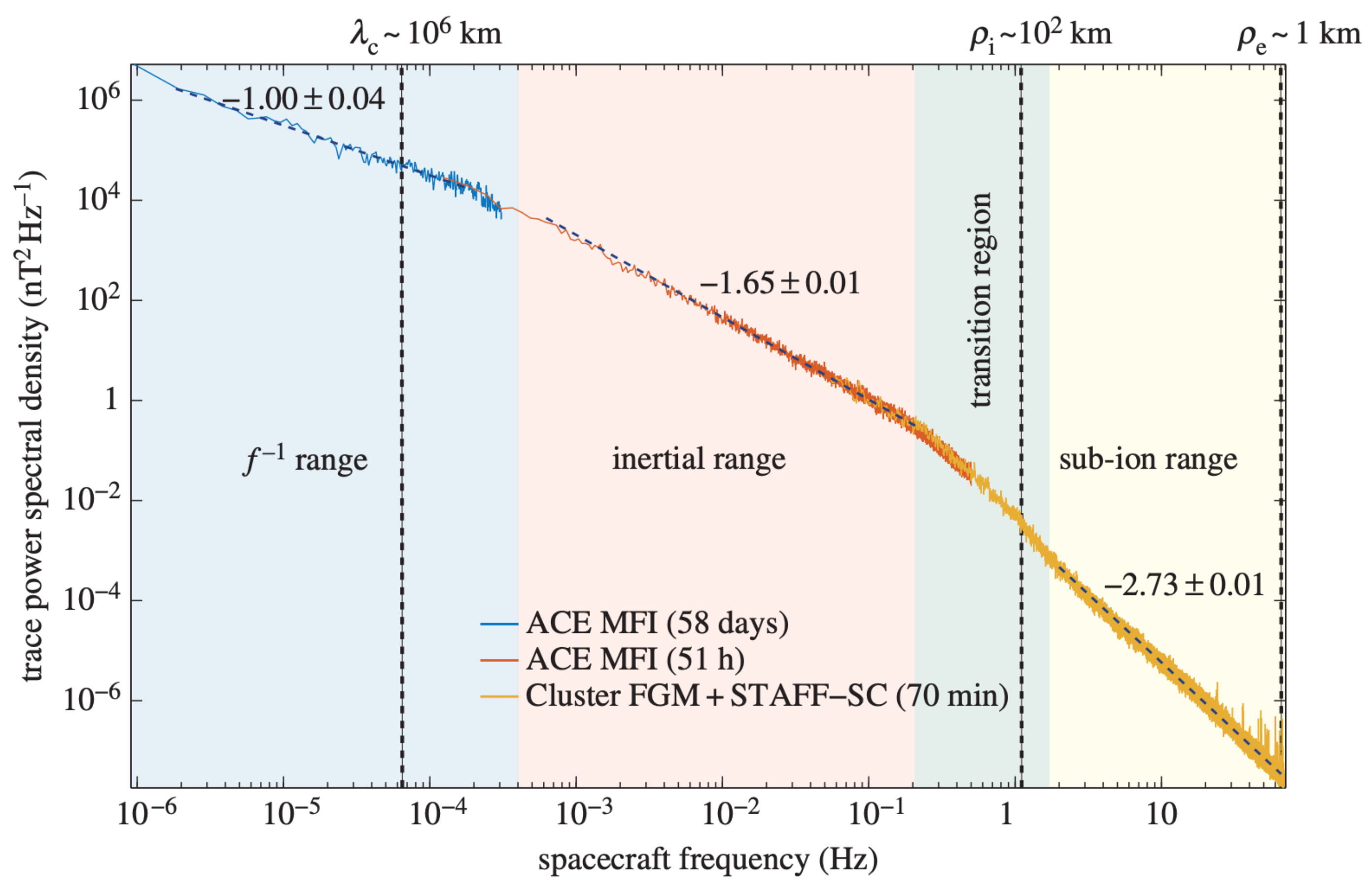


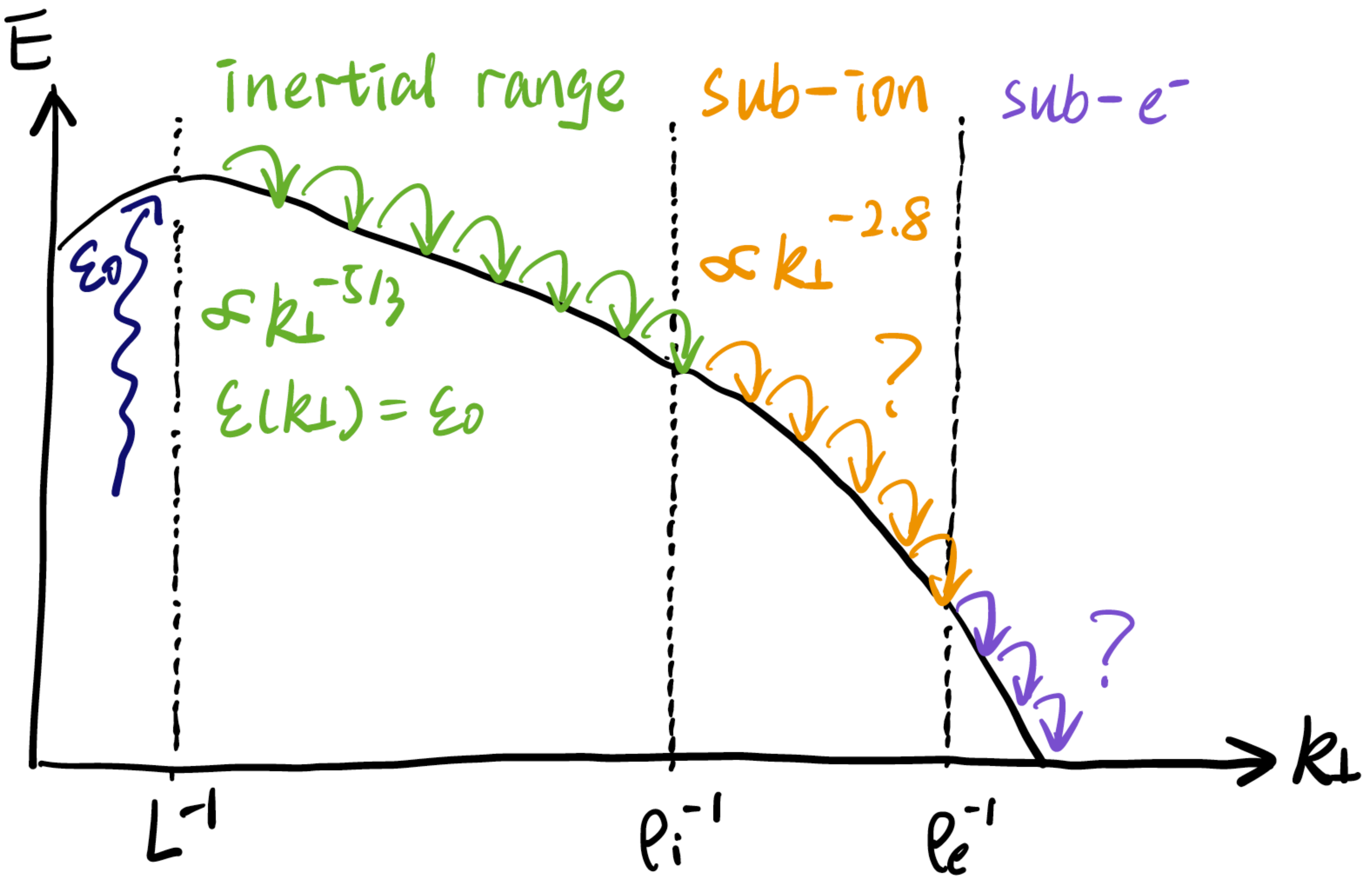
Steepened spectrum; nature of nonlinear cascade changes



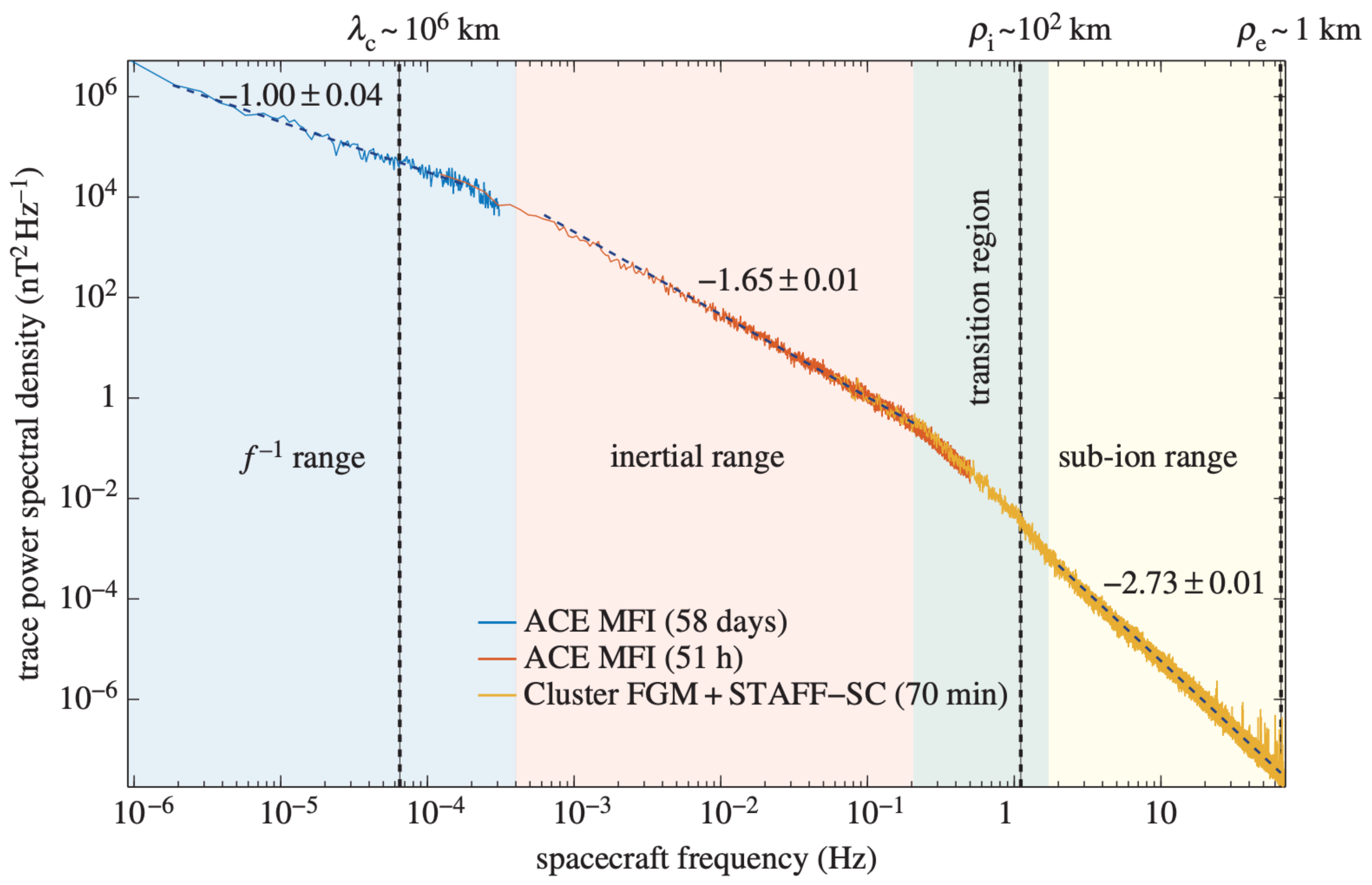


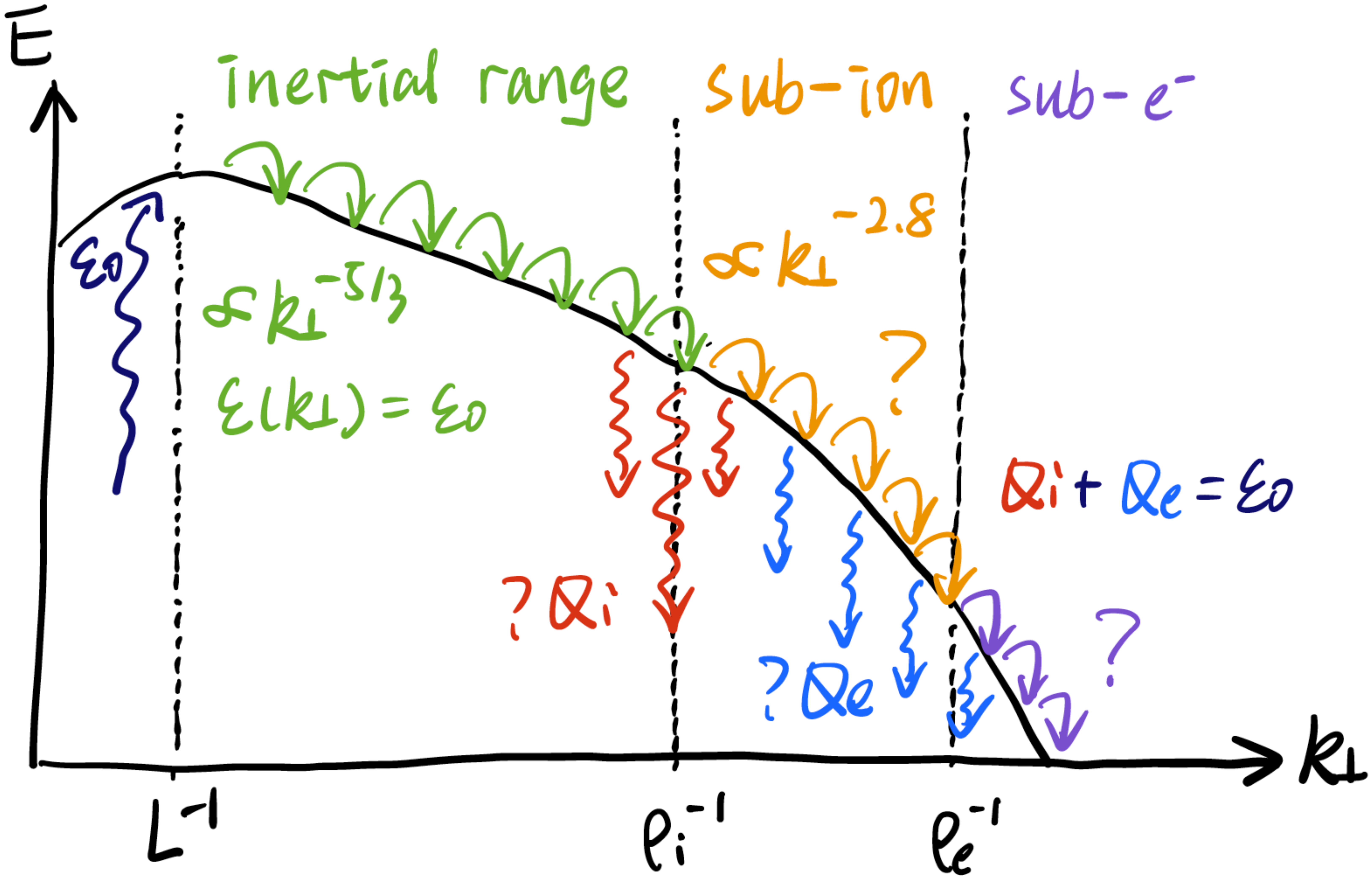
Even steeper spectrum below electron scale



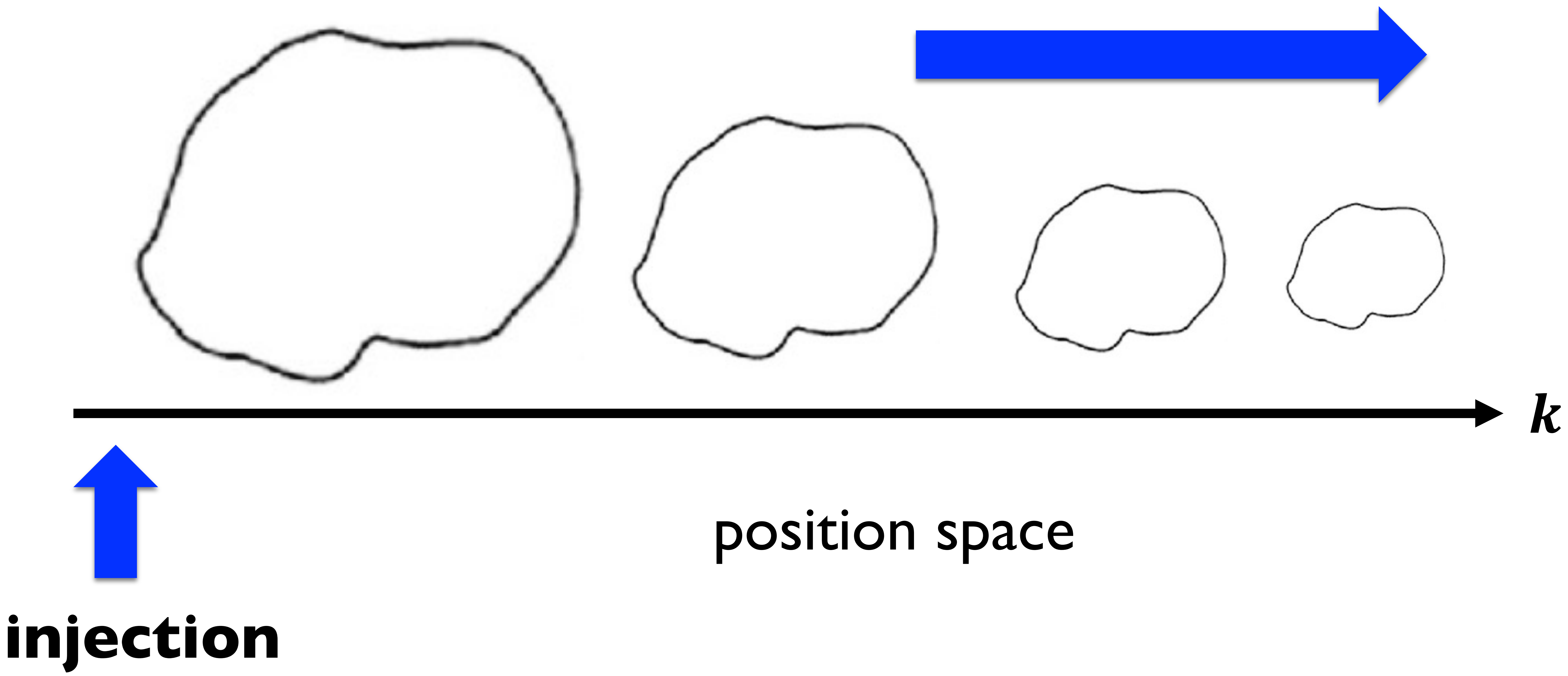


Electromagnetic energy leakage into electron and ion internal energy





Collisional plasma --- “fluid channel” of thermalization



Nonlinear advection

“fluid-channel” of dissipation

at small spatial scales

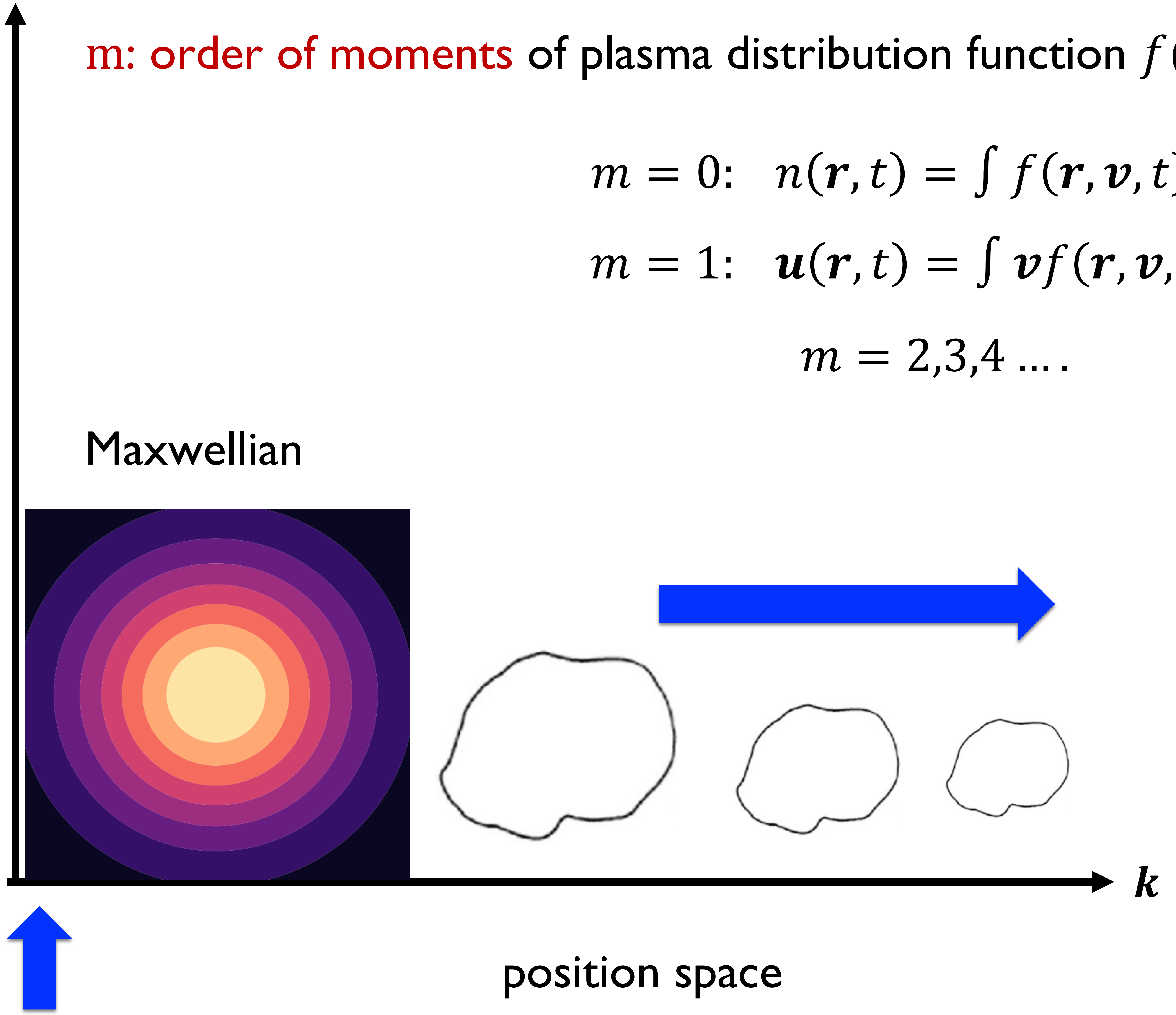
Non-Maxwellian plasma has nontrivial dynamics in velocity space

m: order of moments of plasma distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$

$$m = 0: \quad n(\boldsymbol{r}, t) = \int f(\boldsymbol{r}, \boldsymbol{v}, t) d\boldsymbol{v}$$

$$m = 1: \quad \boldsymbol{u}(\boldsymbol{r}, t) = \int \boldsymbol{v} f(\boldsymbol{r}, \boldsymbol{v}, t) d\boldsymbol{v}$$

$$m = 2,3,4 \dots .$$

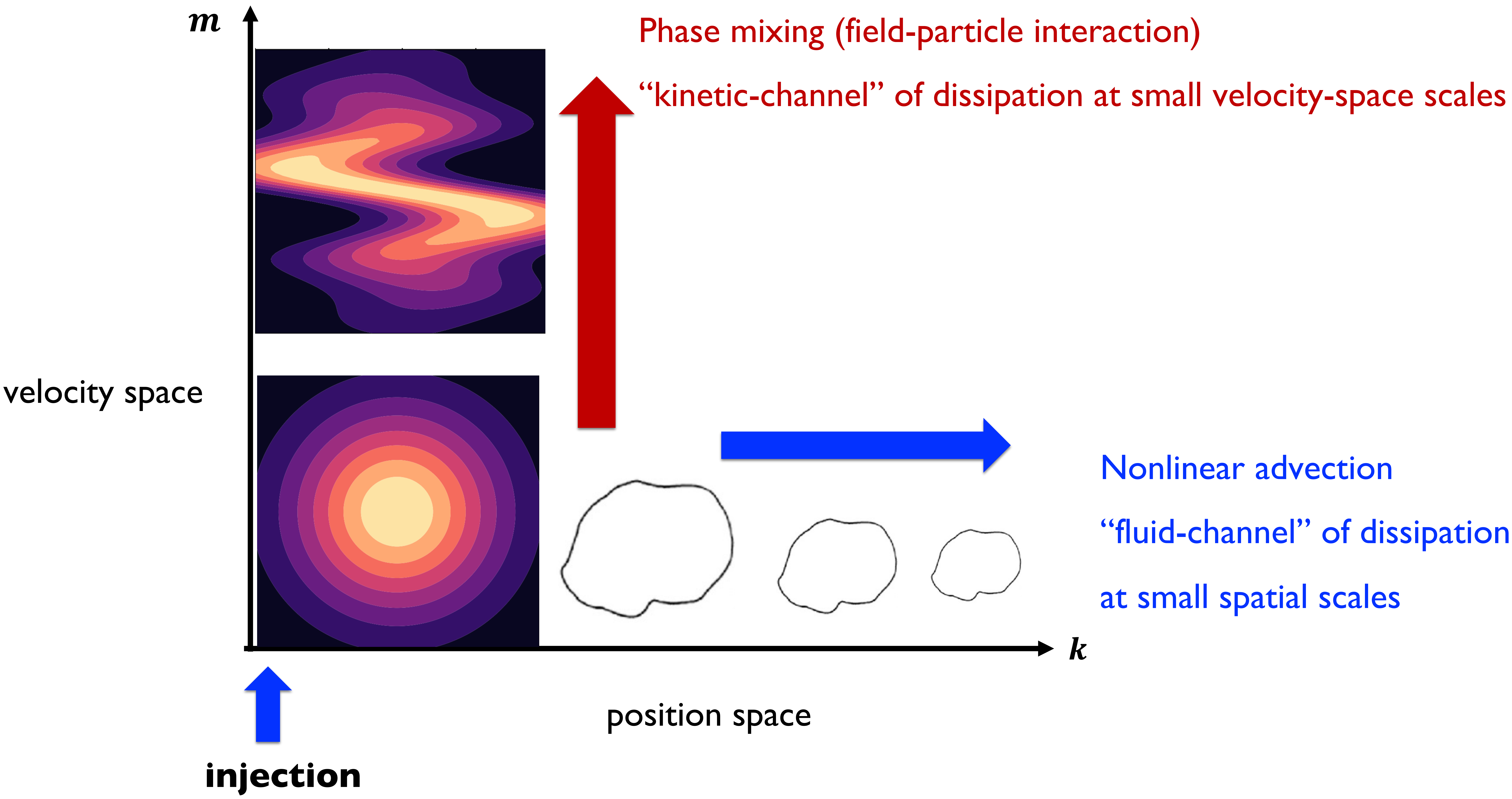


Nonlinear advection

“fluid-channel” of dissipation

at small spatial scales

“kinetic-channel” of dissipation exists for weakly collisional plasma

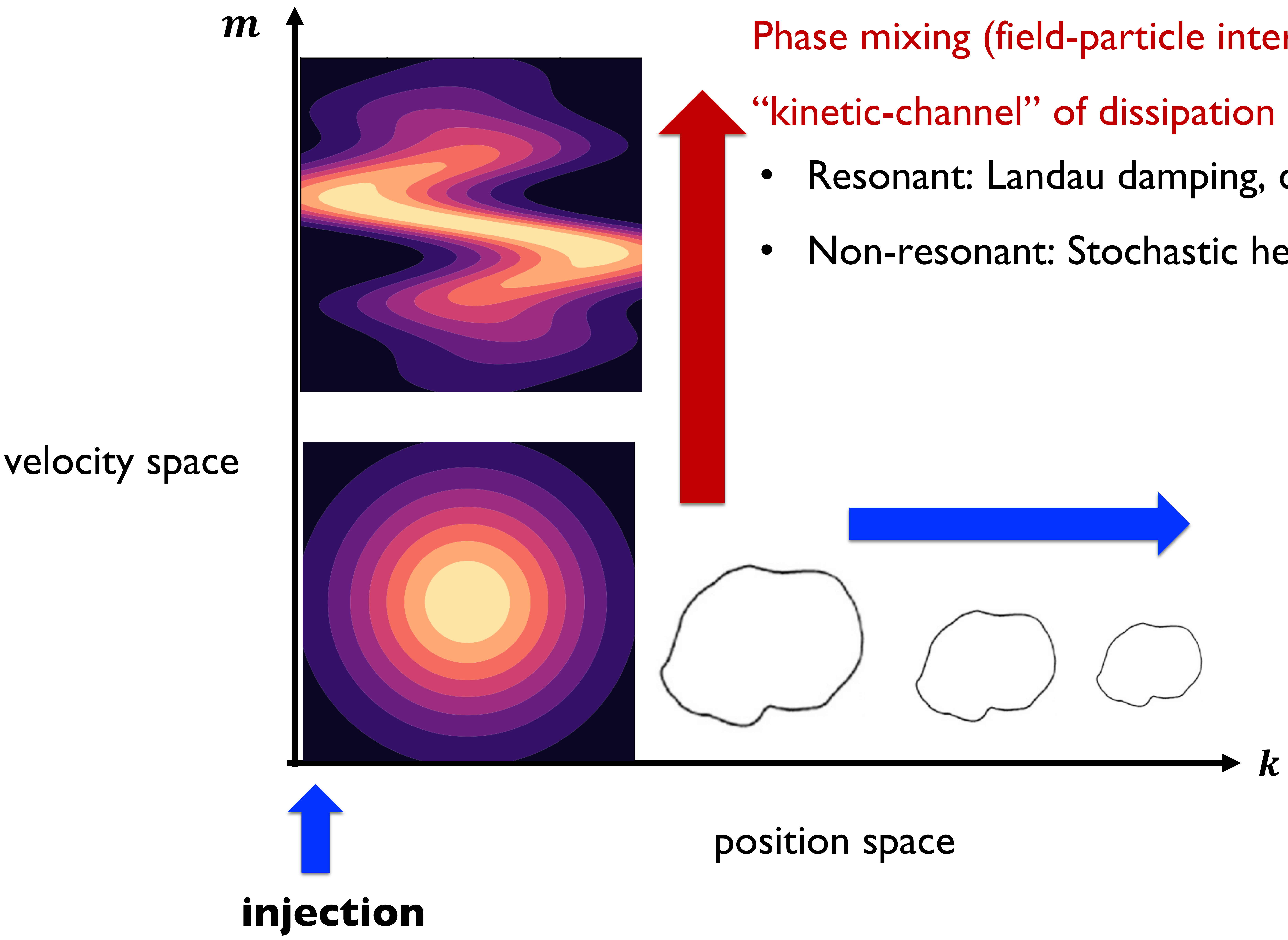


The whole phase-space dynamics is crucial for particle heating

Phase mixing (field-particle interaction)

“kinetic-channel” of dissipation at small velocity-space scales

- Resonant: Landau damping, cyclotron damping, etc.
- Non-resonant: Stochastic heating, magnetic pumping, etc

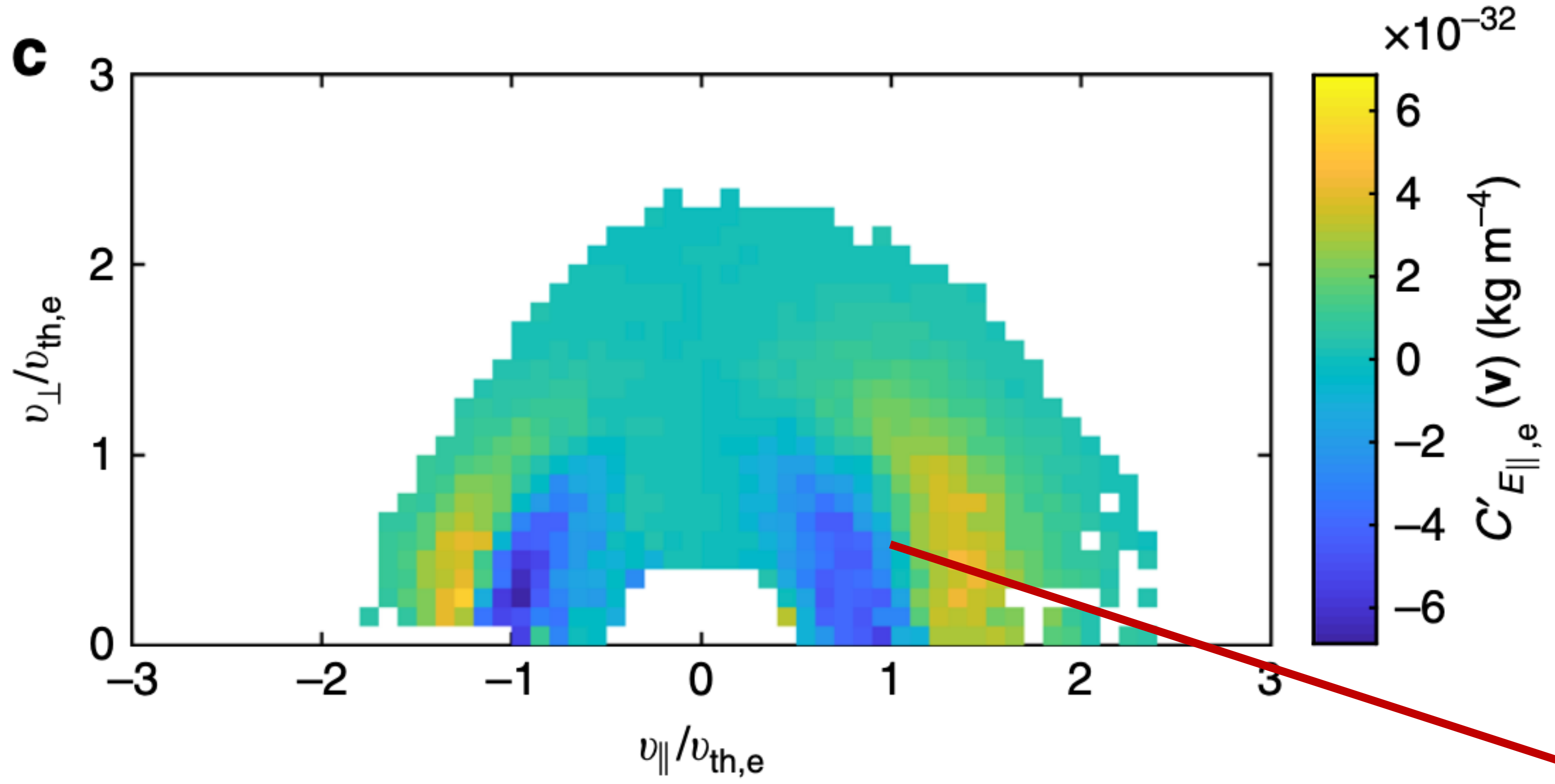


Nonlinear advection

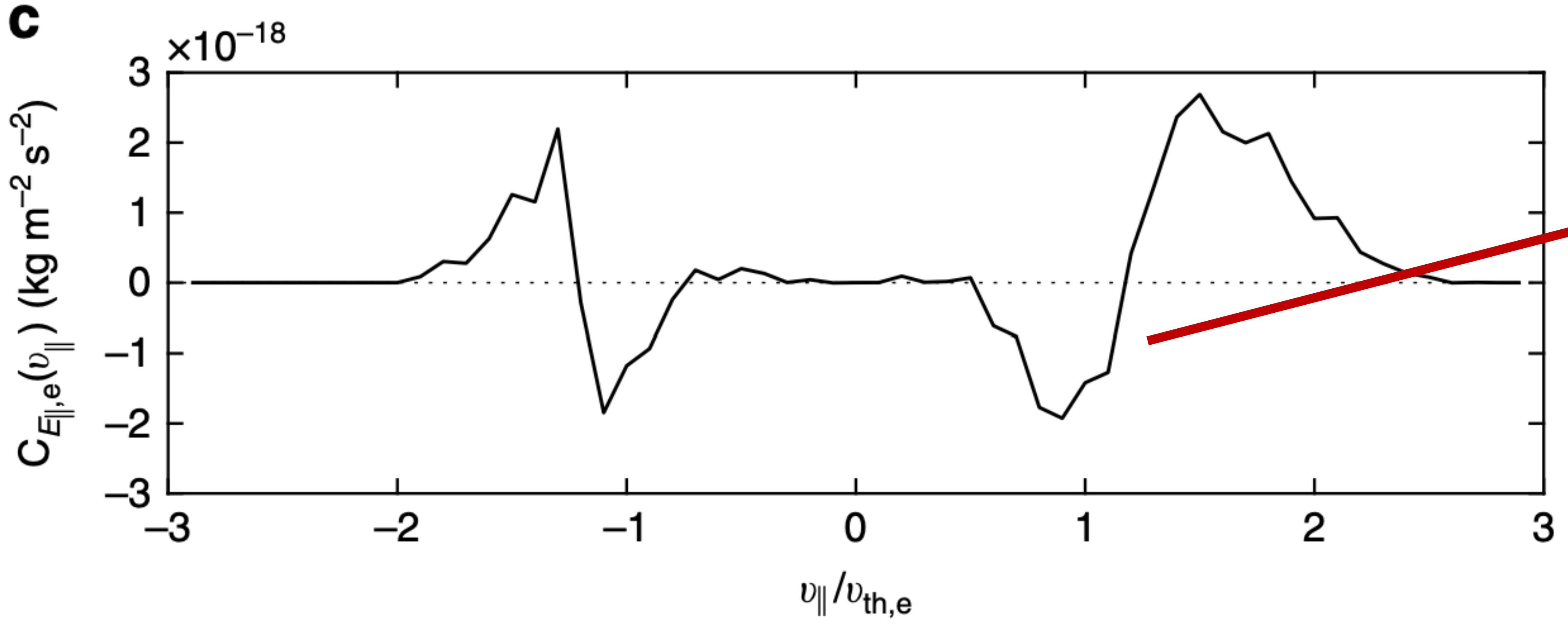
“fluid-channel” of dissipation

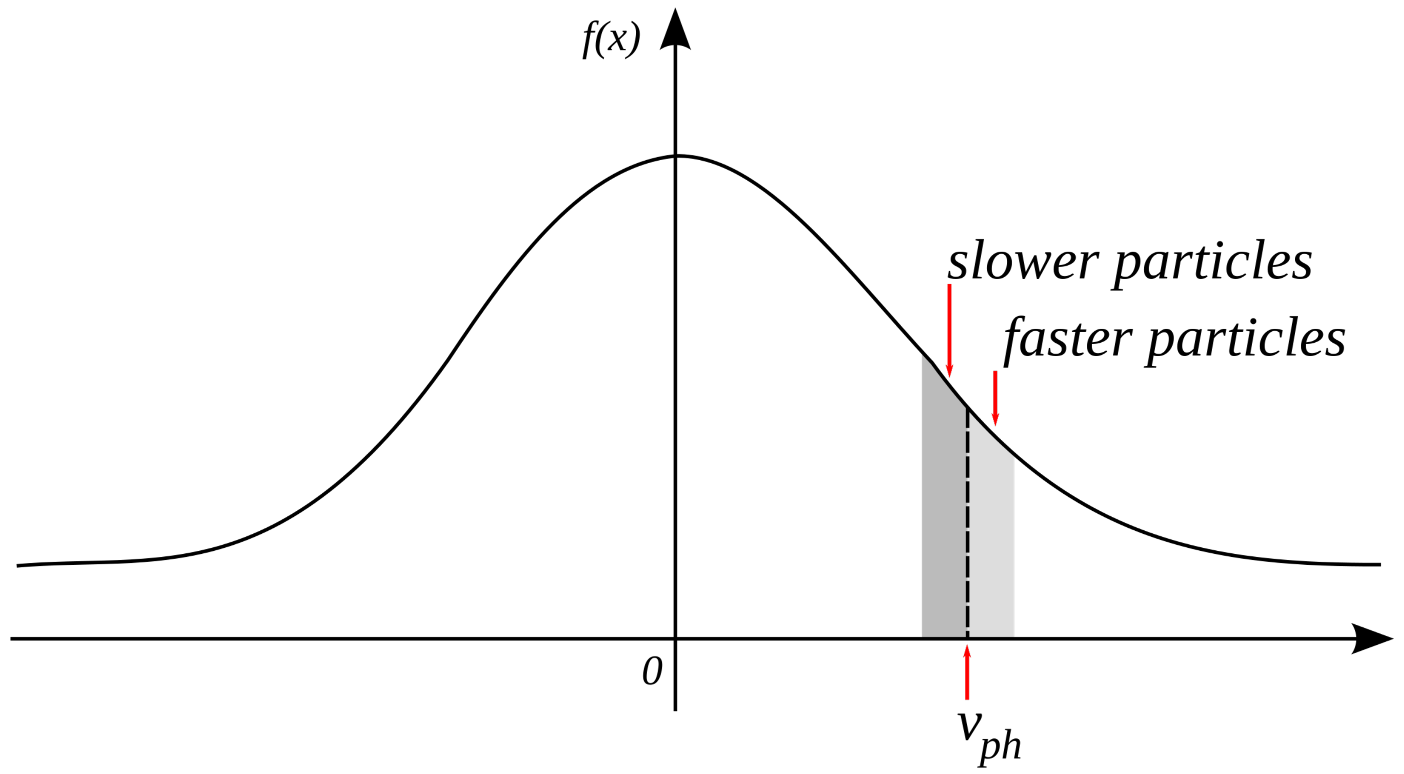
at small spatial scales

Electron Landau damping in Earth's magnetosheath turbulence by MMS



Work done by particles to electromagnetic fluctuations





resonant condition

$$\omega_{\mathrm{KAW}} = k_{\parallel} v_{th,e}$$

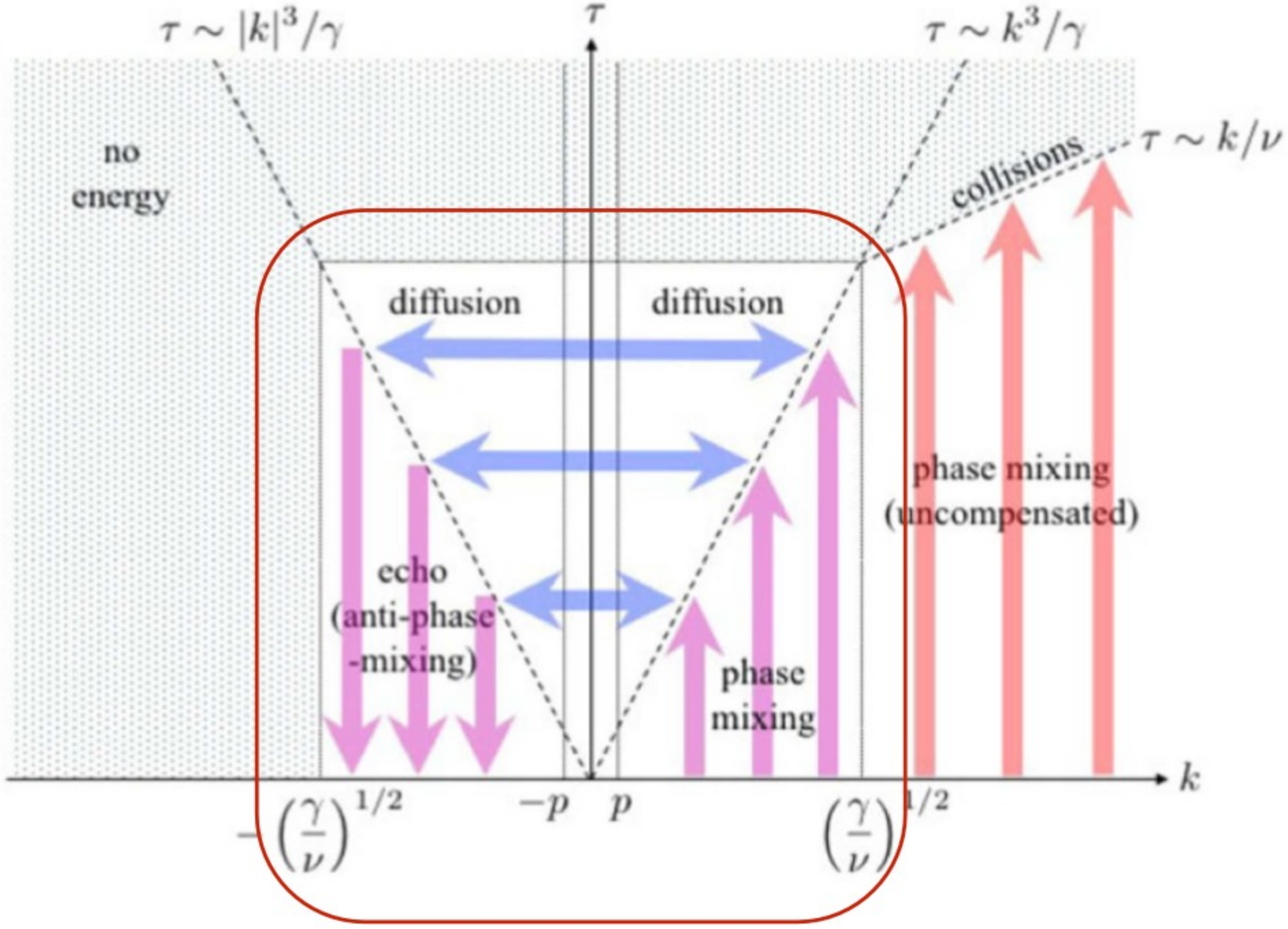
Landau damping of kinetic Alfvén wave (KAW)

Formation of plateau in the particle distribution

MMS magnetosheath. [Chen+(2019) Nature]

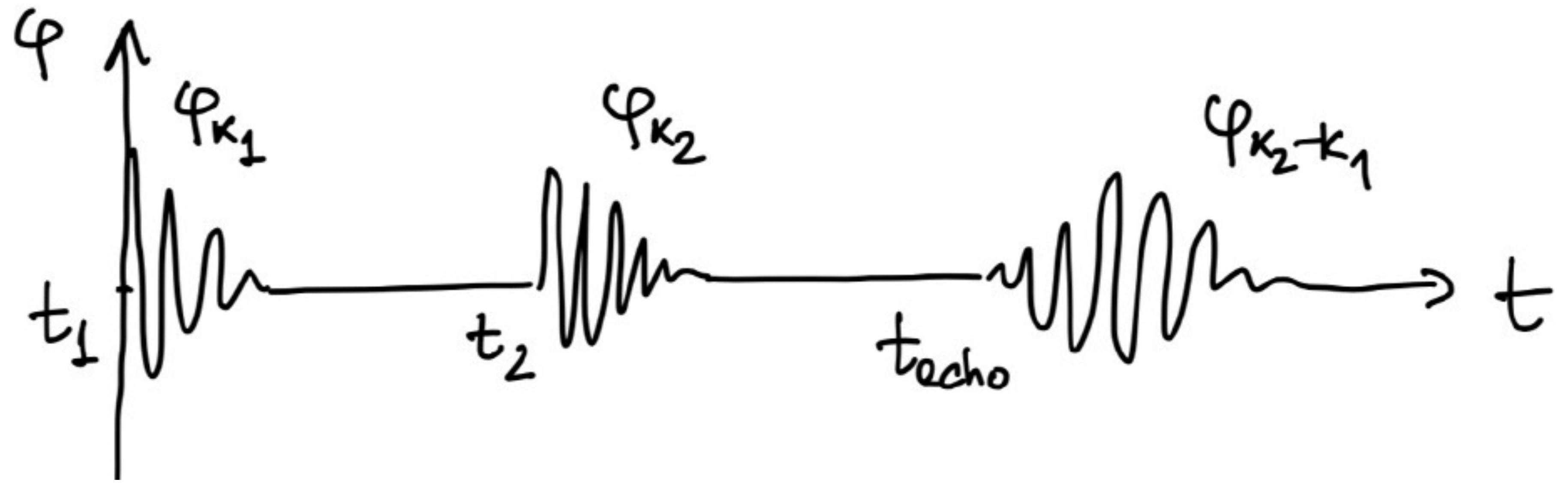
Anti-phase-mixing and plasma echoes

Schekochihin+ 2016; Adkins & Schekochihin 2018



Experimentally observed plasma echo:

[Malmberg+ 1967]



$$\delta f_{k_1} = a_1 e^{-ik_1 v(t-t_1)}, \qquad \delta f_{k_2} = a_2 e^{-ik_2 v(t-t_2)}.$$

Nonlinear coupling:

$$\delta f_{k_2-k_1} \propto \delta f_{k_2} \delta f_{k_1}^* = a_1^* a_2 e^{-ik_2 v(t-t_2)+ik_1 v(t-t_1)}$$

$$= a_1^* a_2 e^{-i[(k_2-k_1)t-(k_2 t_2 - k_1 t_1)]v}$$

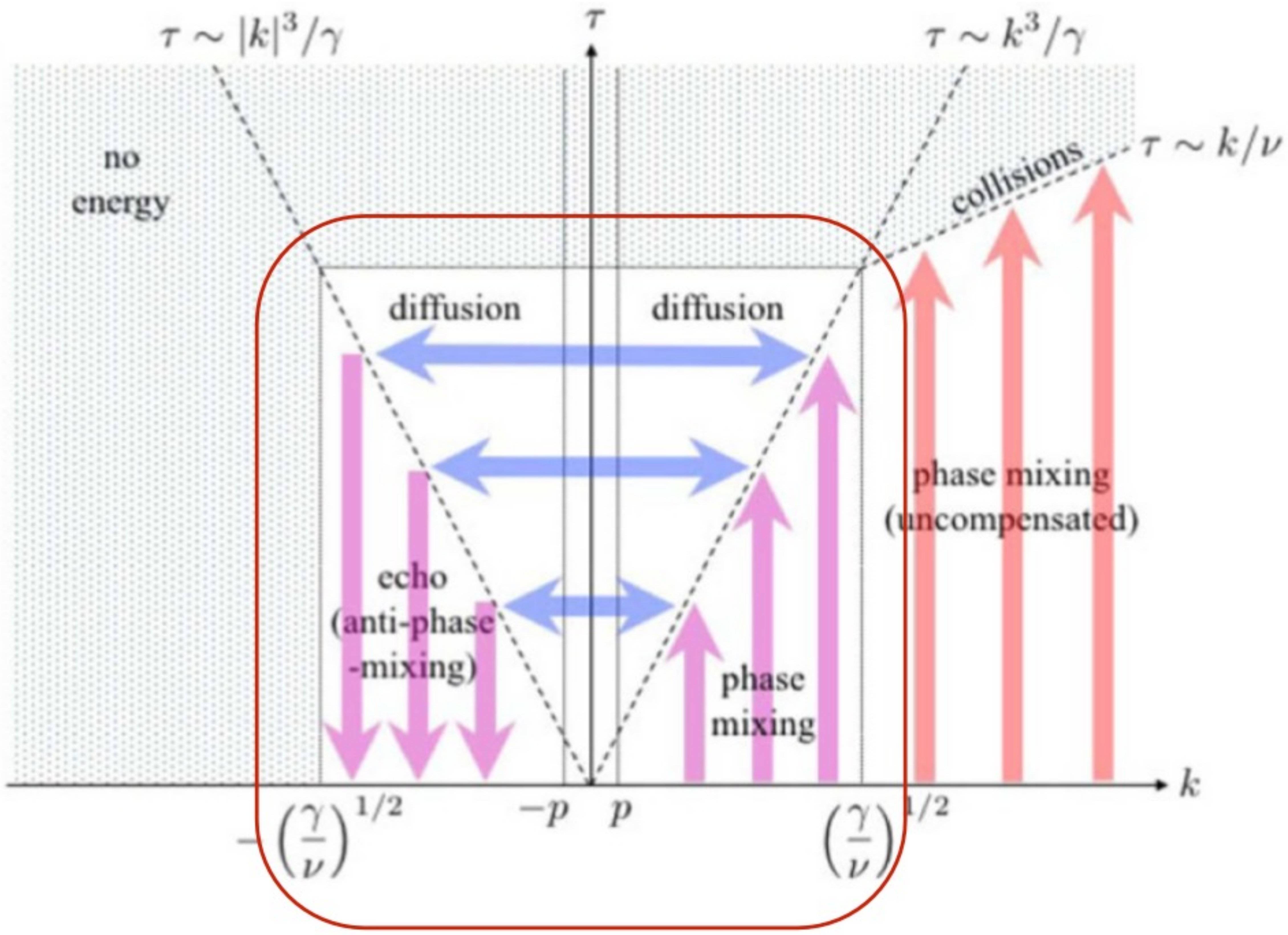
$$\boxed{t_{\text{echo}} = \frac{k_2 t_2 - k_1 t_1}{k_2 - k_1}},$$

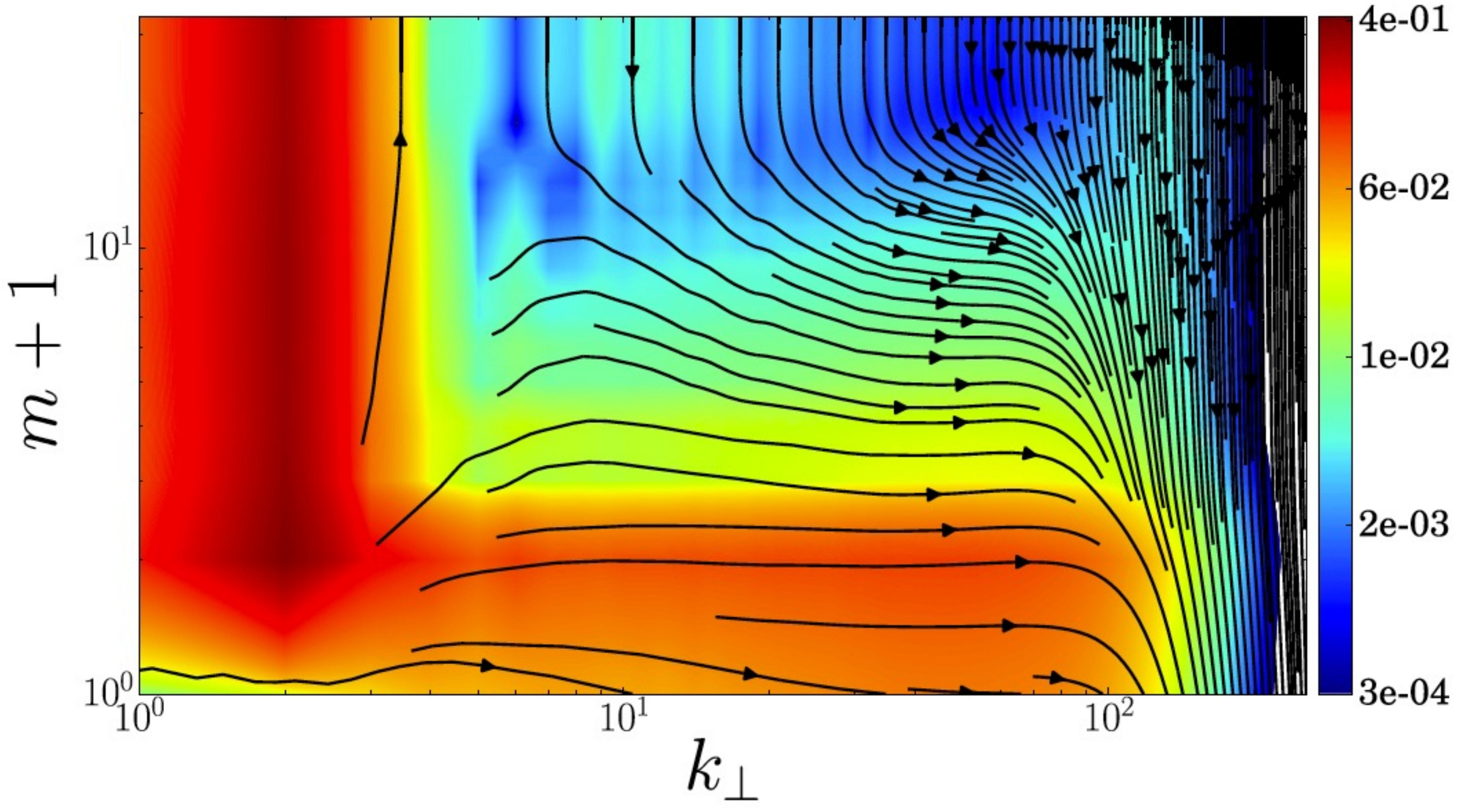
Measurable echo signal $\quad \varphi_{k_2-k_1} \propto \int \mathrm{d}v\, \delta f_{k_2-k_1}$

[adapted from Schekochihin's kinetic lecture notes]

Anti-phase-mixing and plasma echoes

Schekochihin+ 2016; Adkins & Schekochihin 2018





Confirmed by simulations in the inertial range with compressive fluctuations [Meyrand+ 2018]

Also confirmed by *electrostatic*, drift-kinetic simulations in the sub-ion range [Parker+ 2016]

Q: What about in the ion range with electromagnetic fluctuations?

Kinetic Reduced Electron Heating Model (KREHM) framework

Fully kinetic → Gyro-kinetics → (low-β, $\beta_e \sim m_e/m_i$) → KREHM

Simplest yet rigorous eqns containing fluid & kinetic physics [Zocco+ 2011]

Ions become isothermal and electrostatic, only introducing ρ_i scale.

Electrons are described by δf_e (deviation from F_{0e})

m: order of moments

Continuity
$$\frac{1}{n_{0e}}\frac{d\delta n_e}{dt} = -\hat{b}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\parallel$$

Ohm's law
$$\frac{d}{dt}(A_\parallel - d_e^2\nabla_\perp^2 A_\parallel) = -c\frac{\partial\varphi}{\partial z} + \frac{cT_{e0}}{e}\hat{b}\cdot\nabla\left(\frac{\delta n_e}{n_{0e}} + \frac{\delta T_{\parallel e}}{T_{0e}}\right)$$

⋮

δf_e ↓

$m = 0$: δn_e ↓

$m = 1$: $u_{\parallel e}$ $(A_\parallel)$ ↓

$m = 2$: $\delta T_{\parallel e}$ ↓

⋮

Isothermal closure for electrons (no leakage to velocity space)

Fully kinetic → Gyro-kinetics → (low-β, $\beta_e \sim m_e/m_i$) → KREHM

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δf_e

$m = 0$ δn_e

$m = 1$ $u_{\parallel e}$ $(A_\parallel)$

$m = 2$ $\delta T_{\parallel e}$

$\vdots$

Isothermal closure for electrons (no leakage to velocity space)

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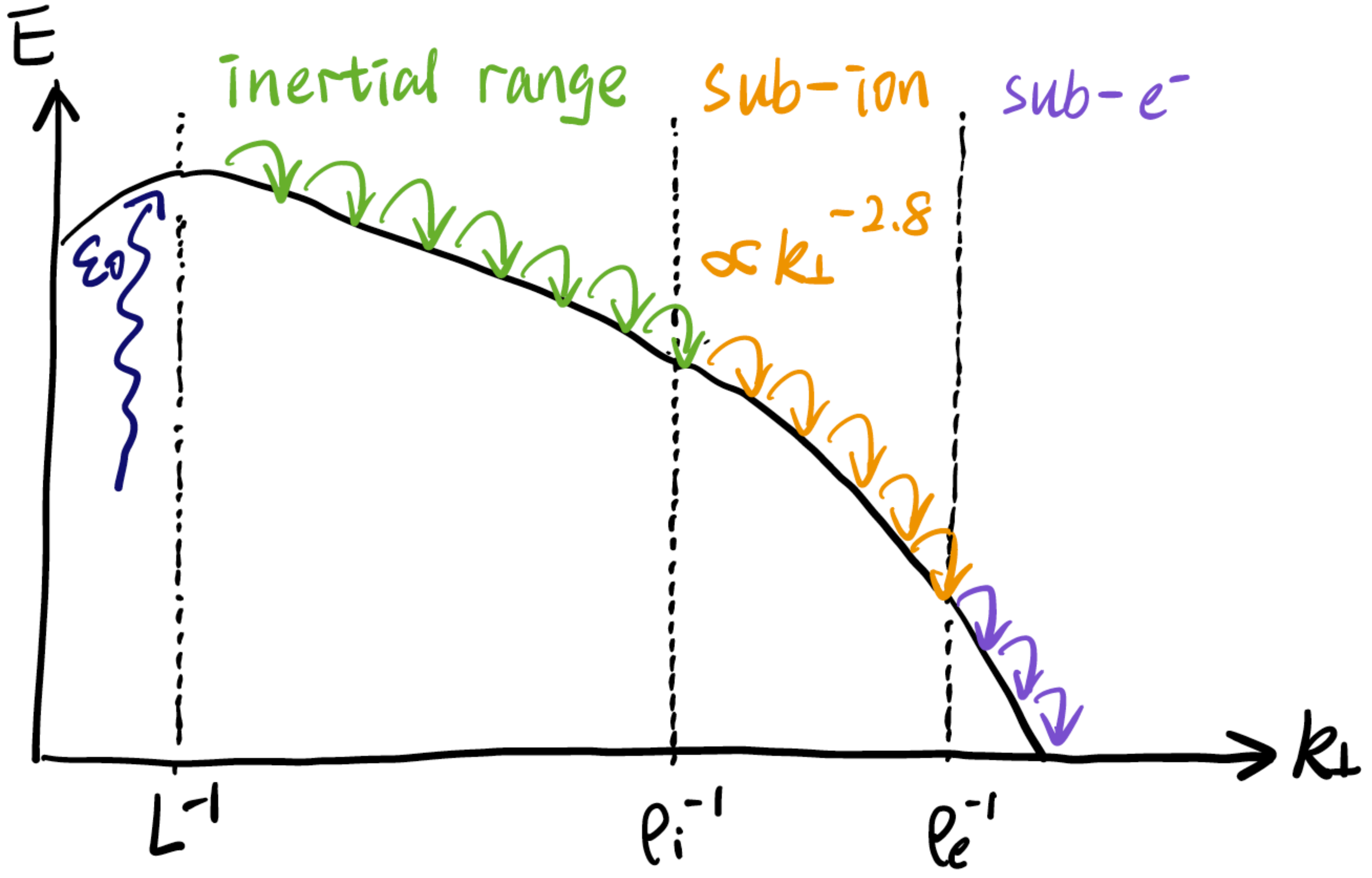

Useful for studying the nature of nonlinear interaction in the position space

Electron kinetic physics

Fully kinetic → Gyro-kinetics → (low-β, $\beta_e \sim m_e/m_i$) → KREHM

Simplest yet rigorous eqns containing fluid & kinetic physics [Zocco+ 2011]

Ions become isothermal and electrostatic, only introducing ρ_i scale.

Electrons are described by δf_e (deviation from F_{0e})

m: order of moments

δf_e → δn_e ($m = 0$) → $u_{\|e}$ $(A_\|)$ ($m = 1$) → $\delta T_{\|e}$ ($m = 2$) → ⋮

Continuity

$$\frac{1}{n_{0e}}\frac{d\delta n_e}{dt} = -\hat{b}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\|$$

Ohm's law

$$\frac{d}{dt}(A_\| - d_e^2\nabla_\perp^2 A_\|) = -c\frac{\partial\varphi}{\partial z} + \frac{cT_{e0}}{e}\hat{b}\cdot\nabla\left(\frac{\delta n_e}{n_{0e}} + \frac{\delta T_{\|e}}{T_{0e}}\right)$$

Kinetic equation

$$\frac{dg_e}{dt} + v_\|\hat{b}\cdot\nabla\left(g_e - \frac{\delta T_{\|e}}{T_{0e}}F_{0e}\right) = C[g_e] + \left(1 - \frac{v_\|^2}{v_{\text{the}}^2}\right)F_{0e}\hat{b}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\|$$

Hermite expansion: $g_e(r, v_\|, t) = \sum_{m=0}^{\infty} H_m(v_\|/v_{\text{the}})g_m(r,t)F_{0e}(v_\|)/\sqrt{2^m m!}$

Using Hermite polynomial

Fully kinetic → Gyro-kinetics → (low-β, $\beta_e \sim m_e/m_i$) → KREHM

Simplest yet rigorous eqns containing fluid & kinetic physics [Zocco+ 2011]

Ions become isothermal and electrostatic, only introducing ρ_i scale.

Electrons are described by δf_e (deviation from F_{0e})

m: order of Hermite polynomials

Continuity

$$\frac{1}{n_{0e}}\frac{d\delta n_e}{dt} = -\hat{b}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\parallel$$

$m = 0$ — δn_e

Ohm's law

$$\frac{d}{dt}(A_\parallel - d_e^2\nabla_\perp^2 A_\parallel) = -c\frac{\partial\varphi}{\partial z} + \frac{cT_{e0}}{e}\hat{b}\cdot\nabla\left(\frac{\delta n_e}{n_{0e}} + \frac{\delta T_{\parallel e}}{T_{0e}}\right)$$

$m = 1$ — $u_{\parallel e}$ $(A_\parallel)$

Kinetic equation

$m = 2$ — $\delta T_{\parallel e}$

$$\frac{dg_m}{dt} = -v_{\rm the}\hat{b}\cdot\nabla\left(\sqrt{\frac{m+1}{2}}g_{m+1} + \sqrt{\frac{m}{2}}g_{m-1} - \delta_{m,1}g_2\right) - \sqrt{2}\delta_{m,2}\hat{b}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\parallel$$

$\delta f_e \to \delta n_e \to u_{\parallel e} \to \delta T_{\parallel e} \to \vdots$

$m \geq 2$: g_m

Ion finite Larmor radius effects + electron drift kinetic physics + KAW

Fully kinetic → Gyro-kinetics → (low-β, $\beta_e \sim m_e/m_i$) → KREHM

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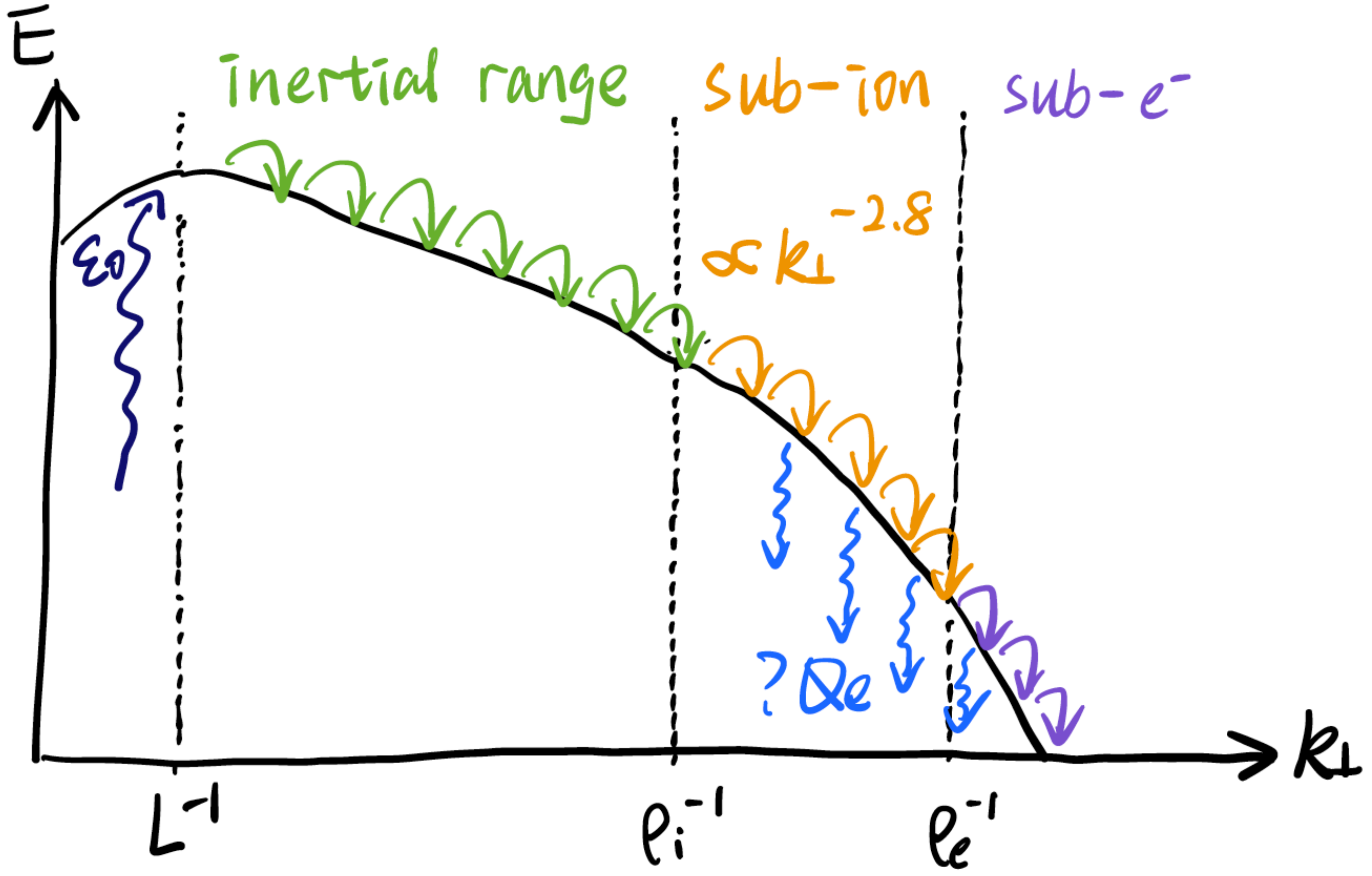

Continuity $$\frac{1}{n_{0e}}\frac{d\delta n_e}{dt} = -\hat{b}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\parallel$$

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Kinetic equation

$$\frac{dg_m}{dt} = -v_{\text{the}}\hat{b}\cdot\nabla\left(\sqrt{\frac{m+1}{2}}g_{m+1} + \sqrt{\frac{m}{2}}g_{m-1} - \delta_{m,1}g_2\right) - \sqrt{2}\delta_{m,2}\hat{b}\cdot\nabla\frac{e}{cm_e}d_e^2\nabla_\perp^2 A_\parallel$$

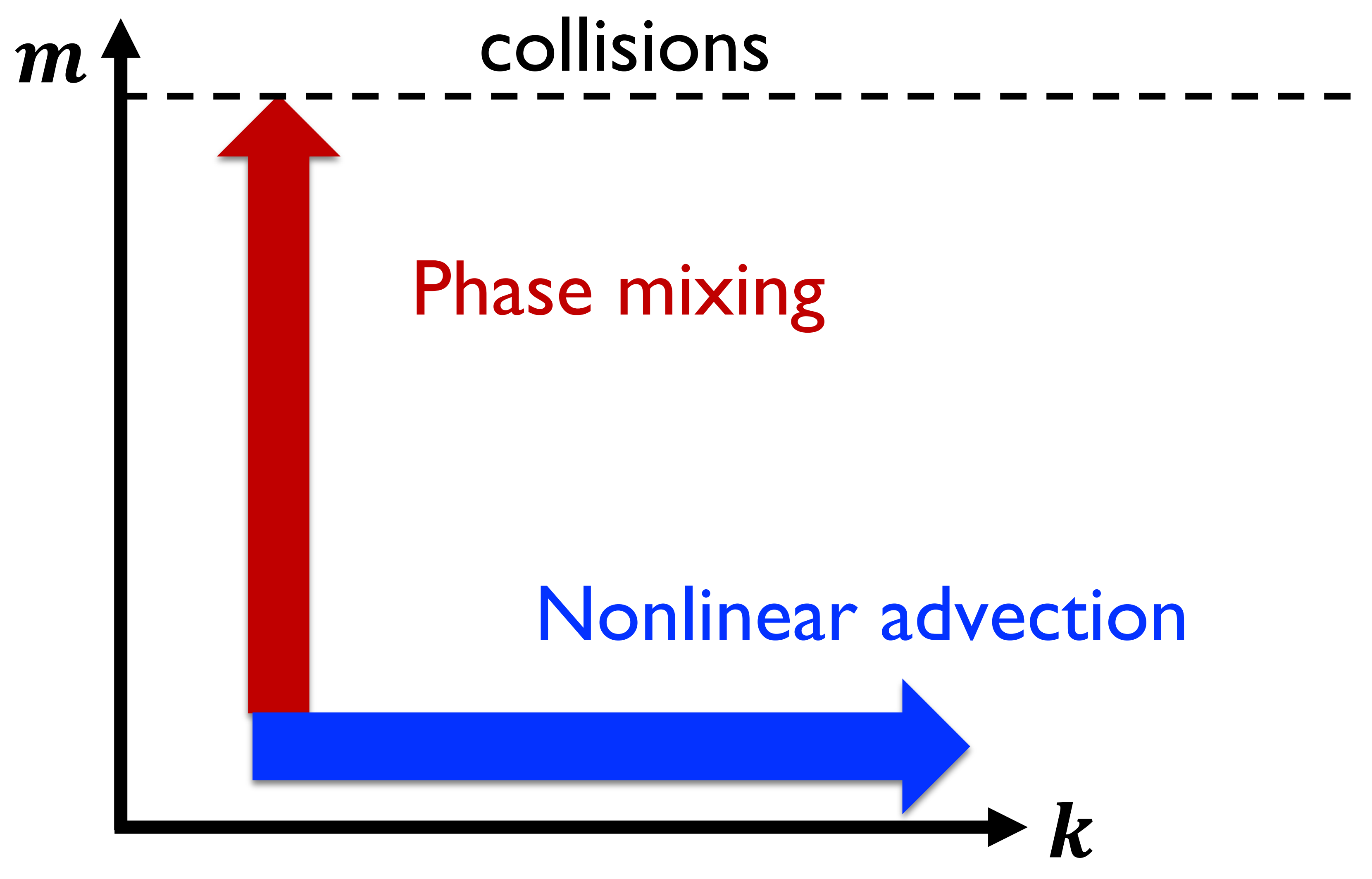
With kinetic electrons --- Phase space cascade and electron heating

$$\frac{dg_m}{dt} = \frac{\partial g_m}{\partial t} + \frac{c}{B_0}\{\varphi, g_m\}$$

Source of free energy in velocity space $\hat{b} \cdot \nabla J_{\|}$

$$\frac{dg_m}{dt} = -v_{\rm the}\hat{\mathbf{b}} \cdot \nabla \left(\sqrt{\frac{m+1}{2}} g_{m+1} + \sqrt{\frac{m}{2}} g_{m-1} - \delta_{m,1} g_2 \right) - \sqrt{2}\delta_{m,2}\hat{\mathbf{b}} \cdot \nabla J_{\|}$$

$$\approx k_{\|} v_{\rm the} g_m/\sqrt{m} \sim \frac{v_{\rm the}}{B_0}\{A_{\|}, g_m\}/\sqrt{m}$$



With kinetic electrons --- Phase space cascade and electron heating

$$\frac{dg_m}{dt} = \frac{\partial g_m}{\partial t} + \frac{c}{B_0}\{\varphi, g_m\}$$

Source of free energy in velocity space $\hat{b}\cdot\nabla J_\parallel$

$$\frac{dg_m}{dt} = -v_{\rm the}\hat{\mathbf{b}}\cdot\nabla\left(\sqrt{\frac{m+1}{2}}g_{m+1} + \sqrt{\frac{m}{2}}g_{m-1} - \delta_{m,1}g_2\right) - \sqrt{2}\delta_{m,2}\hat{\mathbf{b}}\cdot\nabla J_\parallel$$

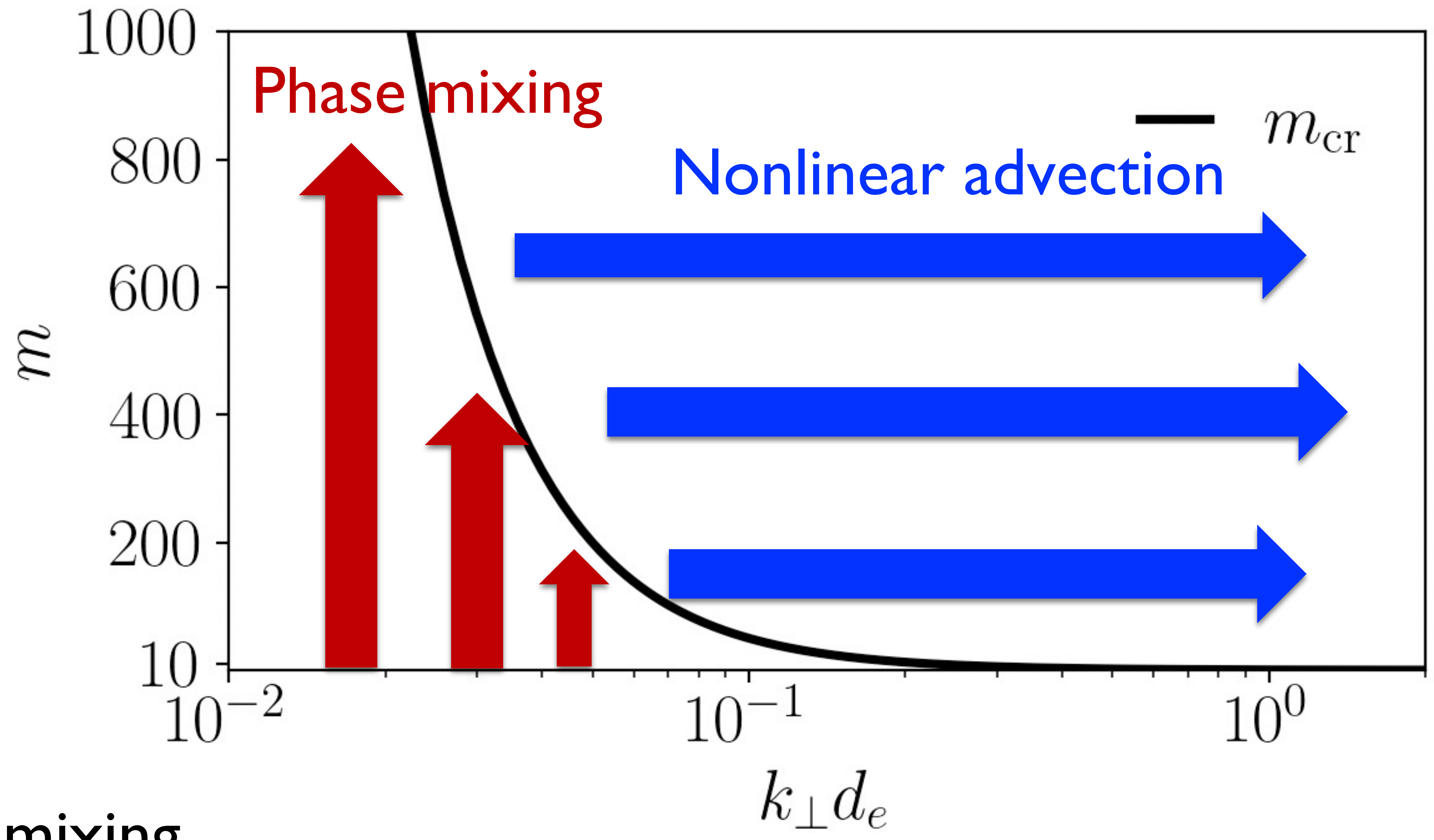
$$\approx k_\parallel v_{\rm the} g_m/\sqrt{m} \sim \frac{v_{\rm the}}{B_0}\{A_\parallel, g_m\}/\sqrt{m}$$

At each scale λ, there is a critical Hermite order m_{cr}:

nonlinear advection rate ~ Phase mixing rate

$$\frac{c}{B_0}\{\varphi, g_{m_{\rm cr}}\} \sim \frac{v_{\rm the}}{B_0}\{A_\parallel, g_{m_{\rm cr}}\}/\sqrt{m_{\rm cr}}$$

$$m_{\rm cr}(\lambda) \sim (\lambda/d_e)^2/(2\tau^2)$$

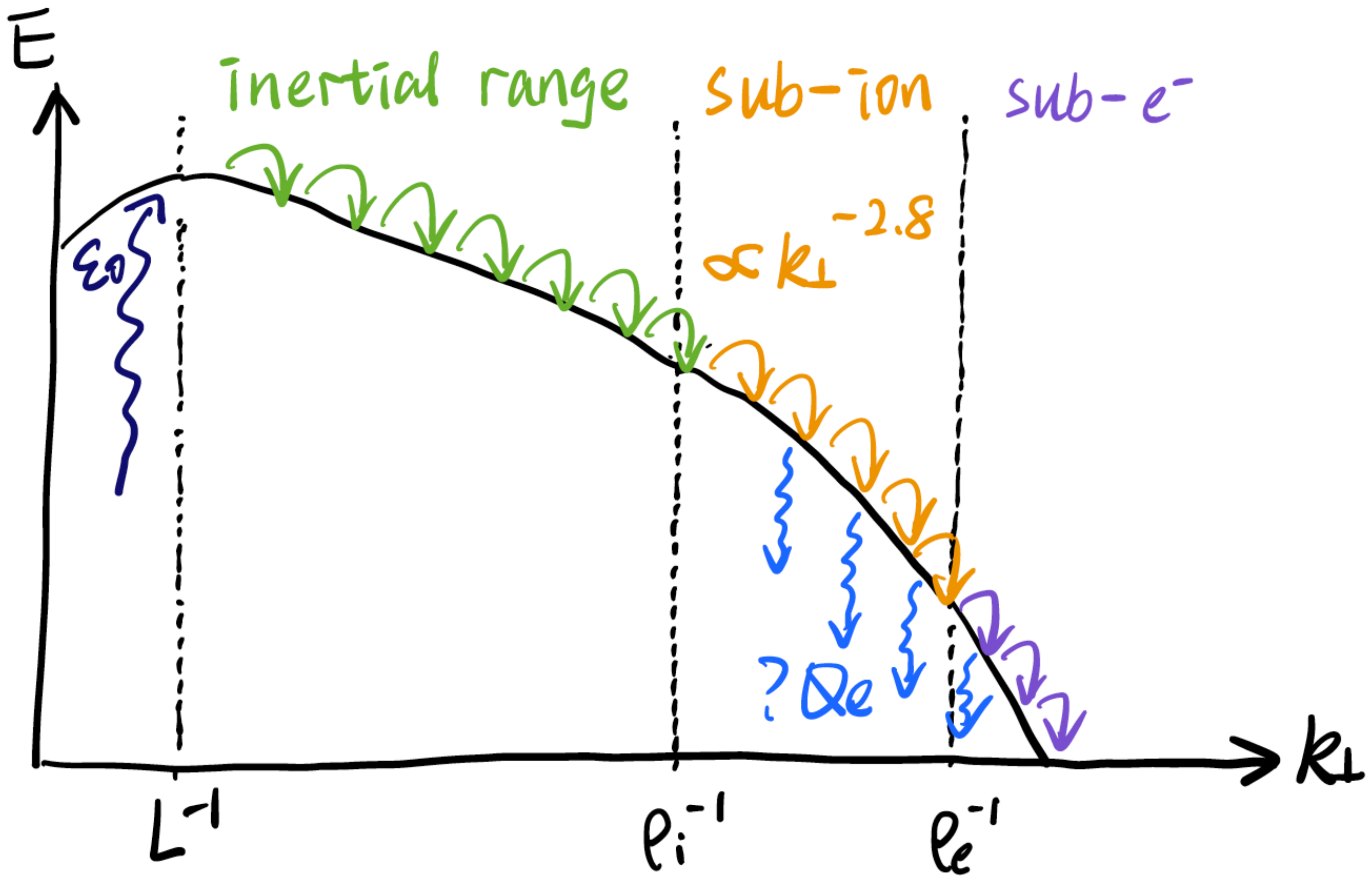


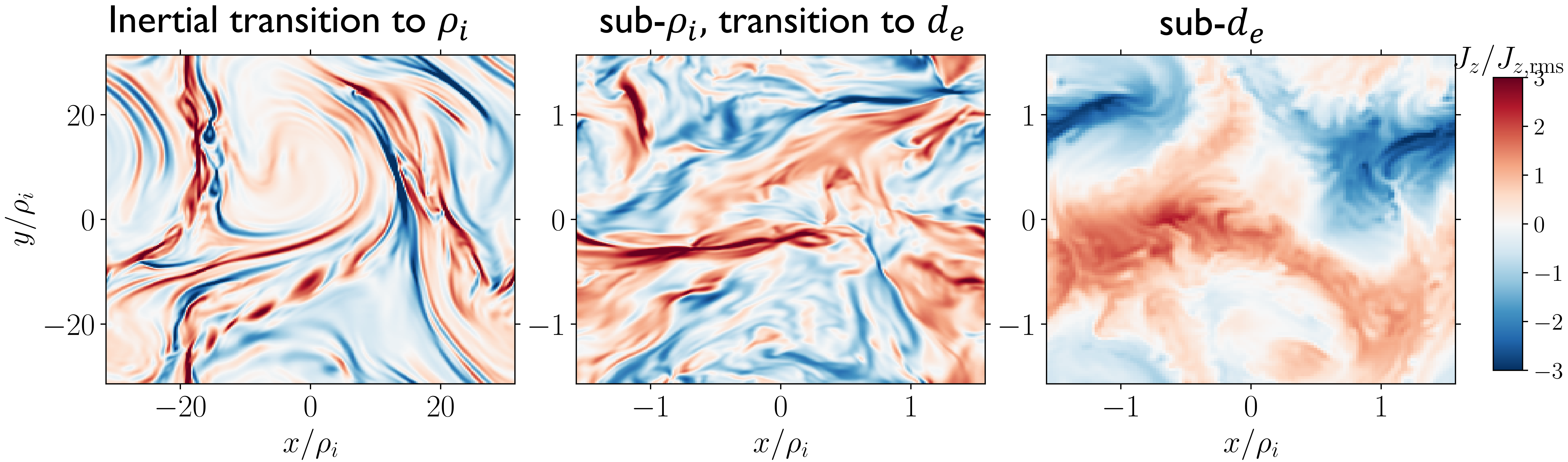
Above m_{cr} plasma echoes are expected to happen and impede phase mixing

In weakly collisional plasmas, the *collisional cut off* $\gg m_{cr}$

Why is efficient electron heating observed in solar wind [Chen+ 2019] and in kinetic simulations[e.g. Howes+2016-2018]?

Simulations solving KREHM eqs

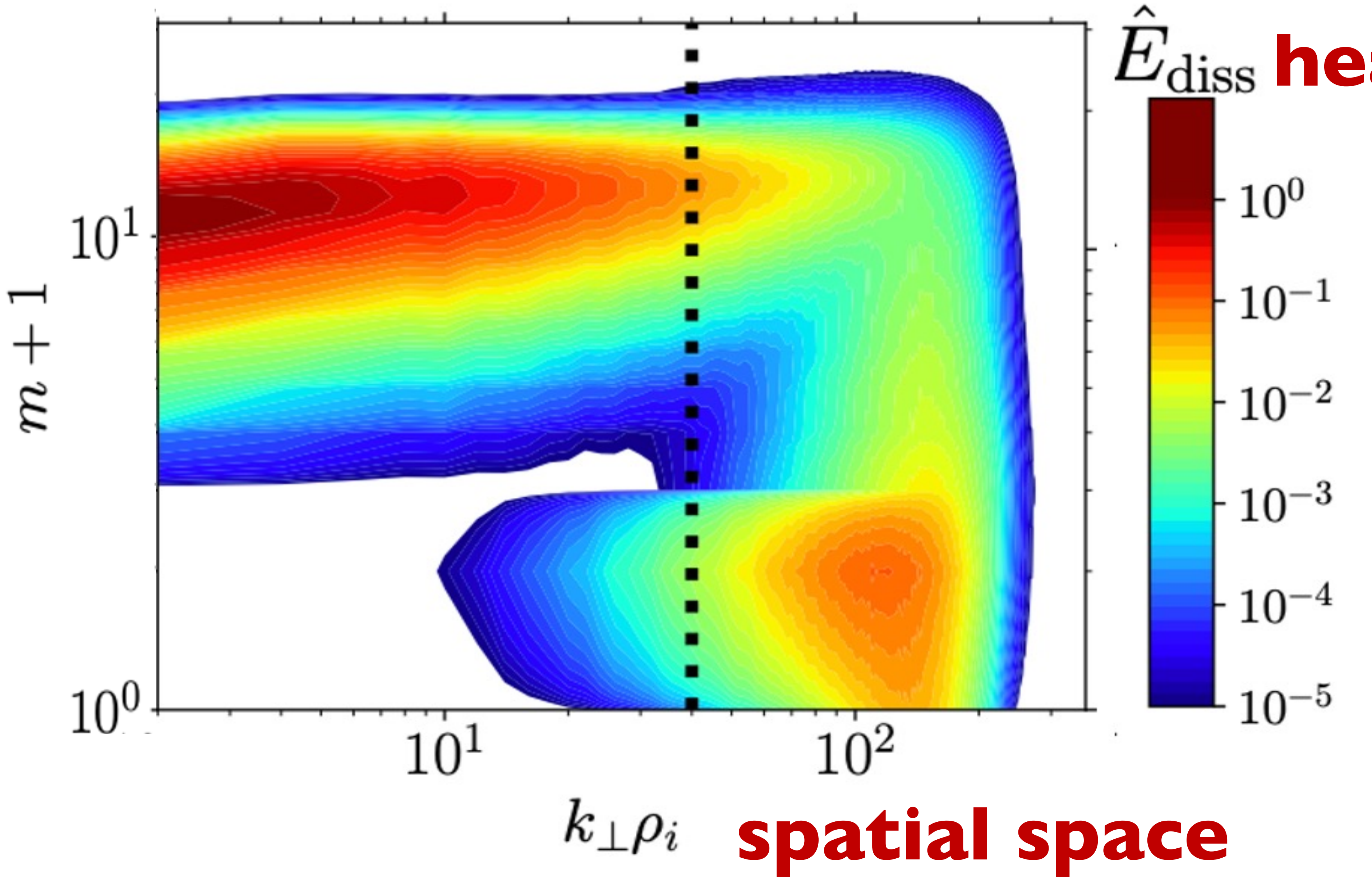




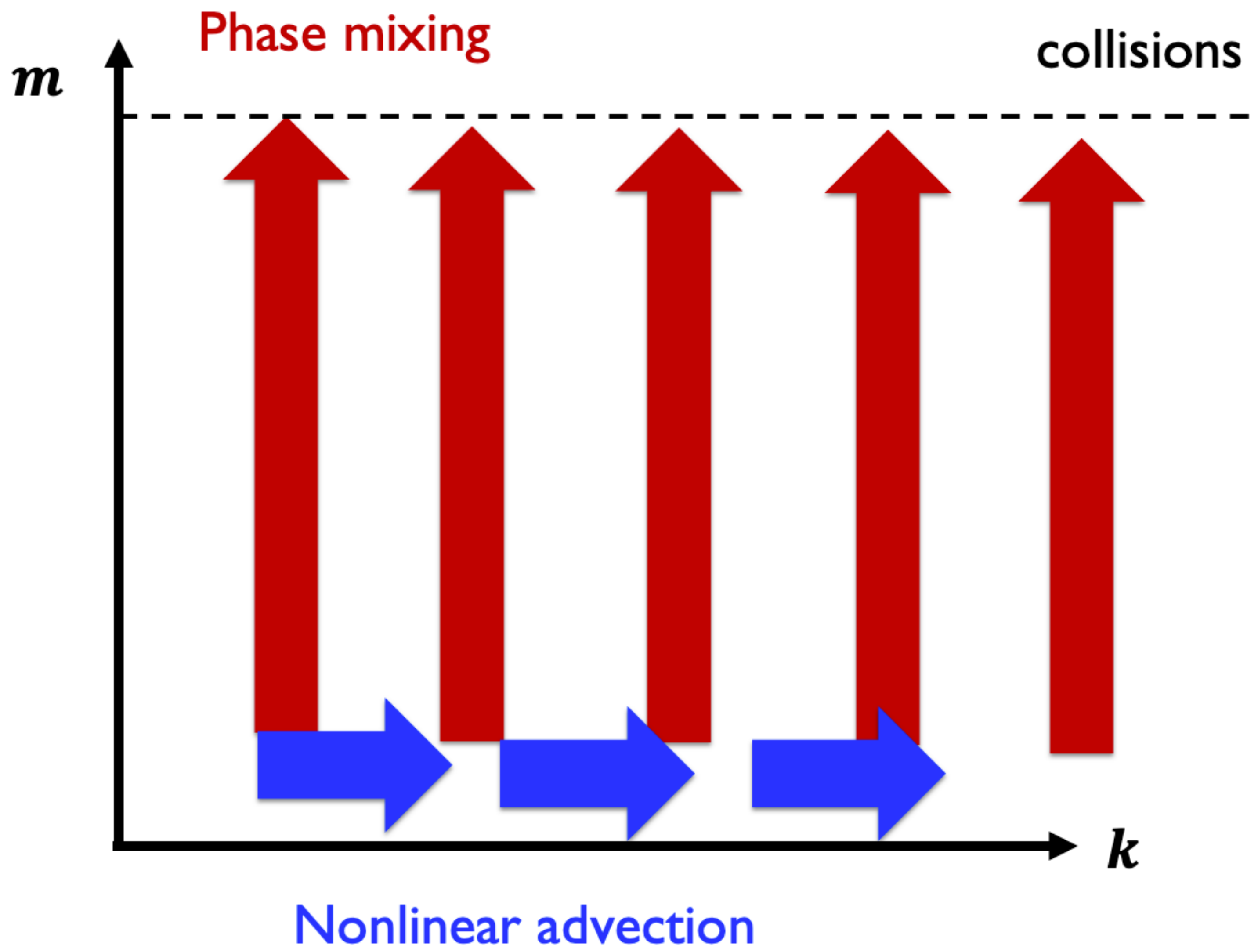
Landau damped EM energy matches electron heating

≈ **electron heating**

velocity space

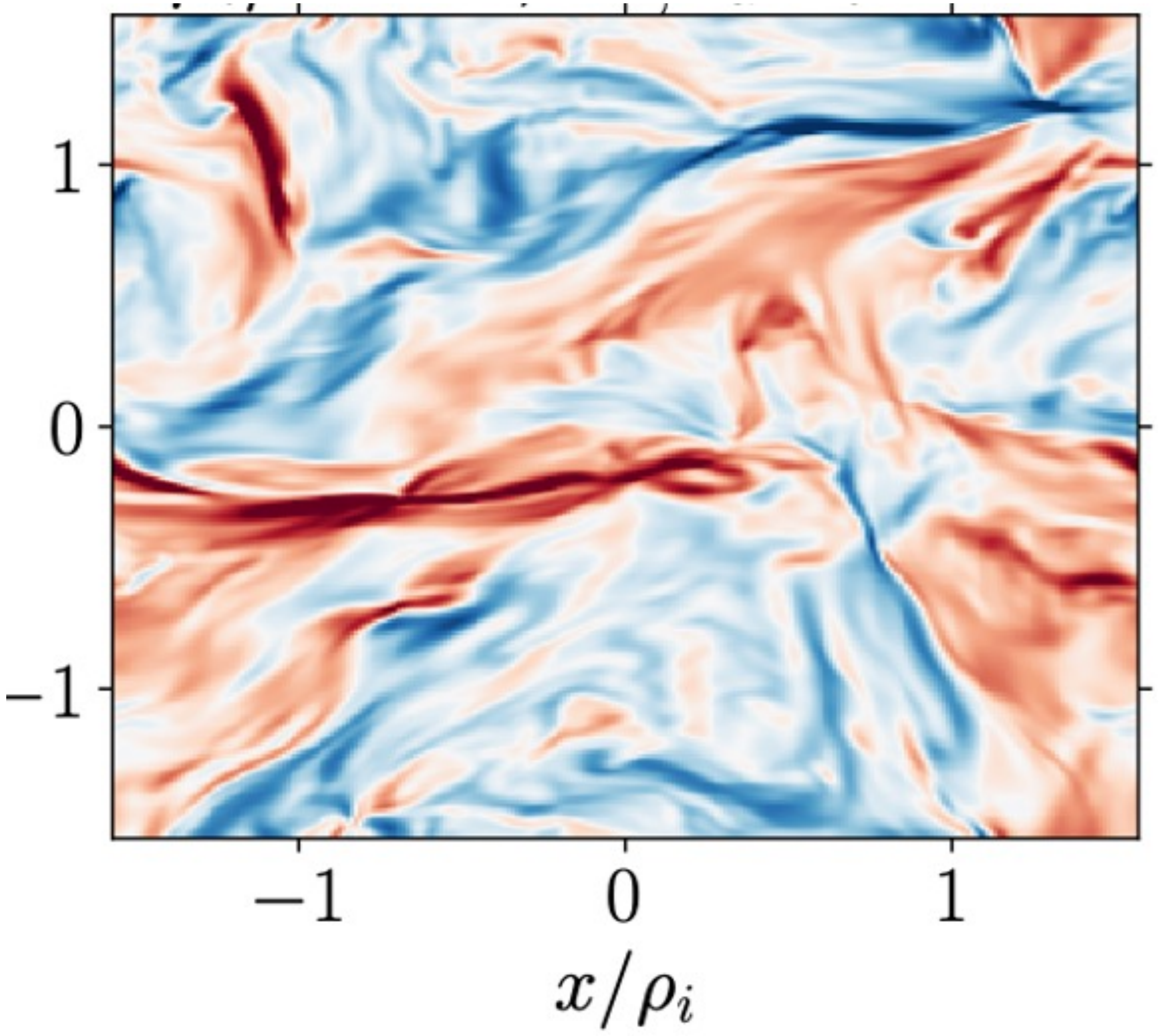


spatial space



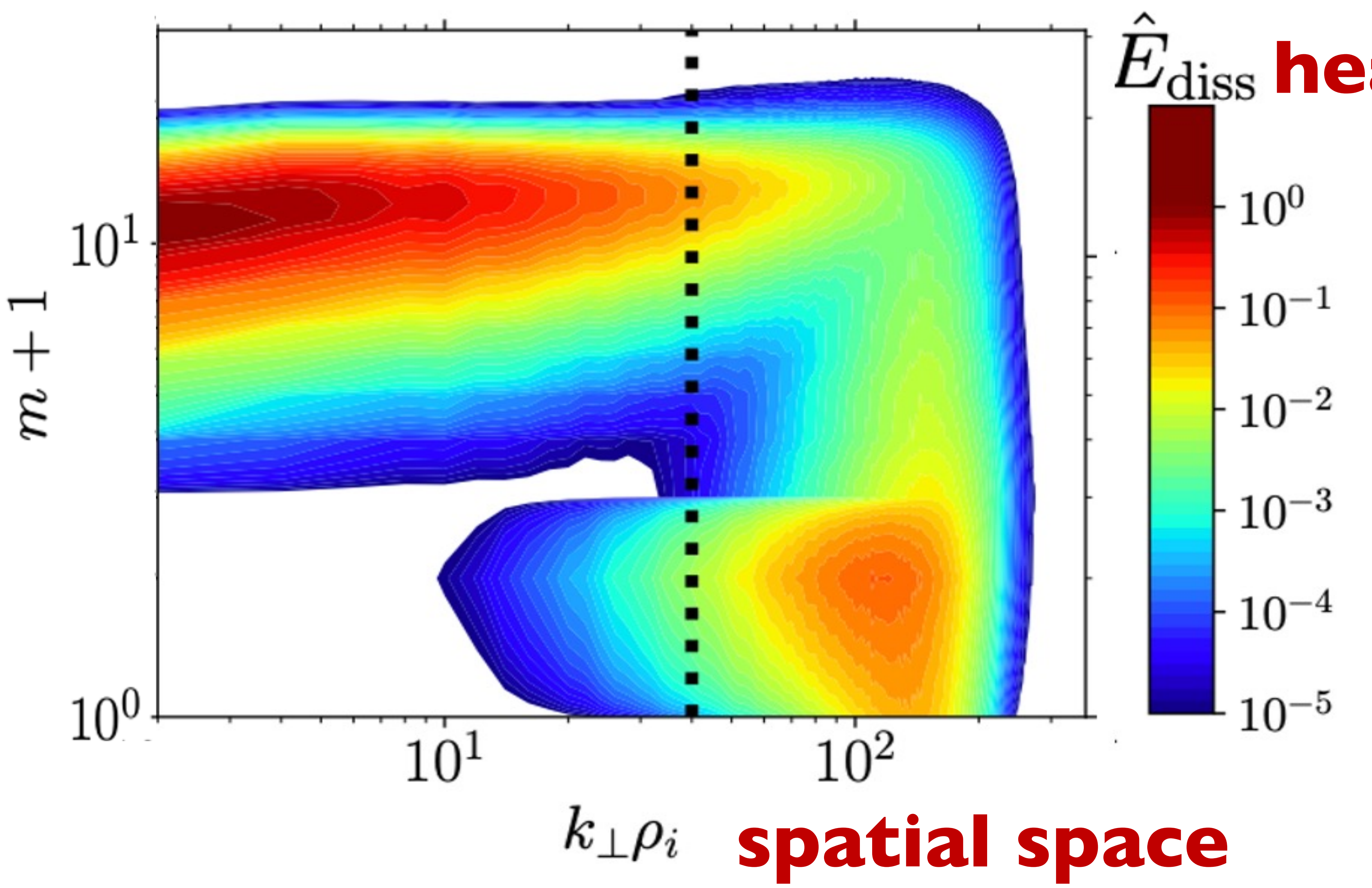
EM fluctuations are Landau-damped at each scale independently → phase mixing domination

sub-ρ_i, transition to d_e



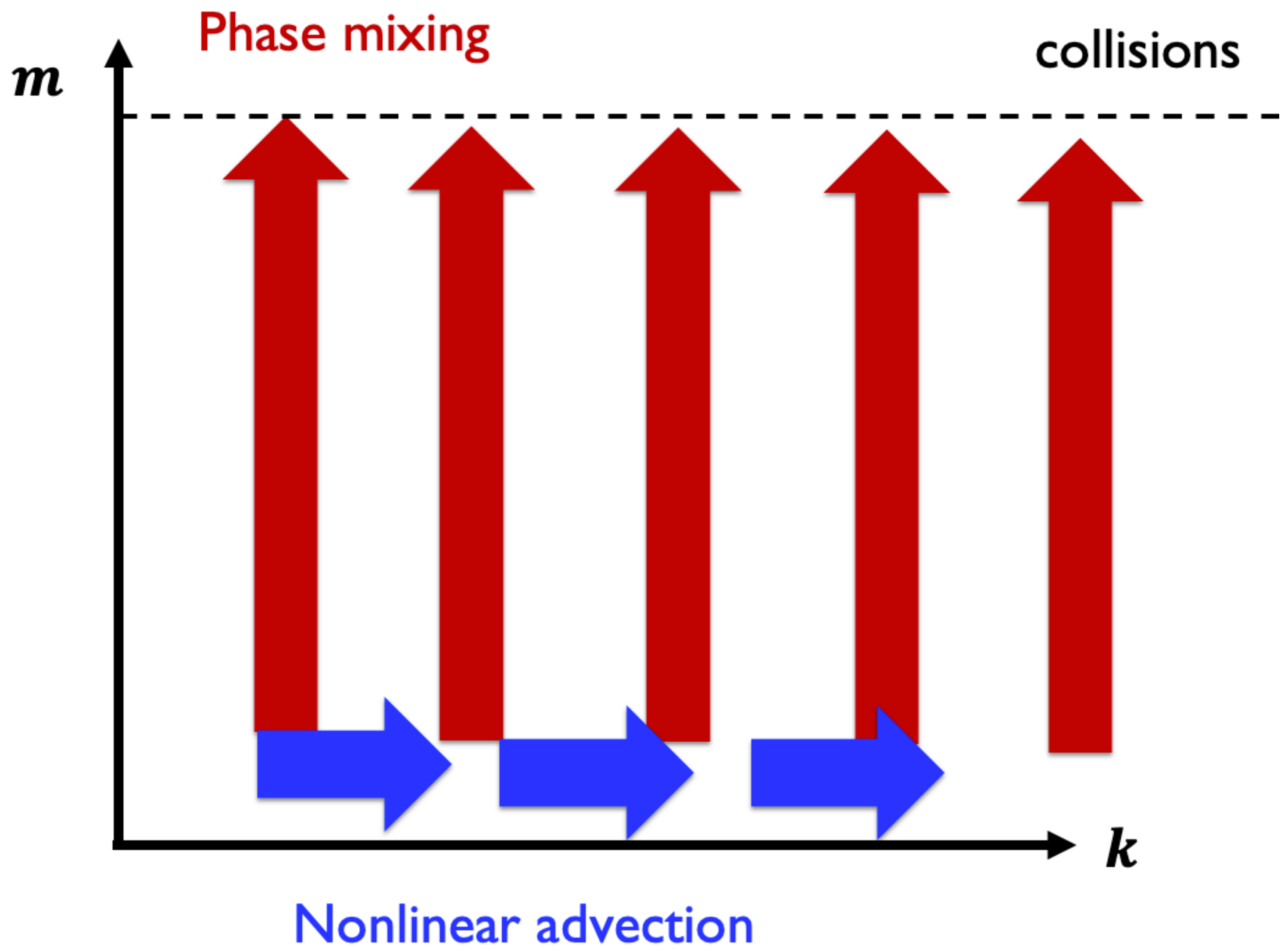
Towards a prescription for electron heating

velocity space



≈ electron heating

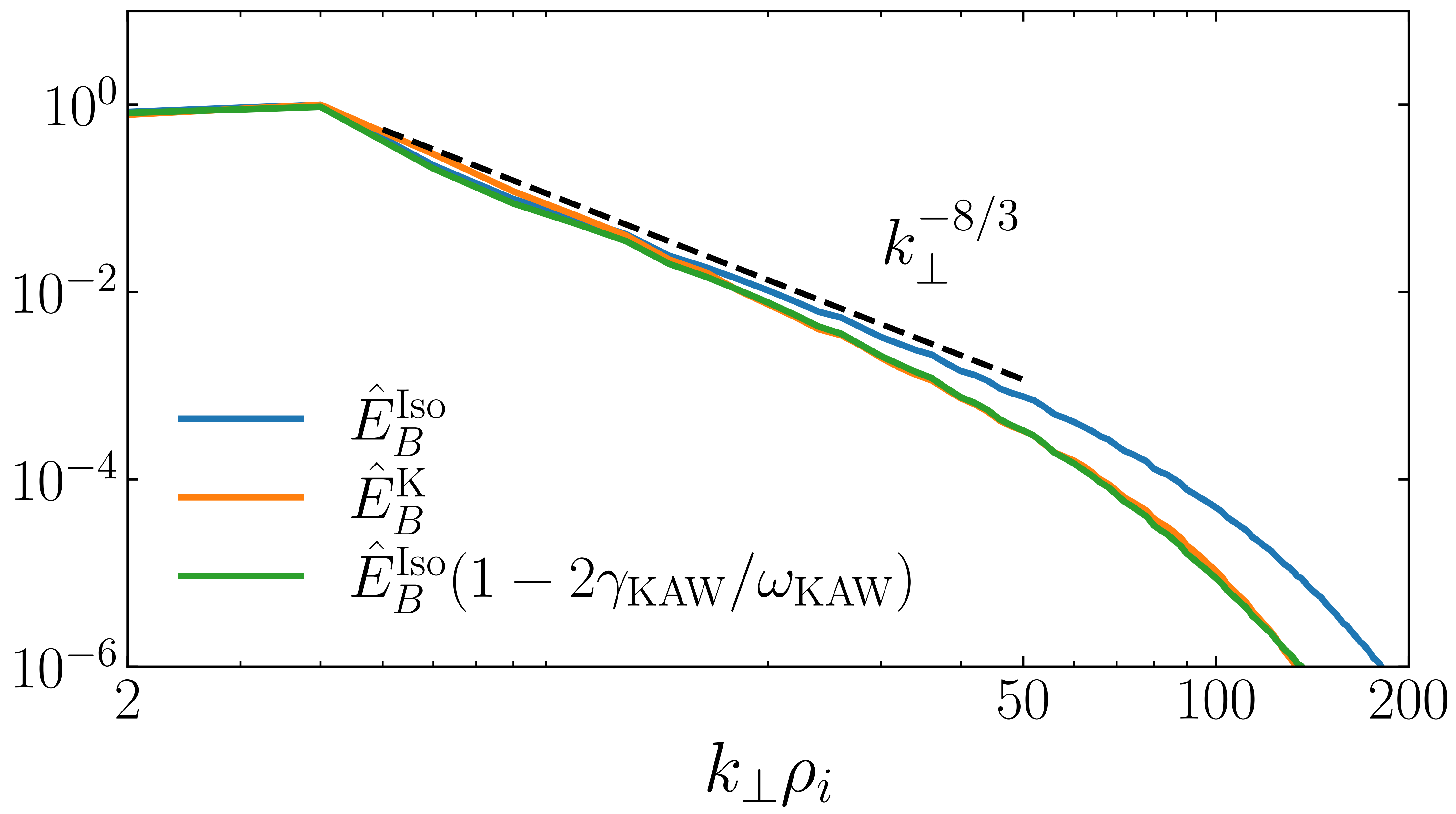
spatial space



EM fluctuations are Landau-damped at each scale independently → phase mixing domination

From the linear dispersion relation of KAWs: damping rate γ_{KAW} and frequency ω_{KAW}

$$\hat{E}_B^{\mathrm{K}}(k_\perp) \approx \hat{E}_B^{\mathrm{iso}}(k_\perp)(1 - 2\gamma_{\mathrm{KAW}}/\omega_{\mathrm{KAW}}).$$



[See also Howes+2008,2011; TenBarge+ 2013]

A zeroth-order solution in the velocity space

$$\boxed{\frac{dg_m}{dt}} = -v_{\rm the}\hat{\mathbf{b}}\cdot\nabla\left(\sqrt{\frac{m+1}{2}}g_{m+1} + \sqrt{\frac{m}{2}}g_{m-1} - \delta_{m,1}g_2\right) - \sqrt{2}\delta_{m,2}\hat{\mathbf{b}}\cdot\nabla J_\|$$

Nonlinear advection

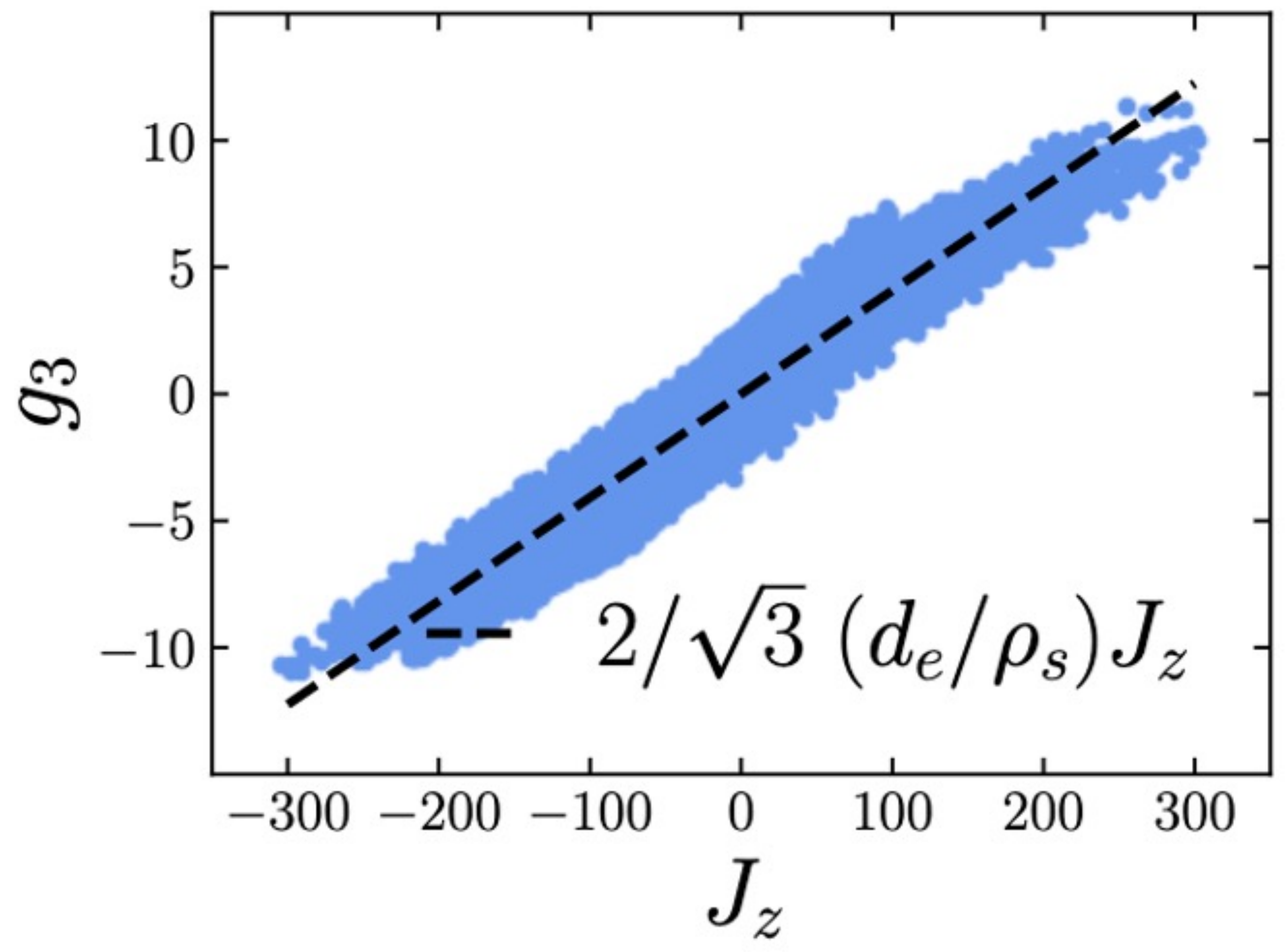
phase mixing

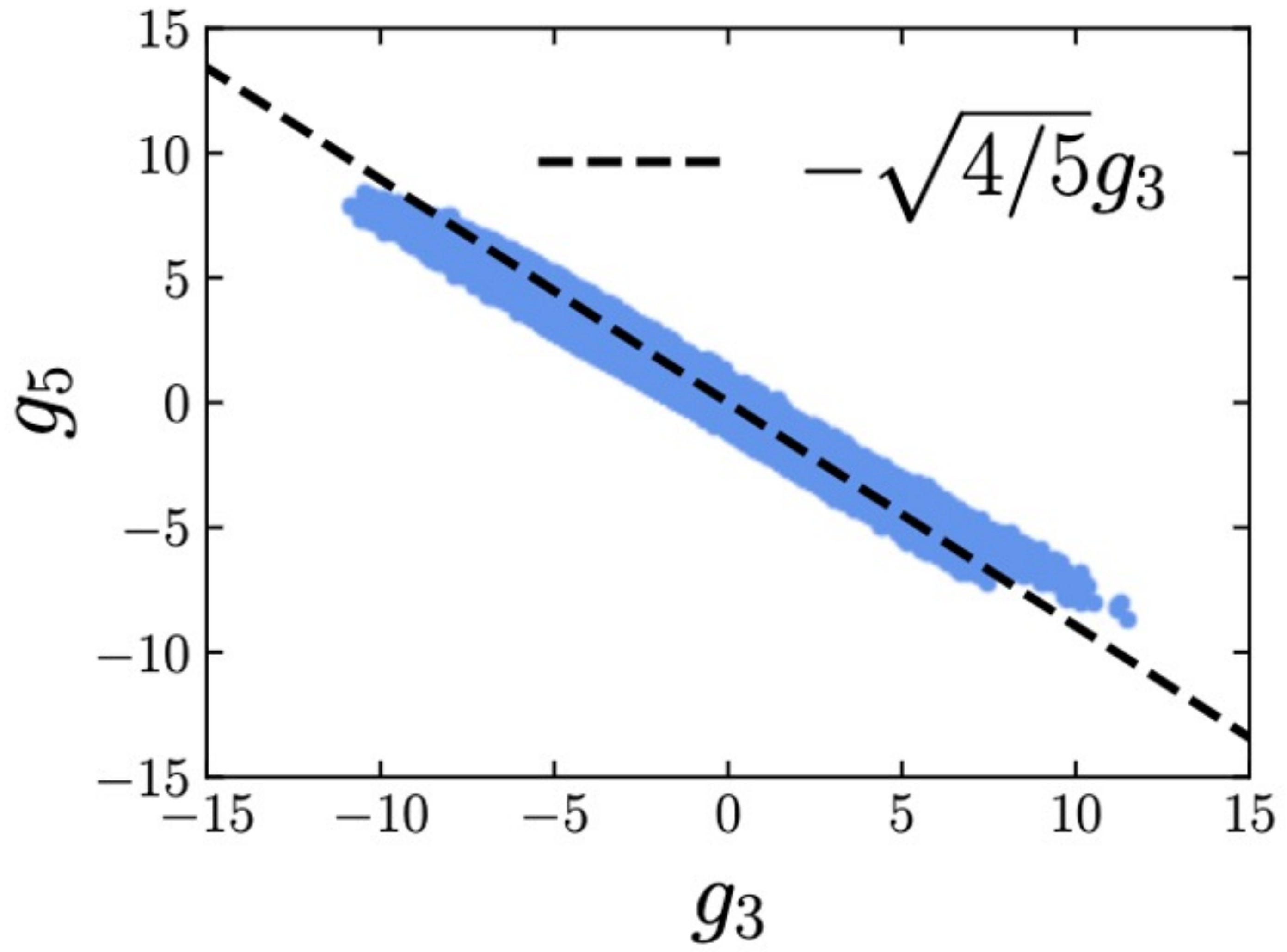
In the phase-mixing dominated regime, *l.h.s.* << *r.h.s.*

To the lowest order:

$$g_{m+1} = -\sqrt{m/(m+1)}g_{m-1}$$

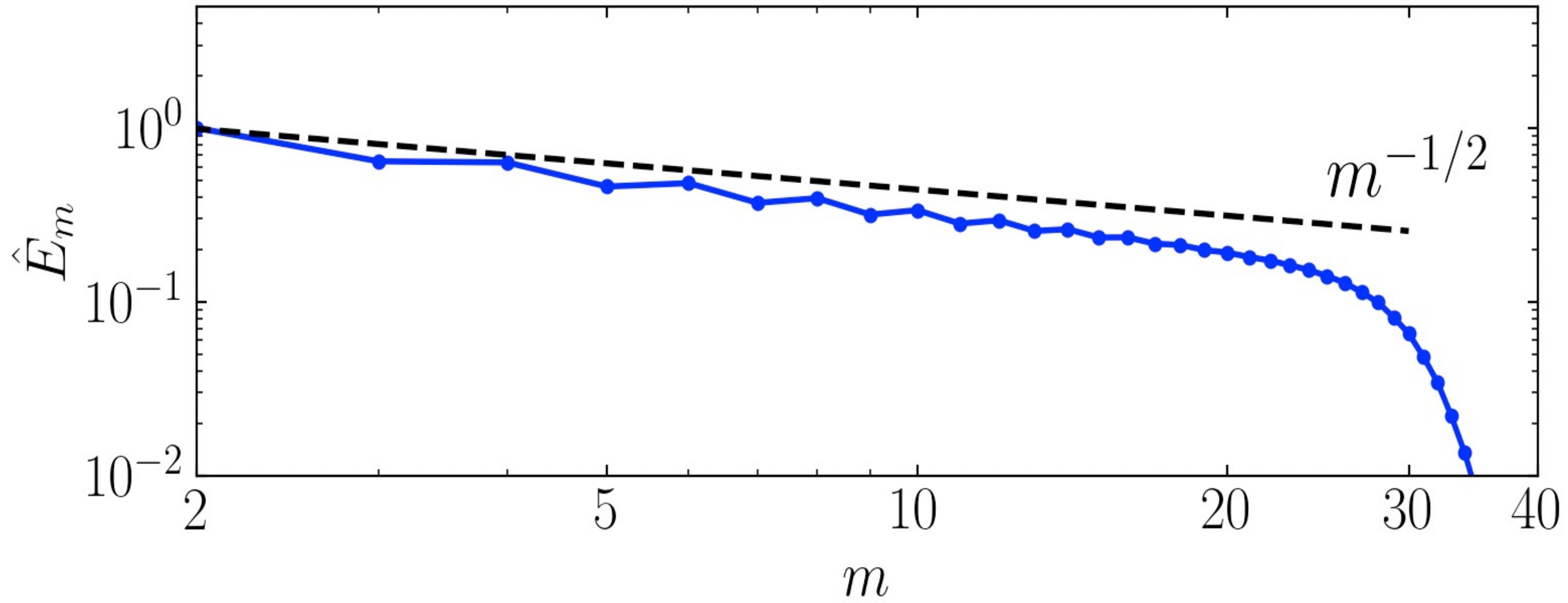
$$g_3 = -\sqrt{2/3}\,(\rho_s/d_e)J_\|$$





The Hermite spectrum of g_e (velocity-space spectrum):

$$E_m \propto m^{-1/2}$$



With kinetic electrons --- Phase space cascade and electron heating

Assuming φ and $A_\parallel$ have the same configuration/gradients

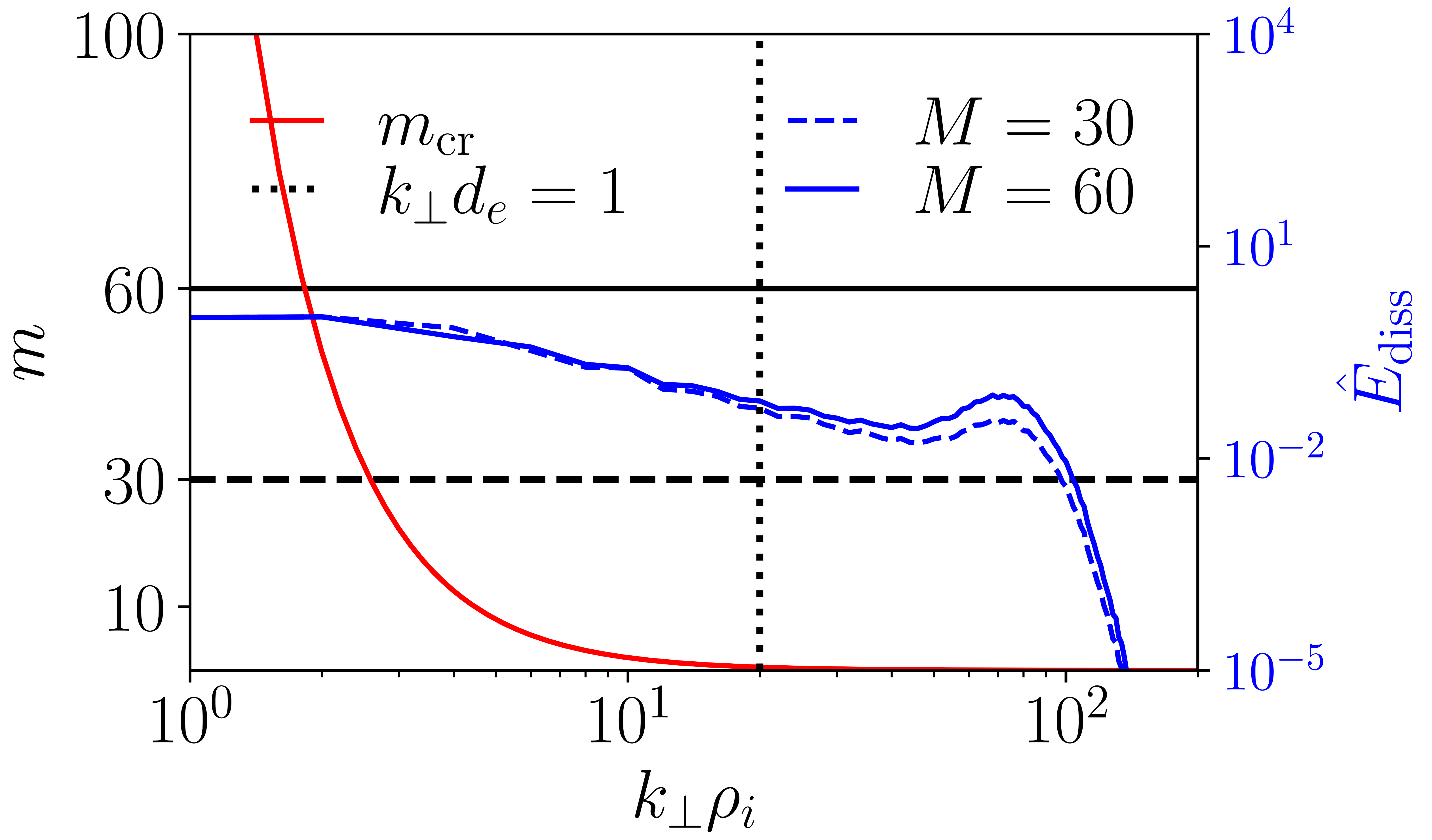
At each scale λ, there is a critical Hermite order m_{cr}:

Nonlinear advection rate~ Phase mixing rate

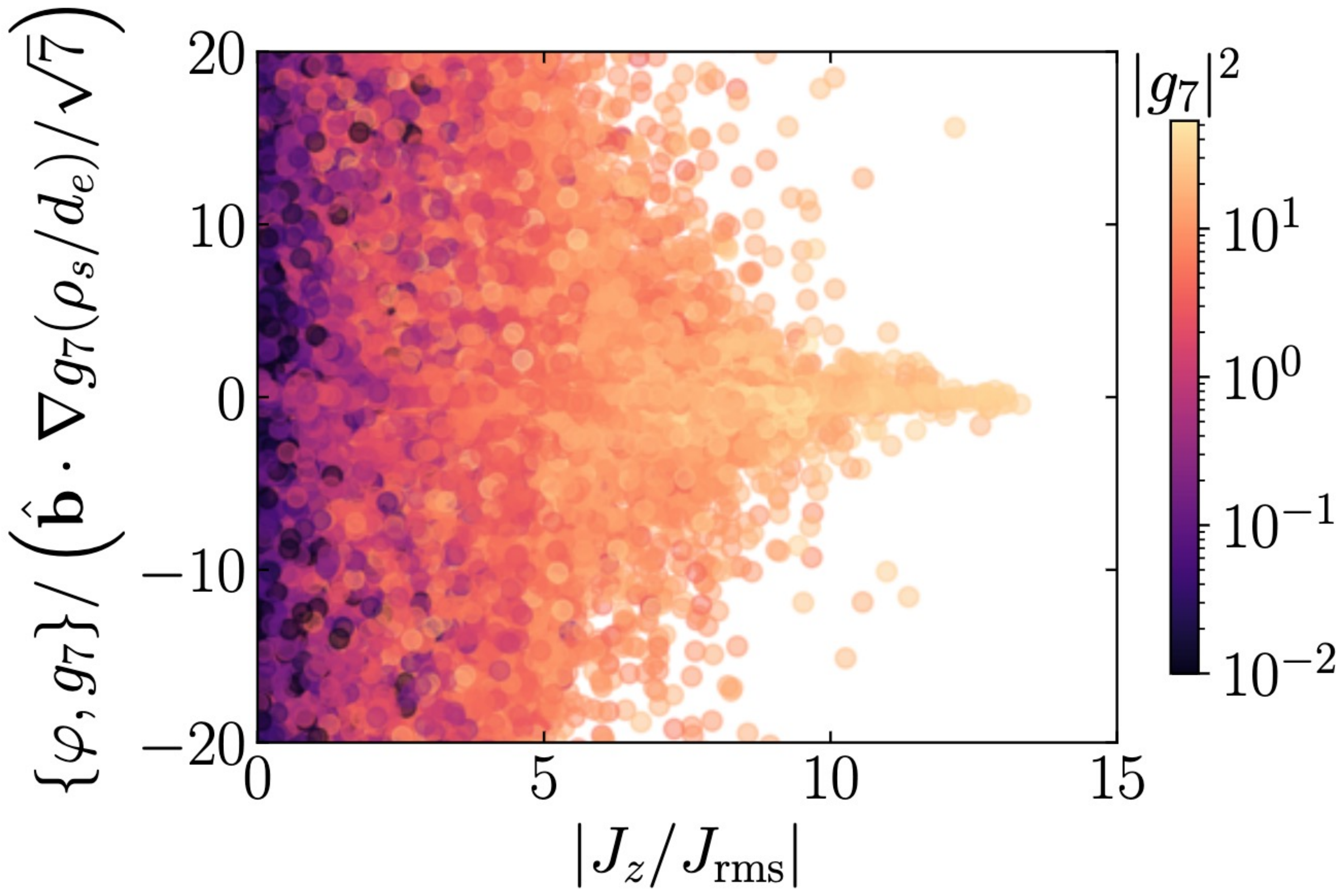
$$\frac{c}{B_0}\{\varphi, g_{m_{\rm cr}}\} \sim \frac{v_{\rm the}}{B_0}\{A_\parallel, g_{m_{\rm cr}}\}/\sqrt{m_{\rm cr}}$$

$$m_{\rm cr}(\lambda) \sim (\lambda/d_e)^2/(2\tau^2)$$

For both cases, there is a sufficiently wide dynamical range that echo could happen.



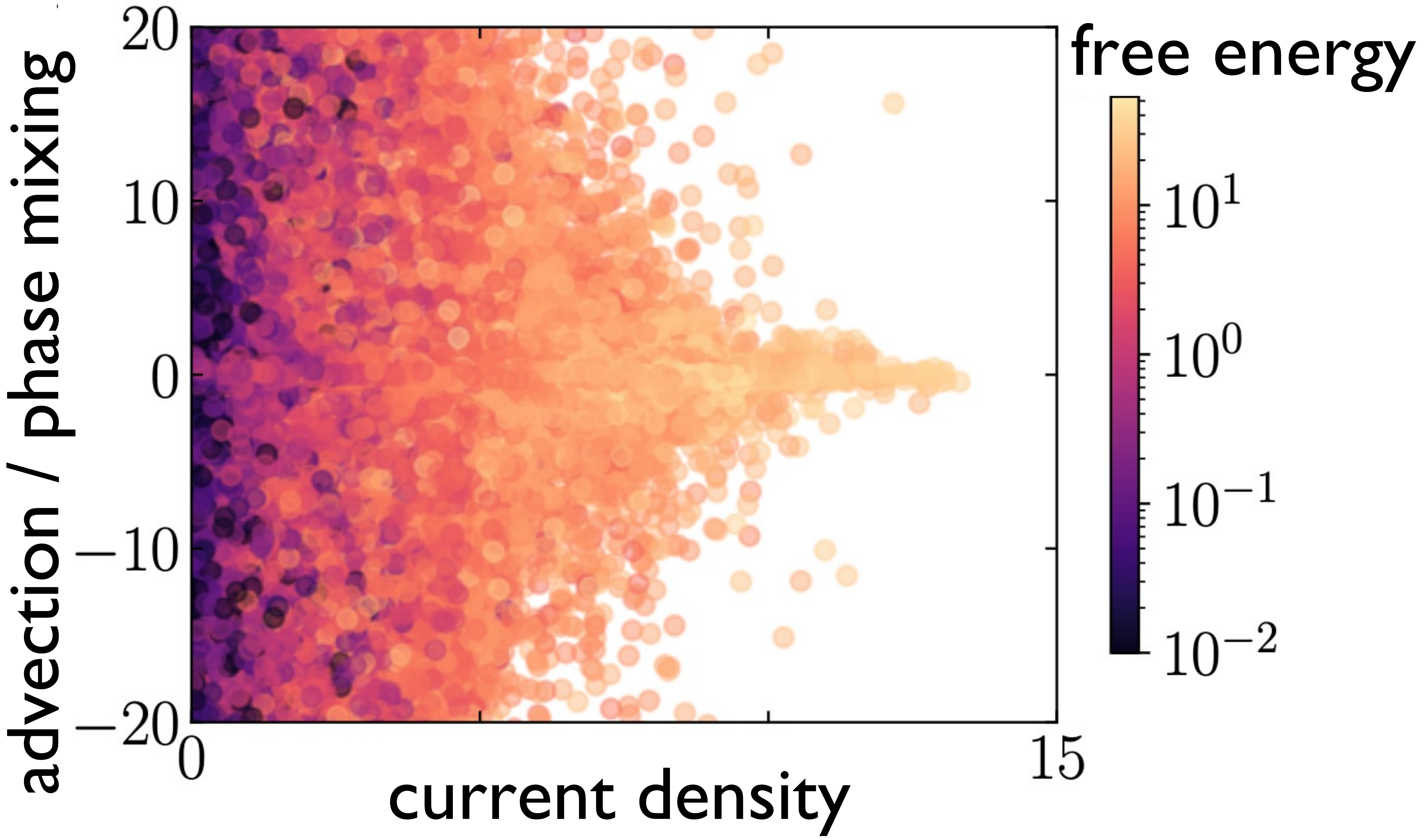
Local weakening of nonlinearity around current sheets enables strong phase mixing



Free energy concentrates around current sheets

At current sheets, nonlinear advection weak, phase mixing strong

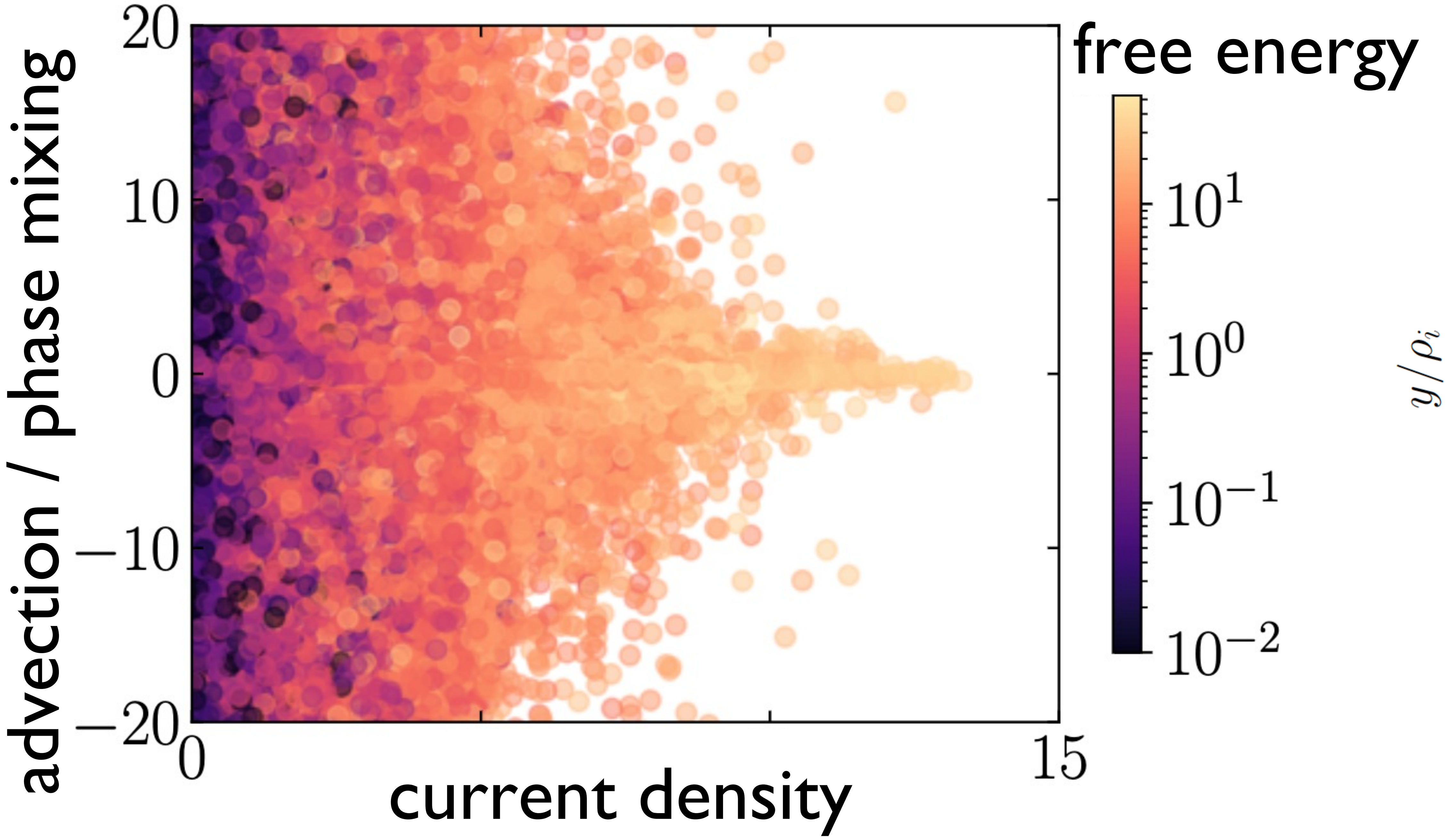
Local weakening of nonlinearity around current sheets enables strong phase mixing



Free energy concentrates around current sheets

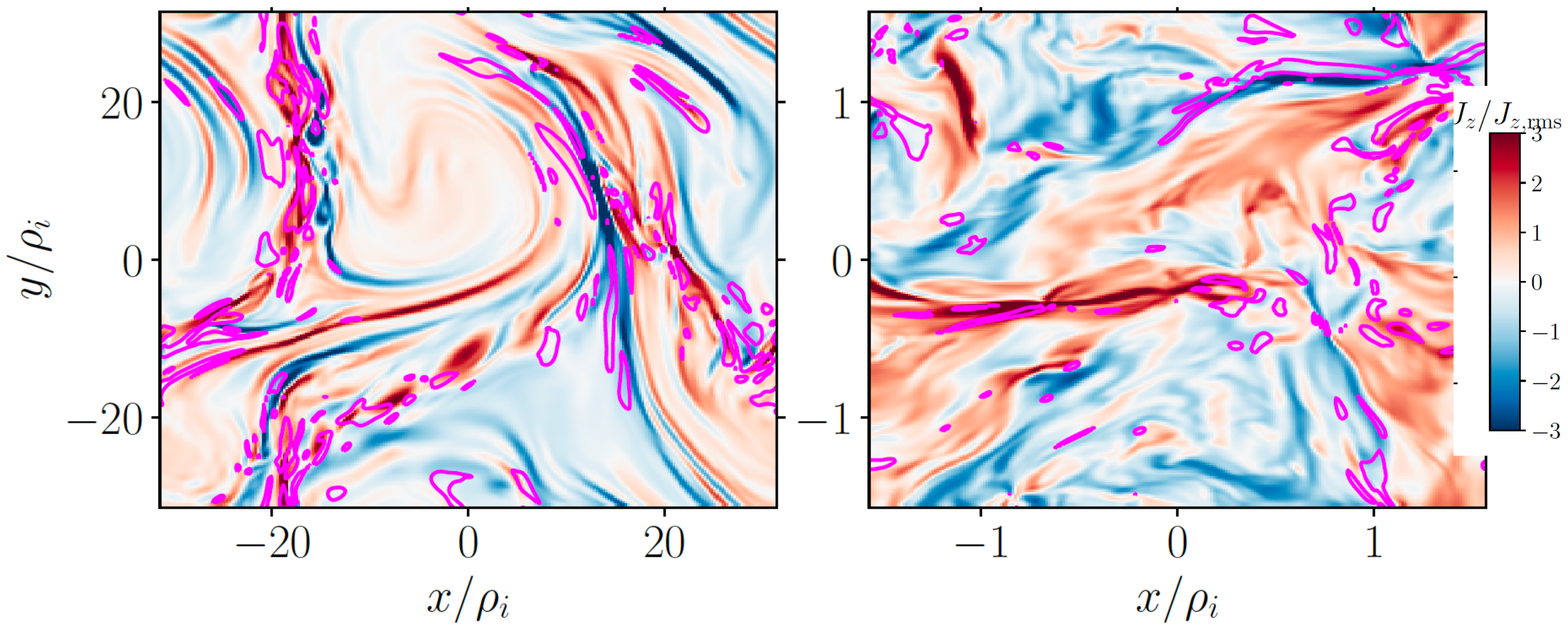
At current sheets, nonlinear advection weak,
phase mixing strong

Heating occurs around current sheets



Free energy concentrates around current sheets

At current sheets, nonlinear advection weak, phase mixing strong



Magenta contours indicates regions with strong electron heating

Heating overlaps with current sheets

Summary

In kinetic turbulence (strongly magnetized regime)

- The kinetic channel of energy dissipates dominates the fluid channel → Electron heating is caused by Landau damping of KAWs (not Ohmic heating)
- Free energy is concentrated around current sheets *(velocity-space physics)*, where nonlinearity is weakened *(position-space physics)*. Plasma echoes are impeded.