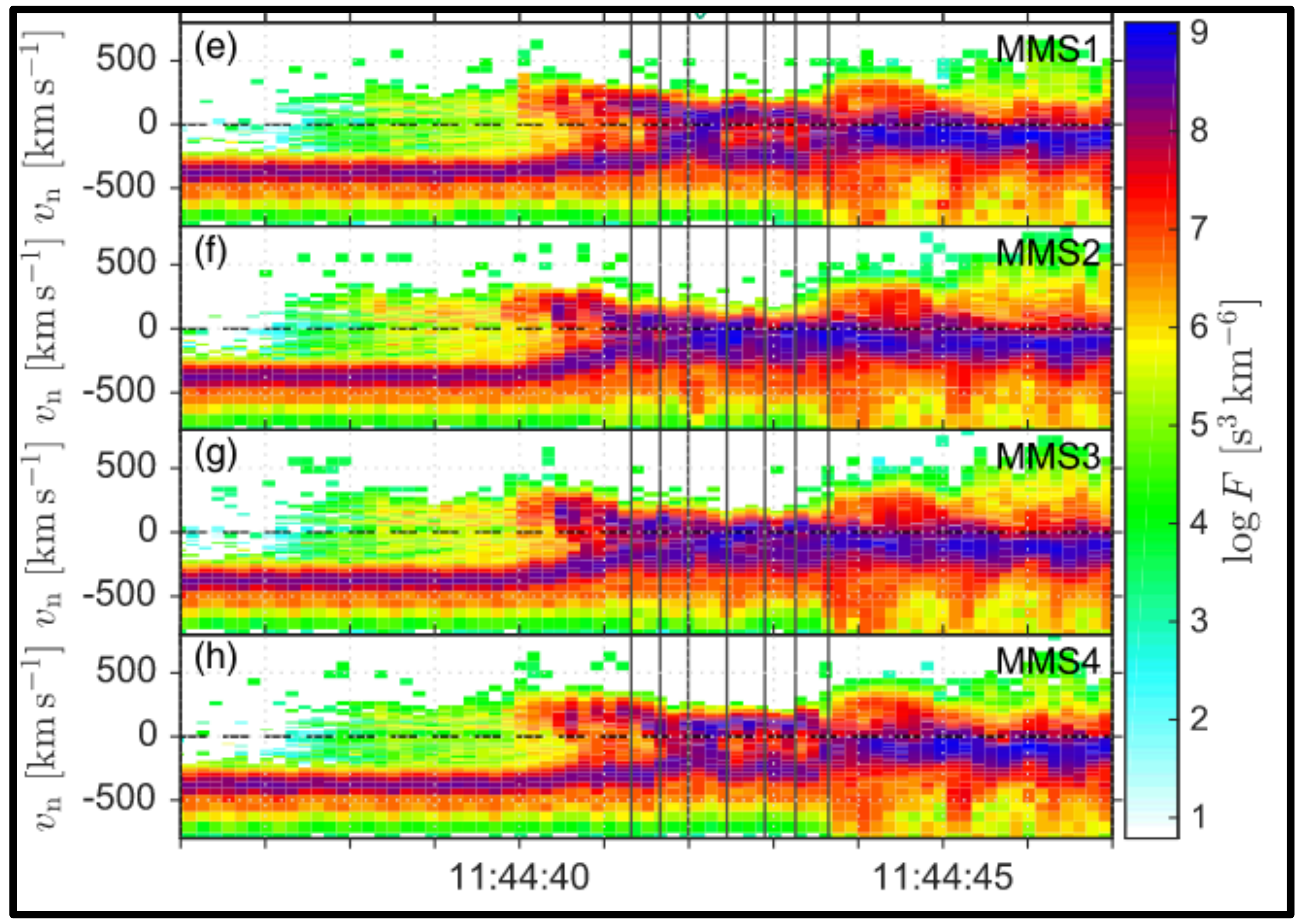
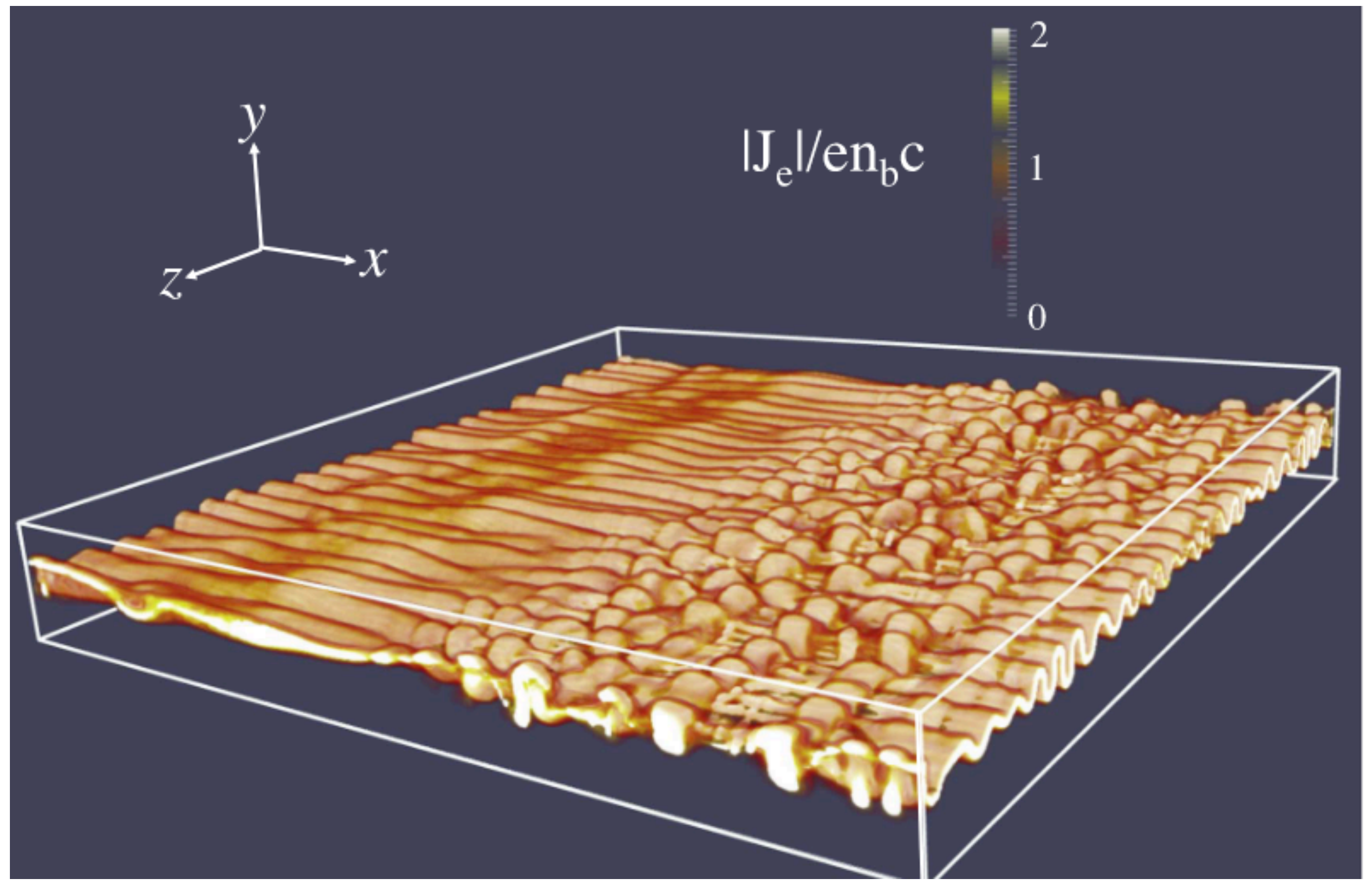
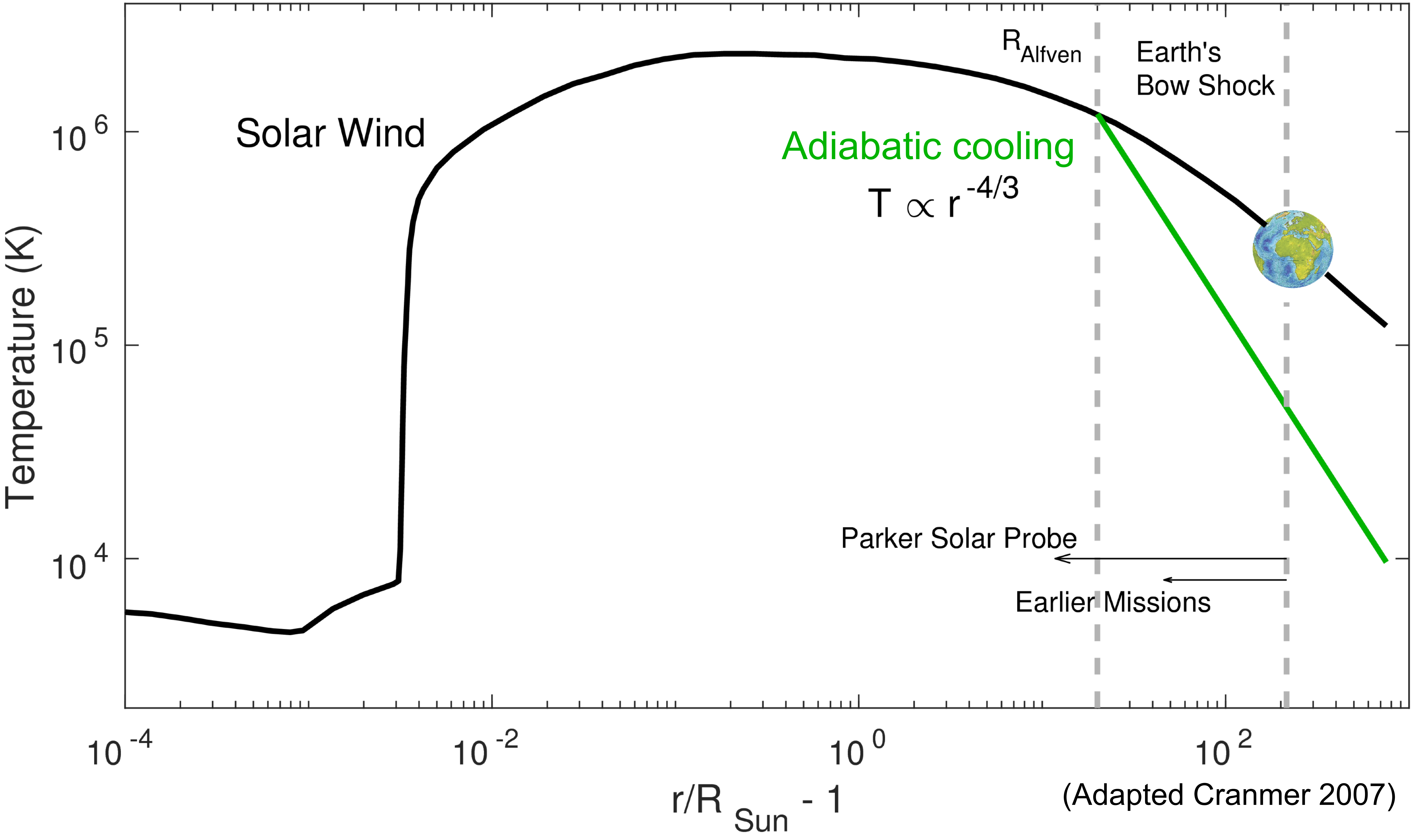
Field-particle correlation is a technique for determining what energy transfer processes are actually important in a given element of plasma

(Johlander 2016)





(Werner 2017)

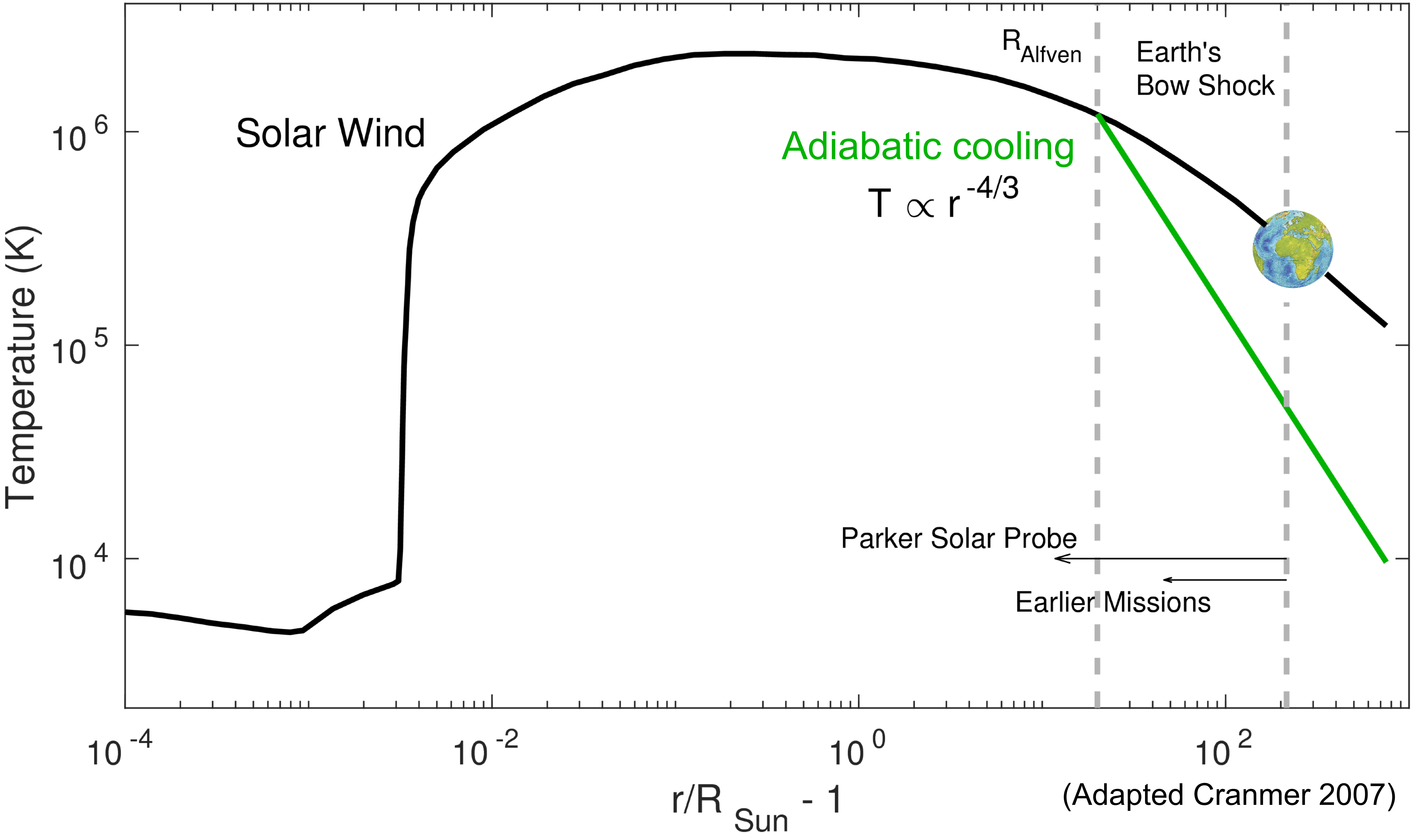


(Adapted Cranmer 2007)

Field-particle correlation is a technique for determining what energy transfer processes are actually important in a given element of plasma

- Understanding the dynamics as a function of time, not just in the asymptotic limit is critical
- Need to distinguish the secular evolution from oscillatory motion
- Distinguishing between multiple processes occurring simultaneously critical for understanding the physics

Measurements are primarily **in situ**



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What is the Field-Particle Correlation (FPC)?

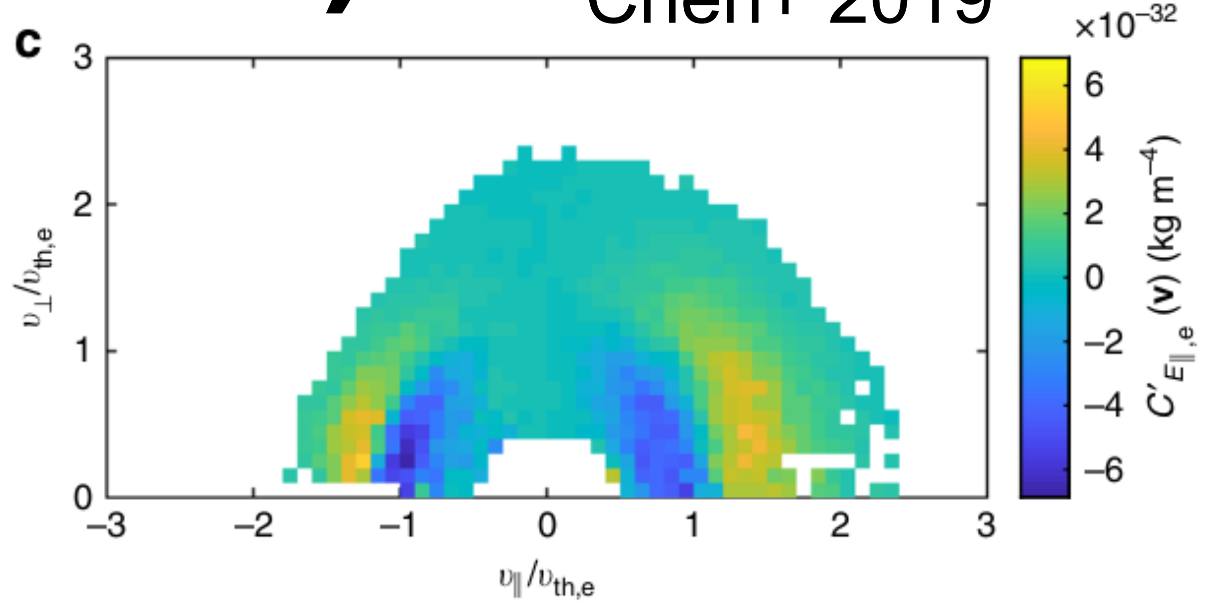
Starting with the Vlasov equation, we multiply by the energy for the given species ($m_s v^2/2$), to obtain an expression for the evolution of the phase space energy density

$$\frac{\partial w_s}{\partial t} = -\mathbf{v}\cdot\nabla w_s - q_s\frac{v^2}{2}\mathbf{E}\cdot\frac{\partial f_s}{\partial v} - q_s\frac{v^2}{2}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_s}{\partial v}$$

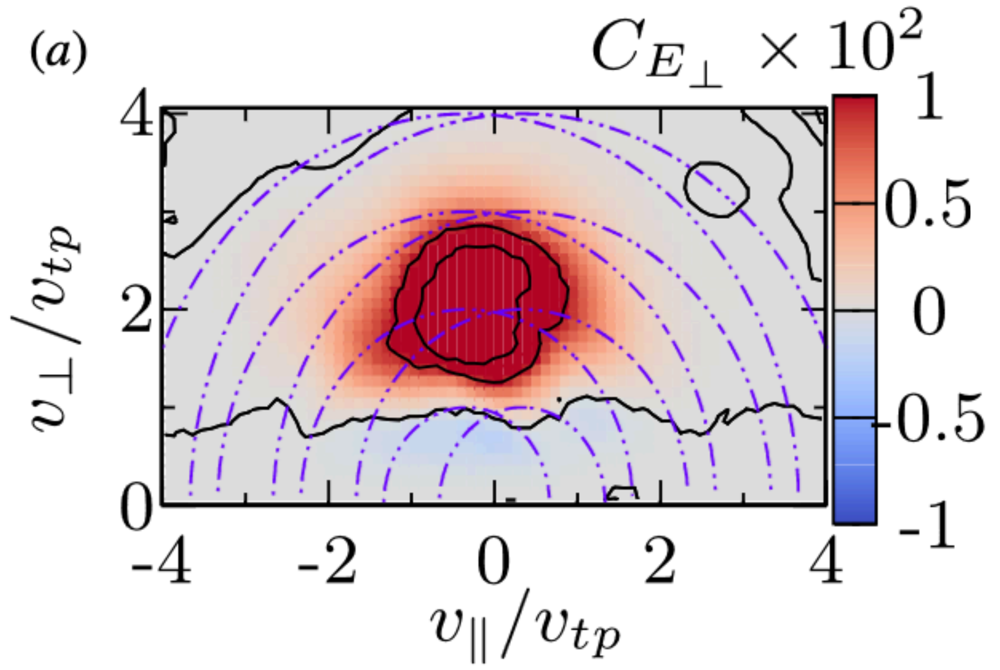
where $w_s(\mathbf{r},\mathbf{v},t) = m_s v^2 f_s(\mathbf{r},\mathbf{v},t)/2$

(Klein+ 2016, Howes+2017)

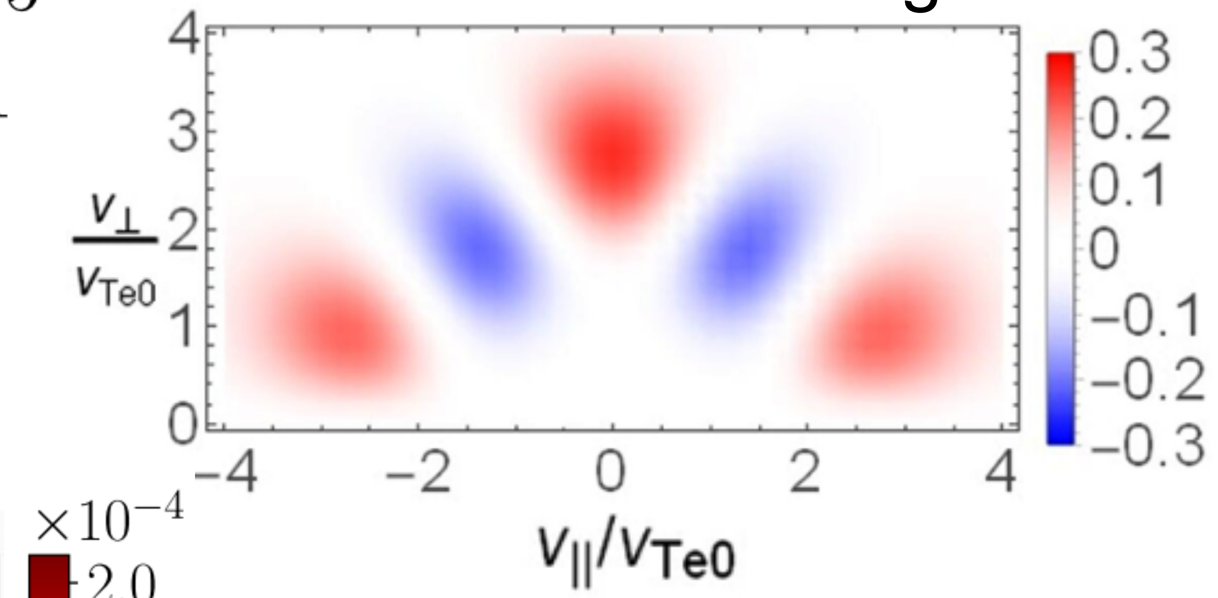
Chen+ 2019

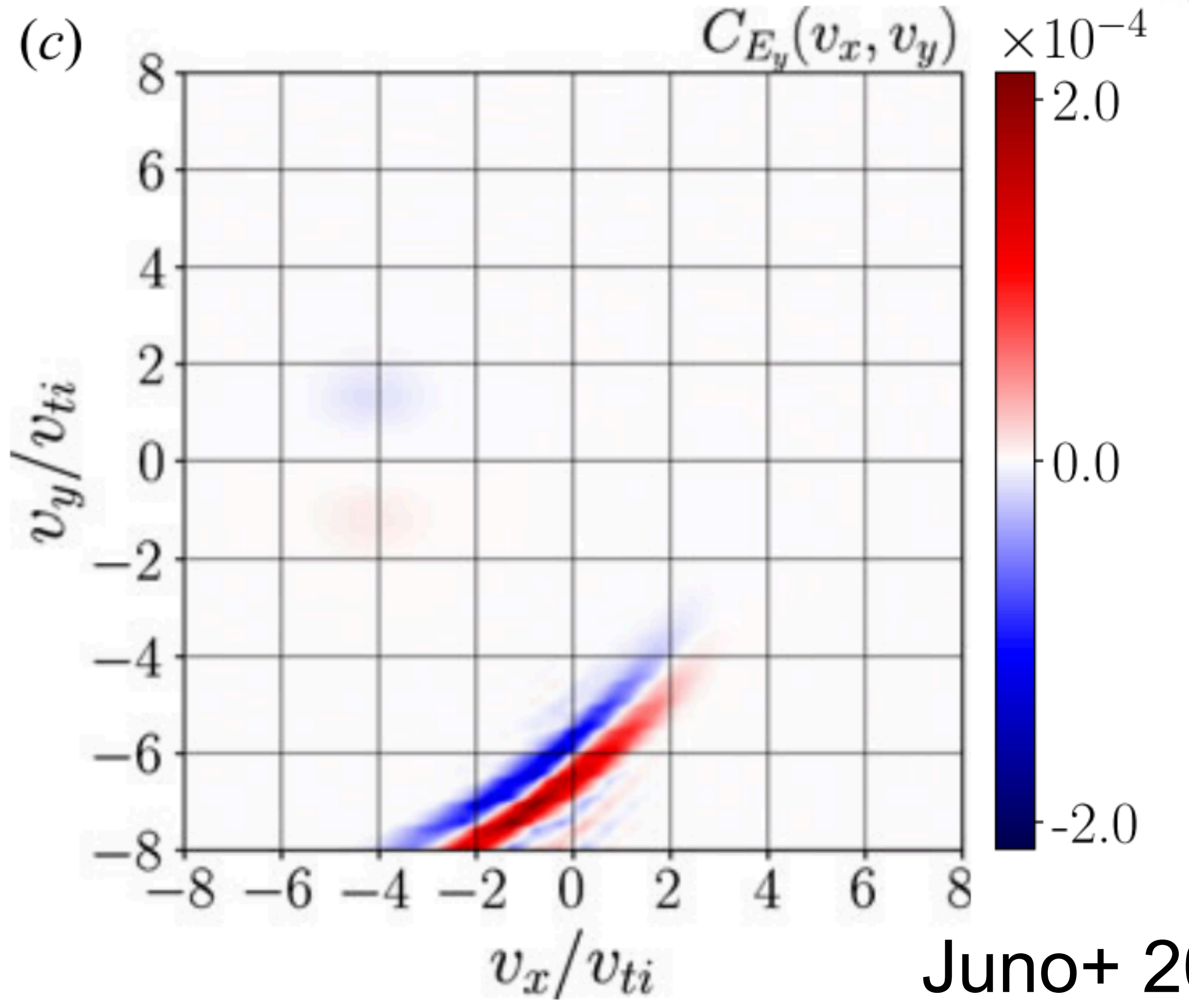


Klein+ 2020



Montag+ 2022





Juno+ 2021

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- Contributes zero net energy - transfers energy within physical and phase space

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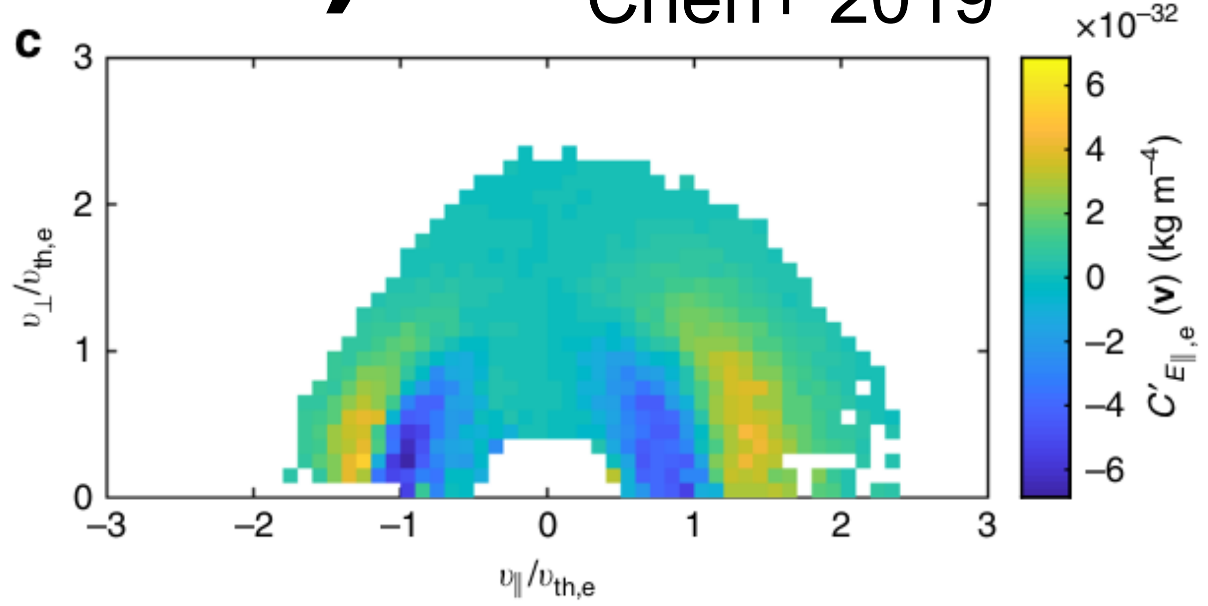

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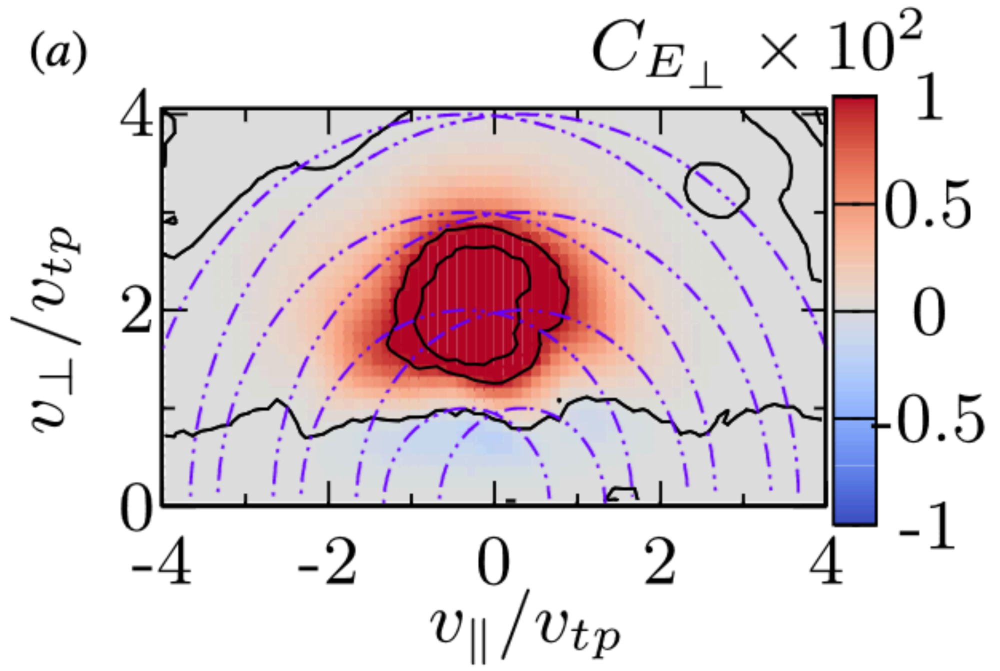

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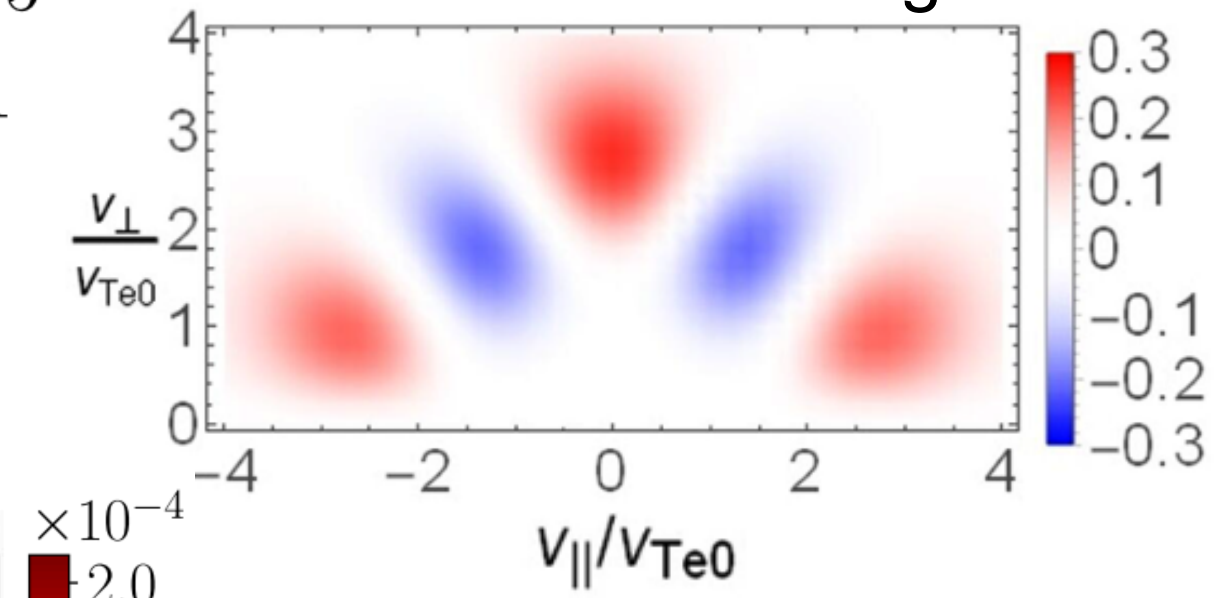

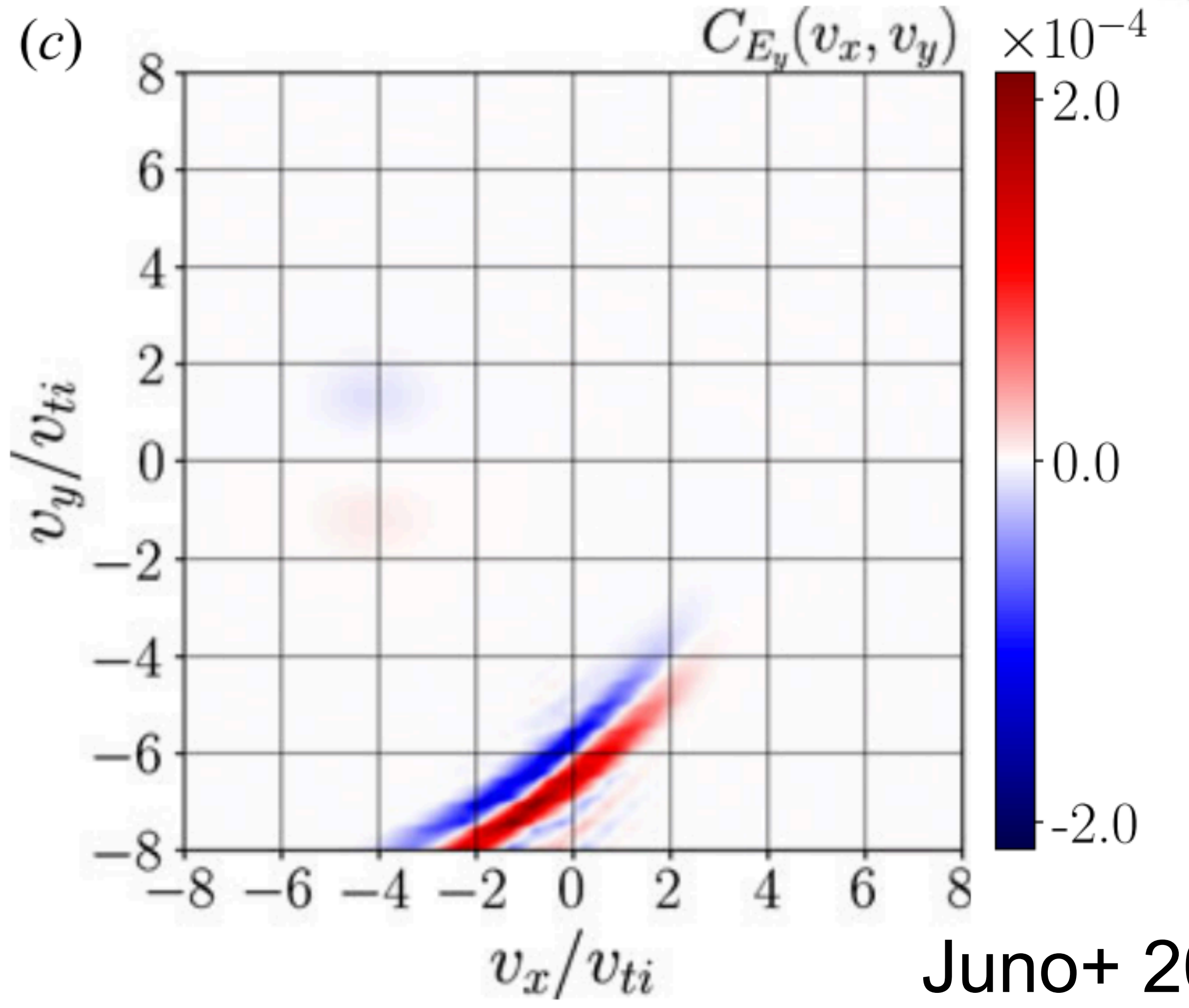

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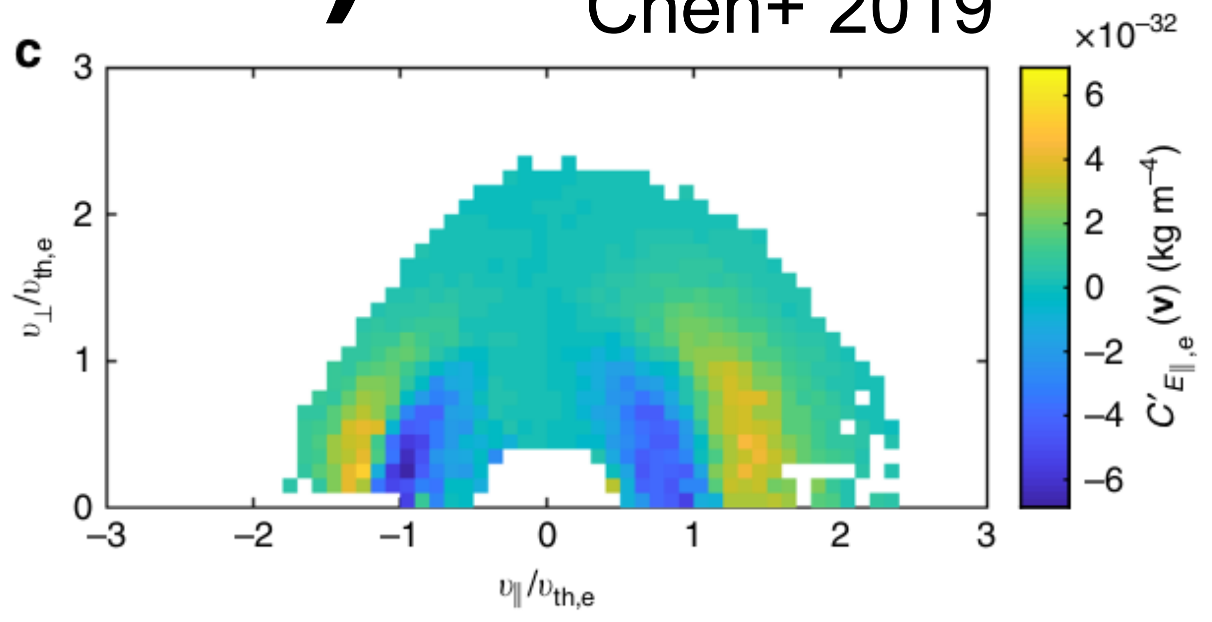

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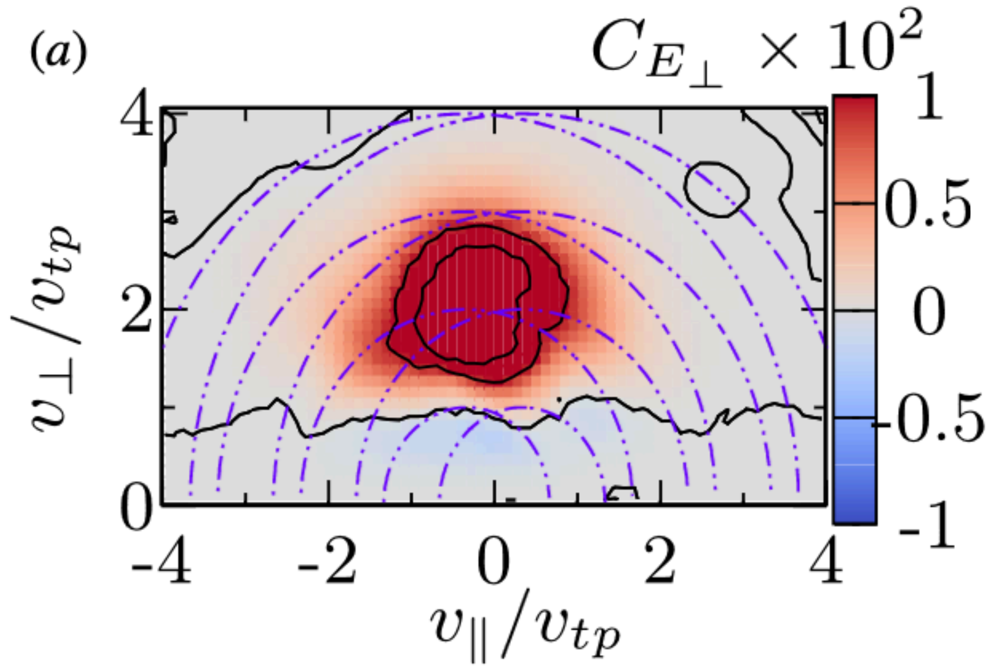

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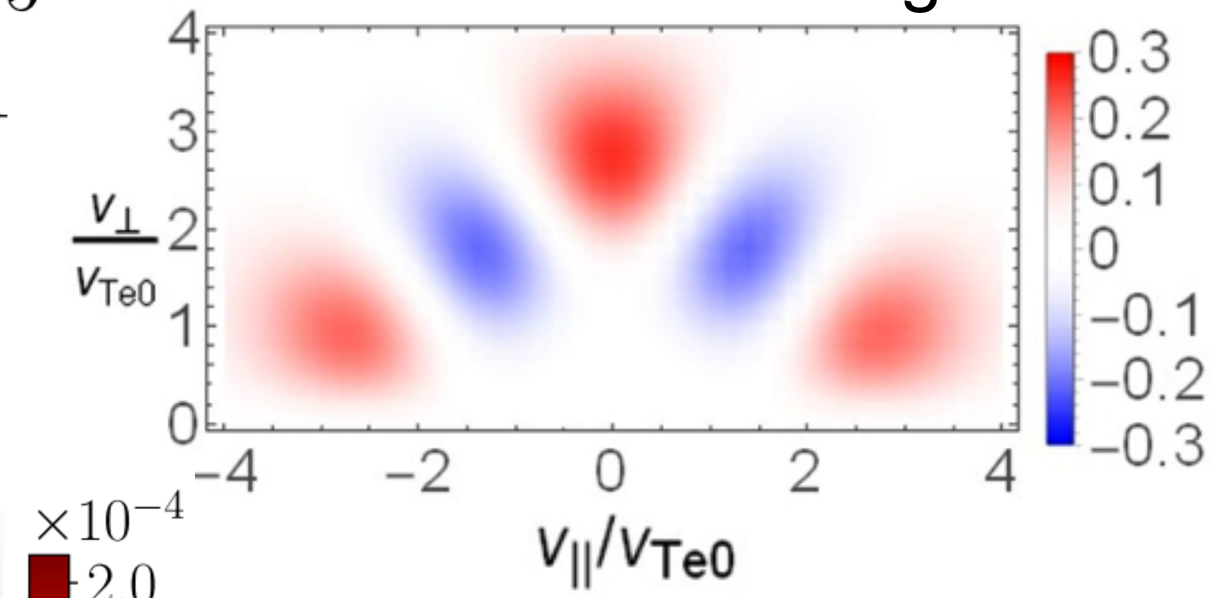

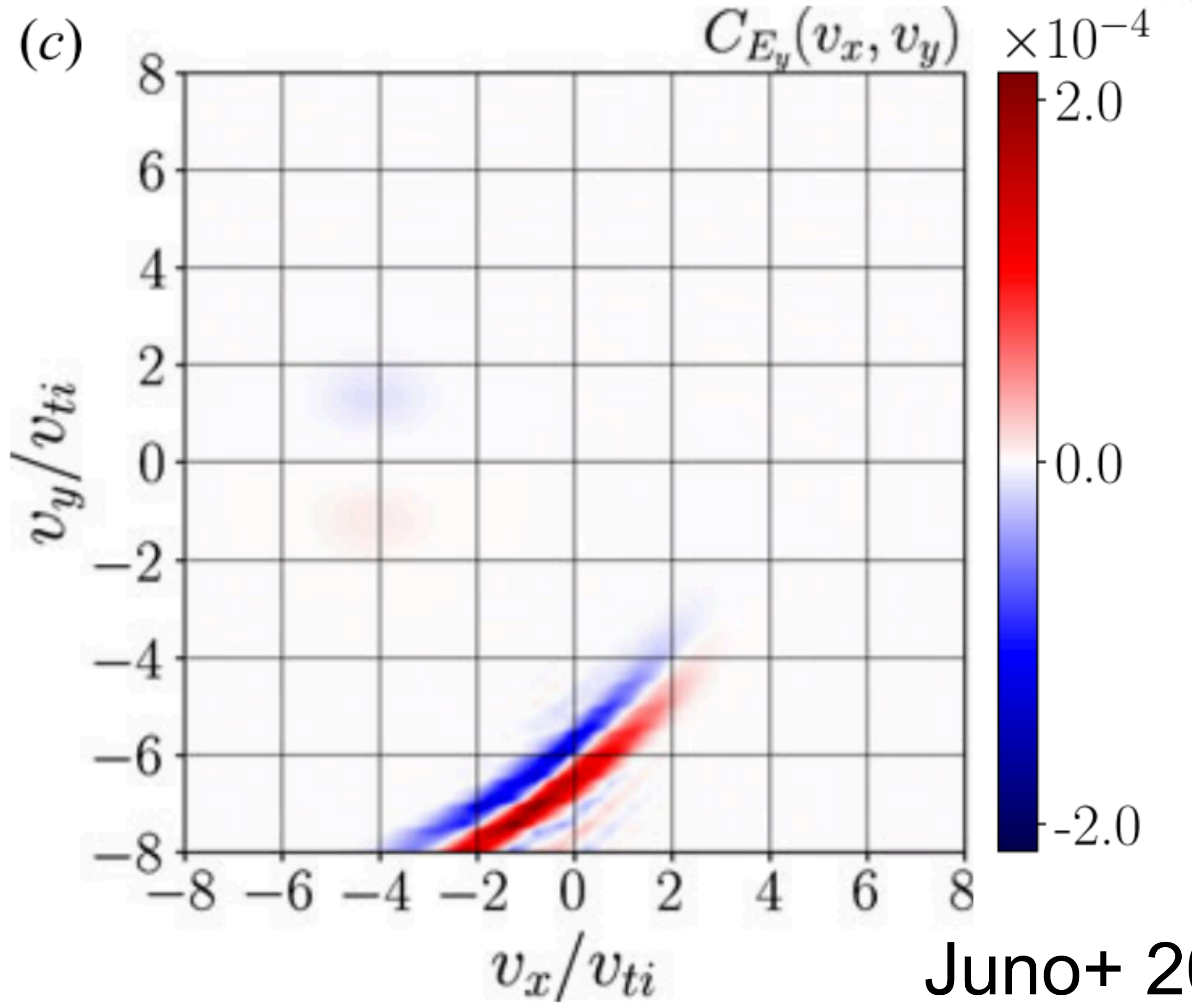

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It can be shown that when integrated over physical space and velocity space, this term is proportional to j·E

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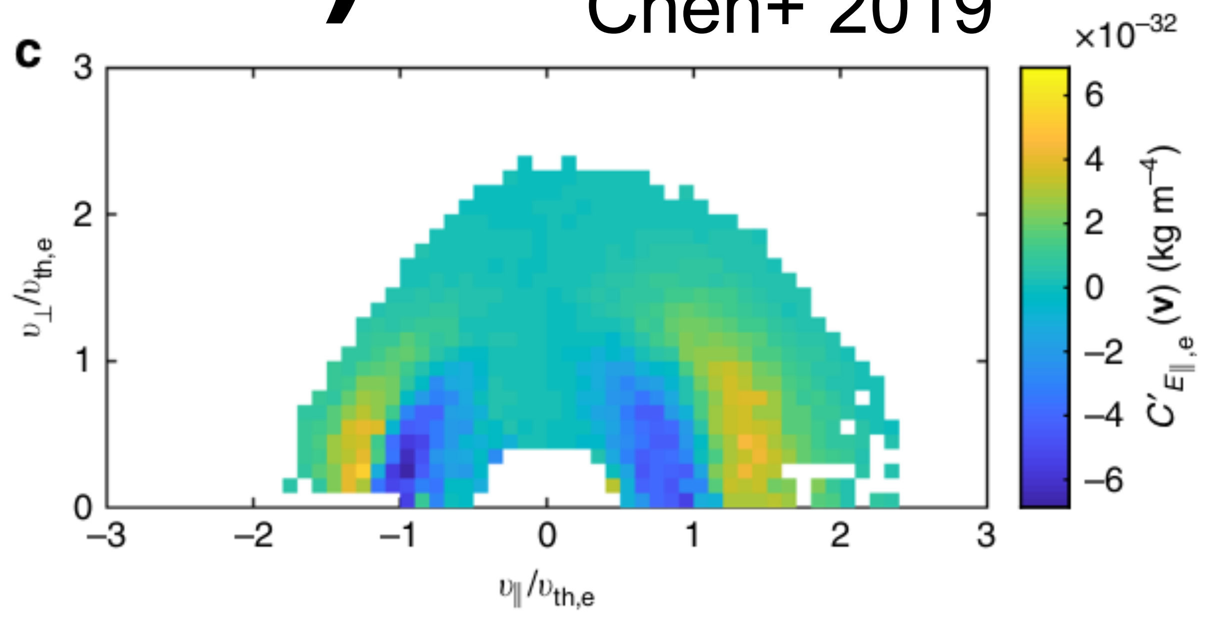
Chen+ 2019



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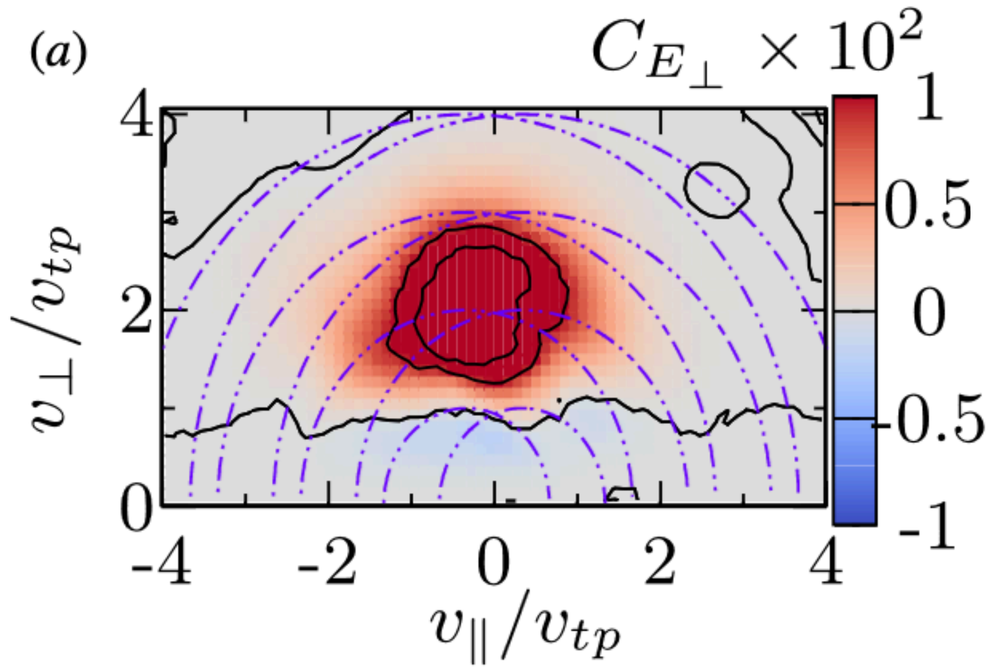

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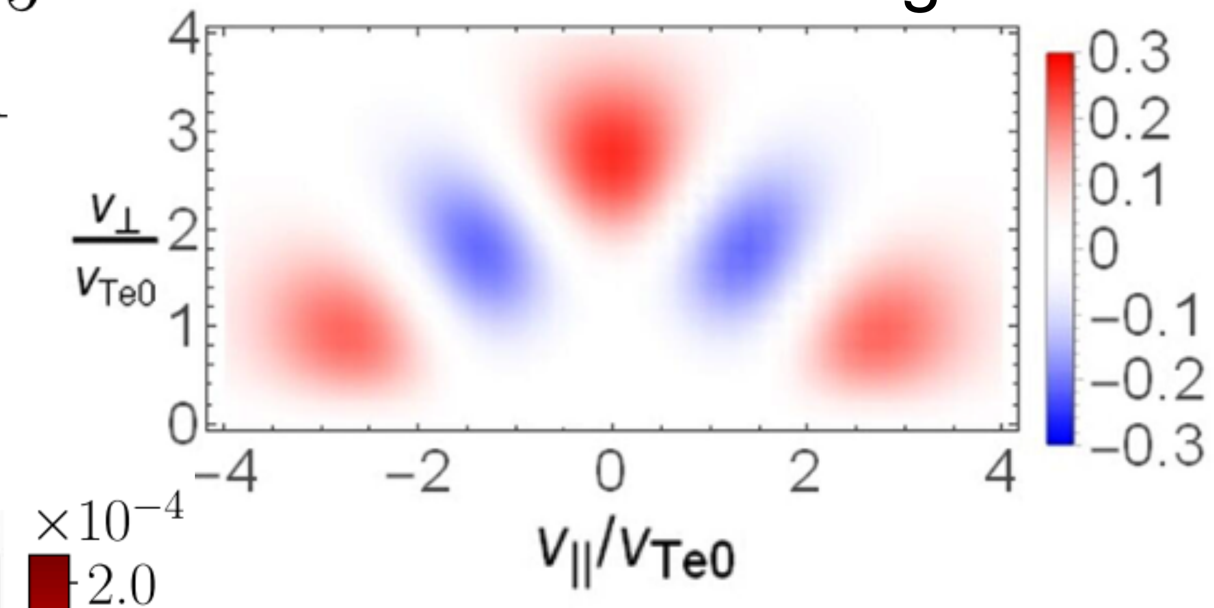

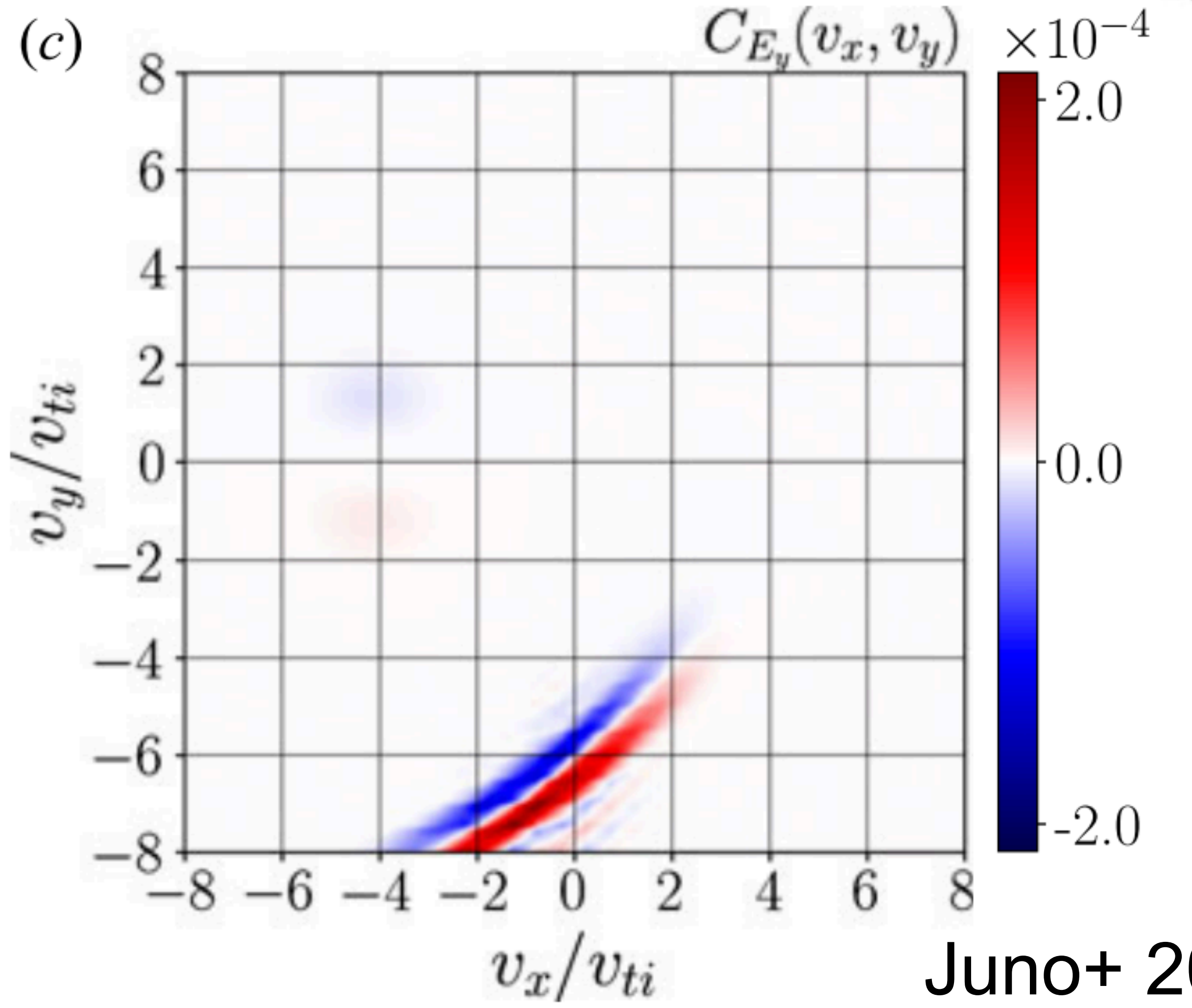

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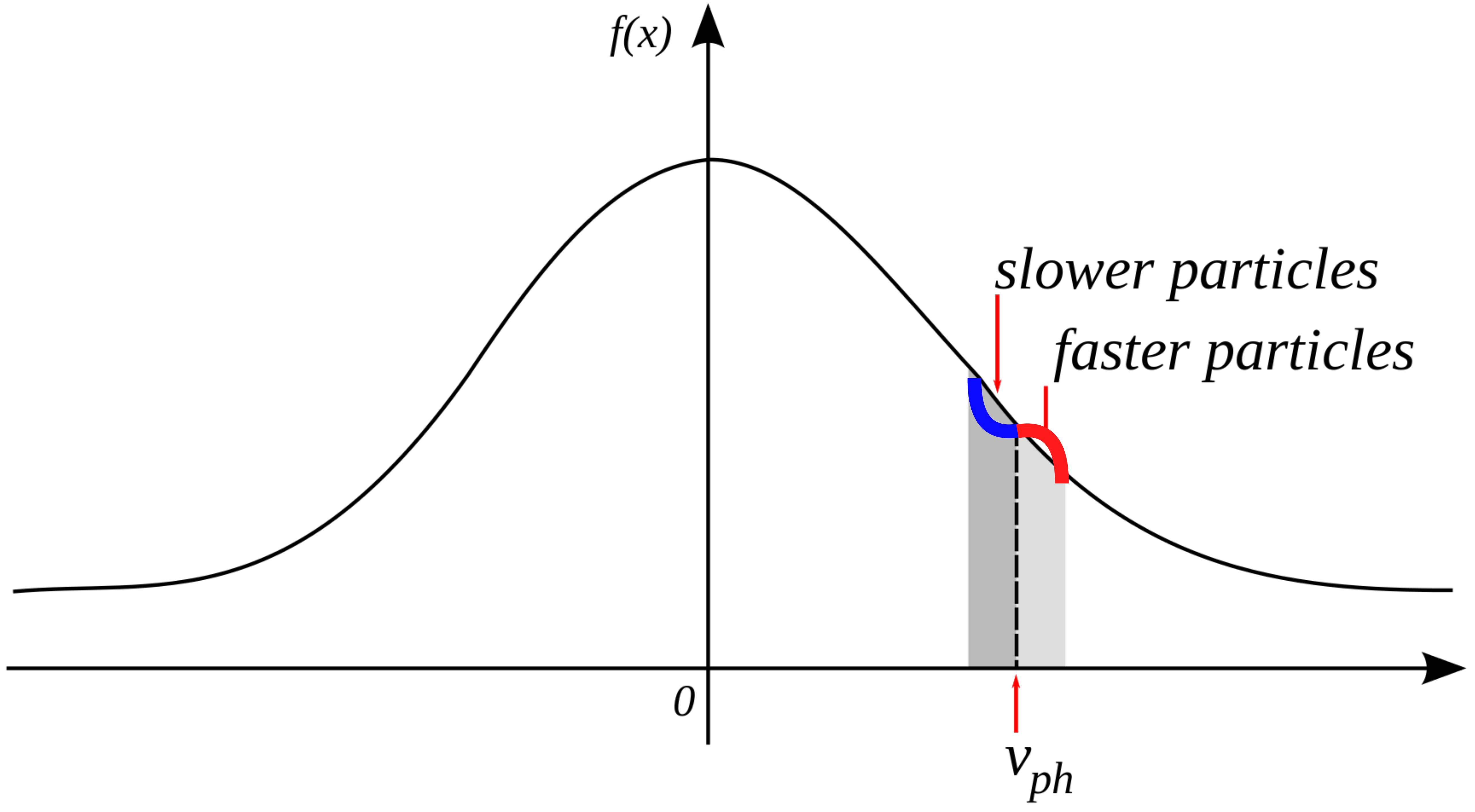
The Field-Particle Correlation (FPC) signal of Landau damping



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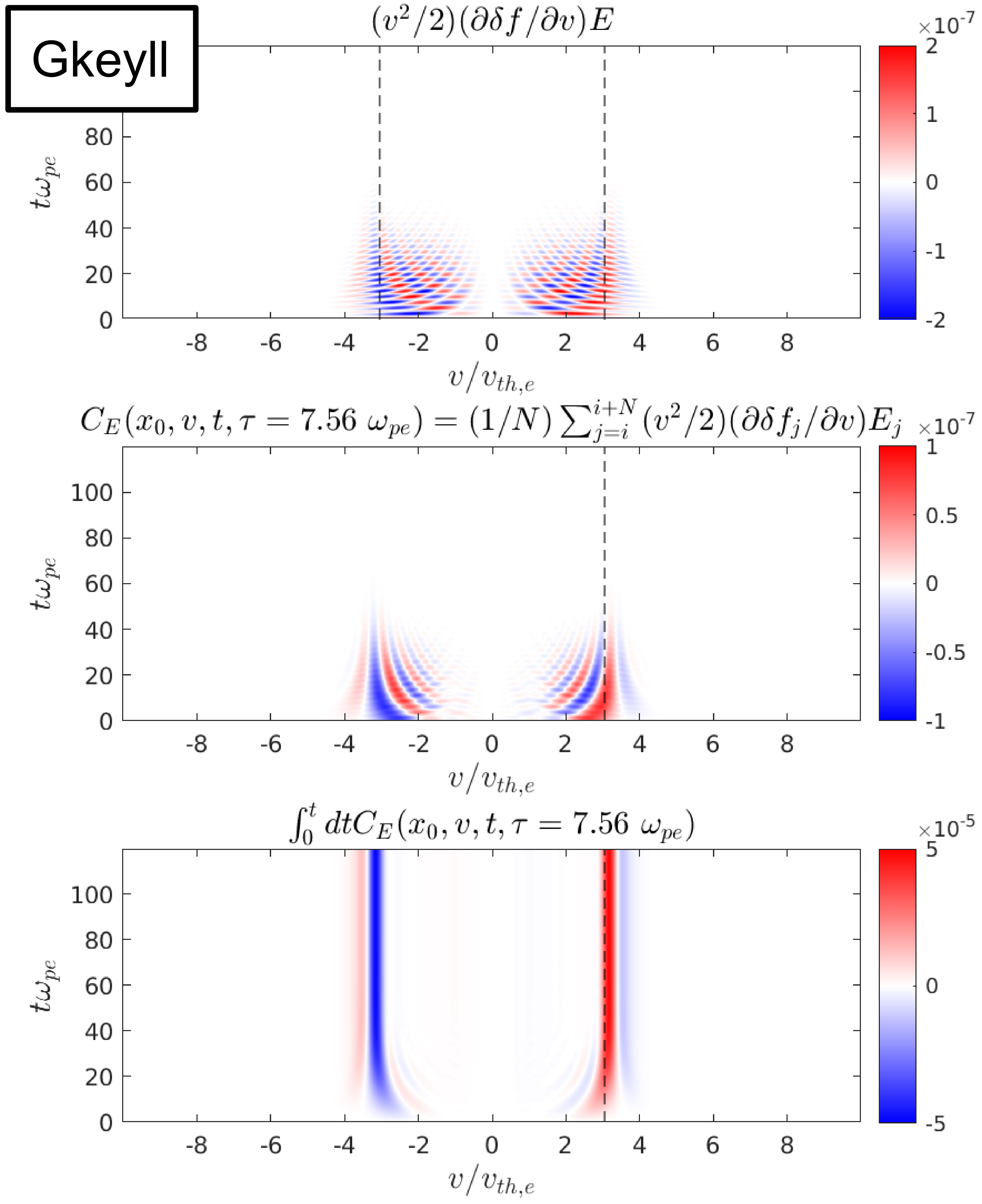

Obtaining the Frequency-Resolved Field-Particle Correlation (FR-FPC)

Start with the field-particle correlation

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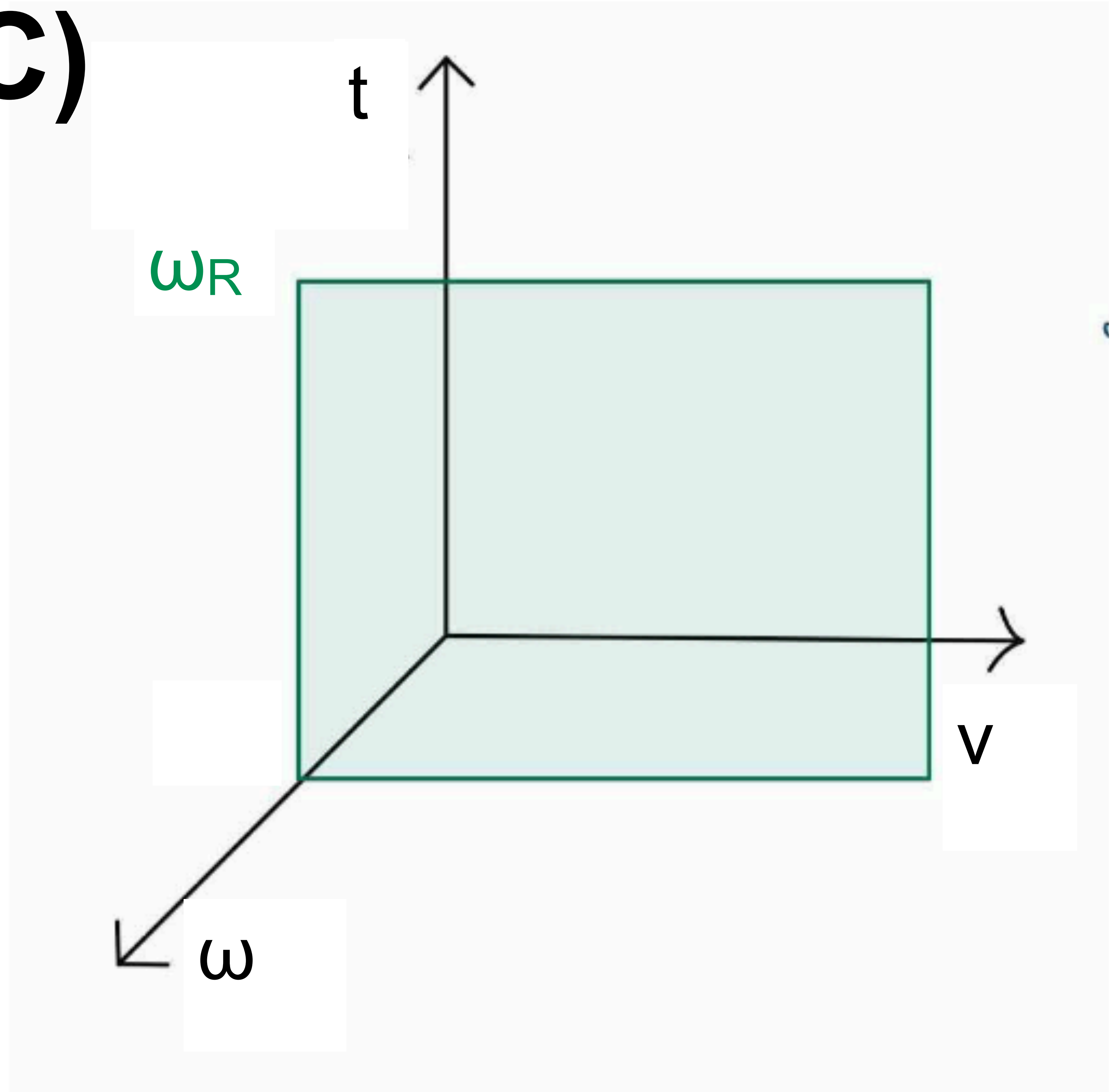

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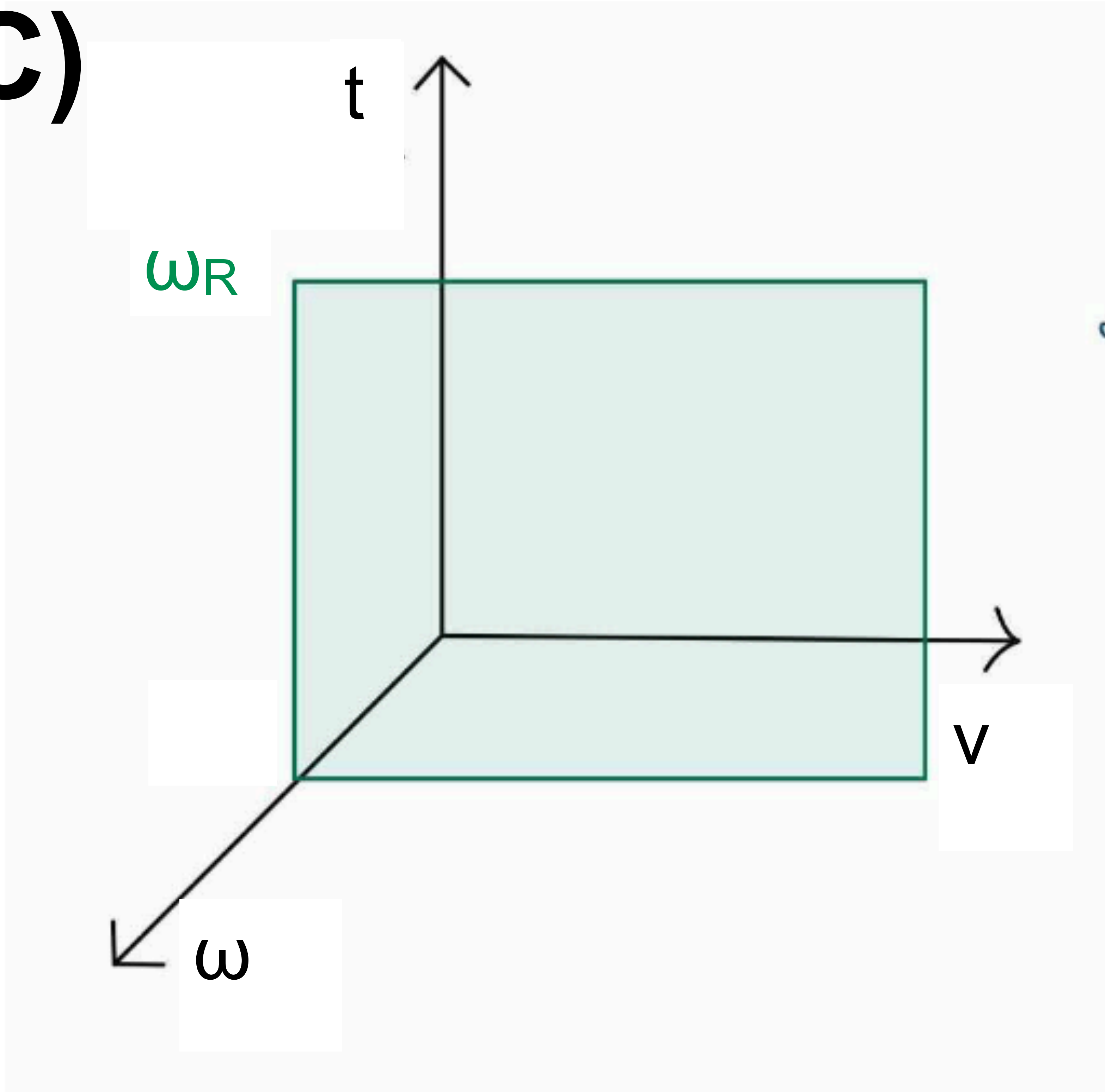

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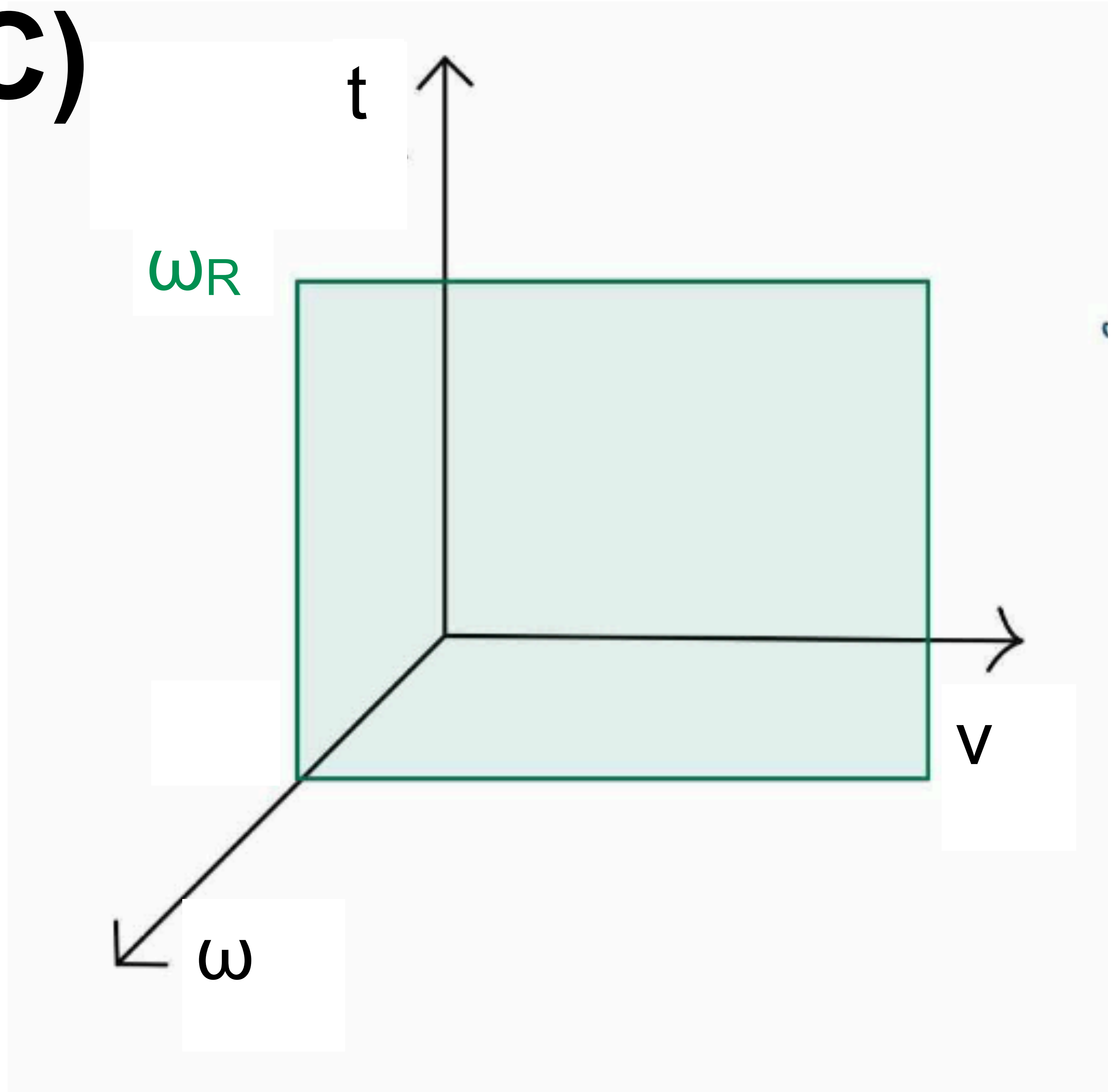
$$E = E_0 e^{\gamma t} e^{-i\omega_R t}$$

Imagine that we have a single wave with frequency ω_R, damped at some rate γ, and plug that into the FPC expression



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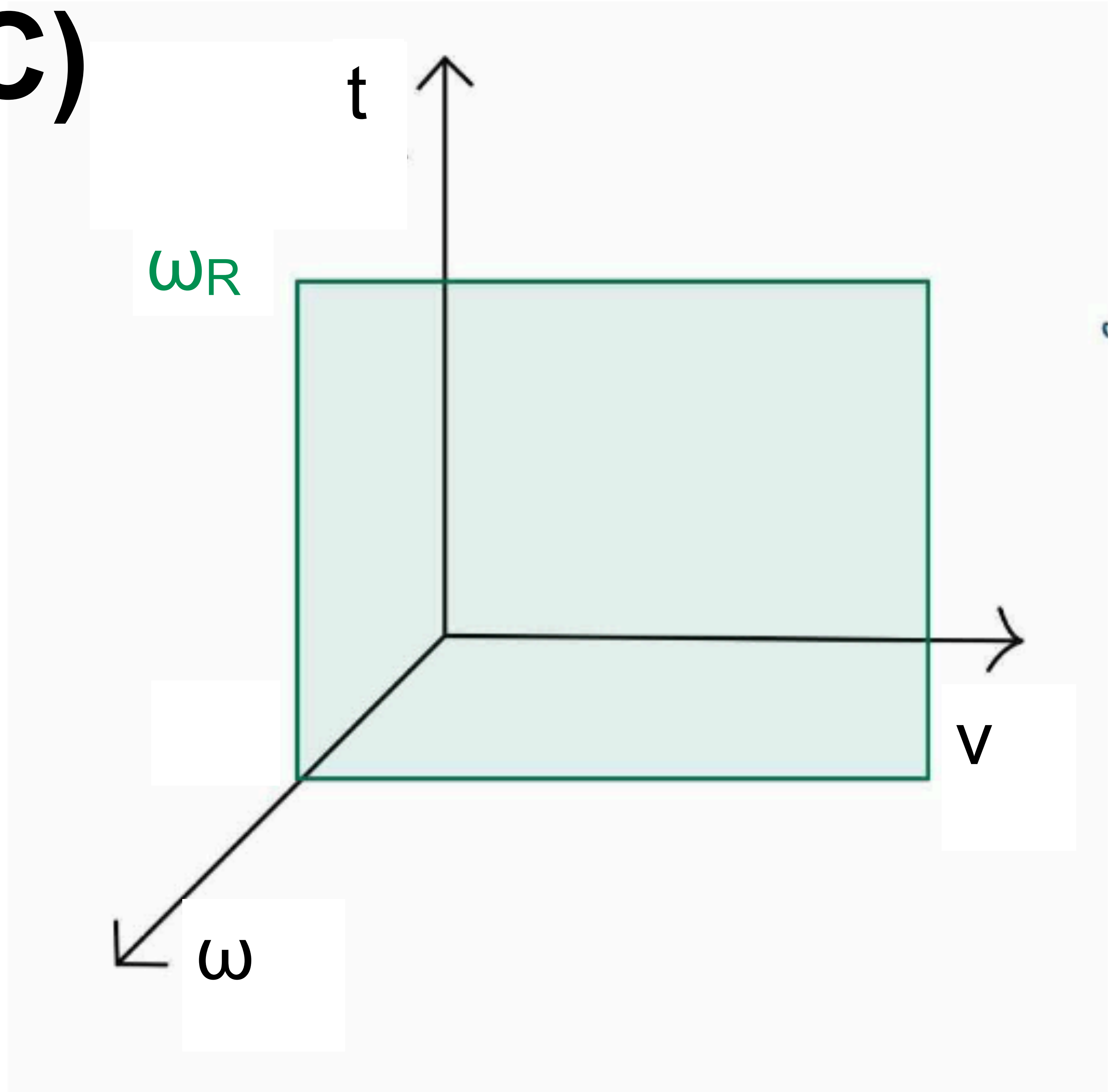

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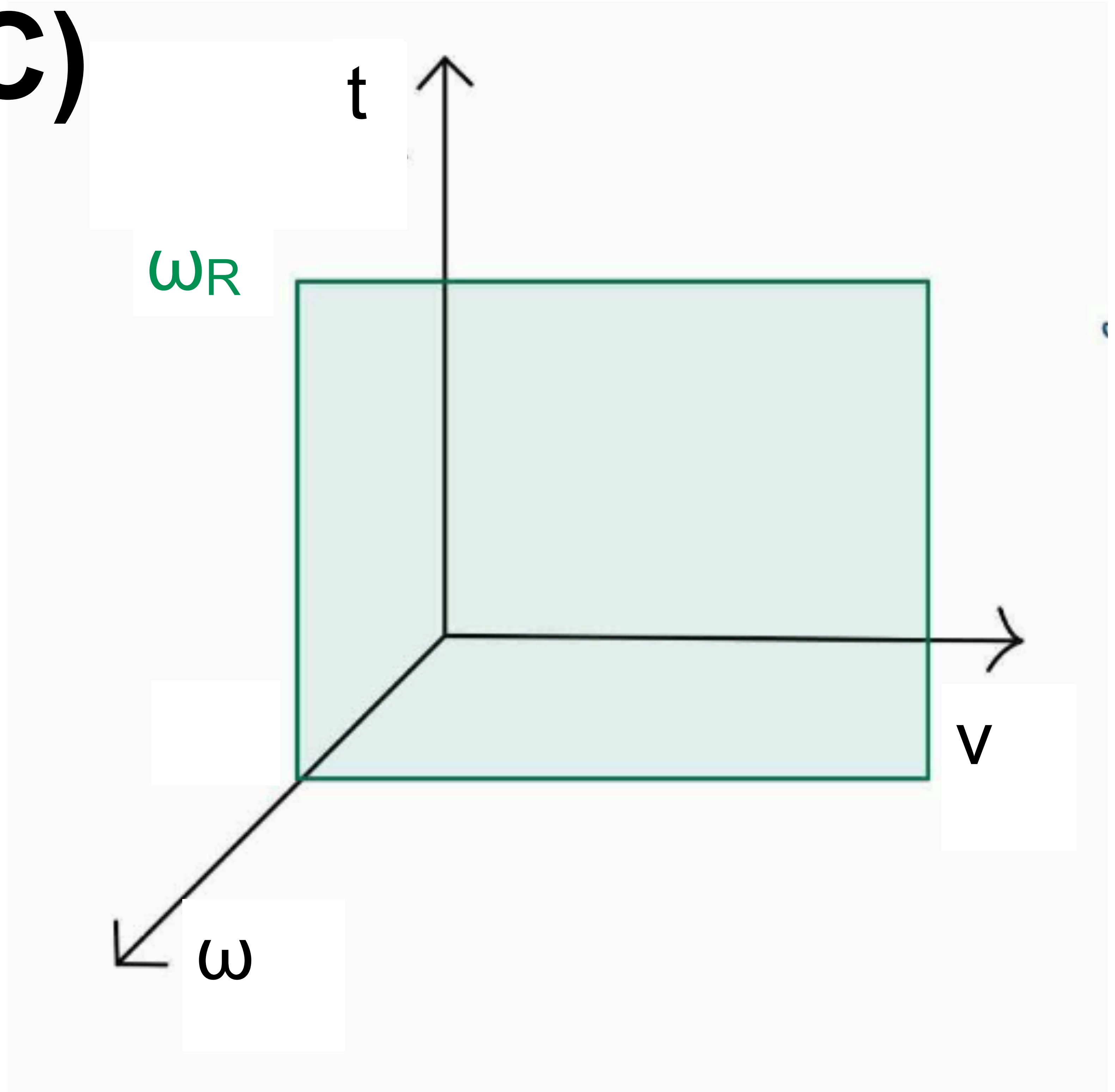

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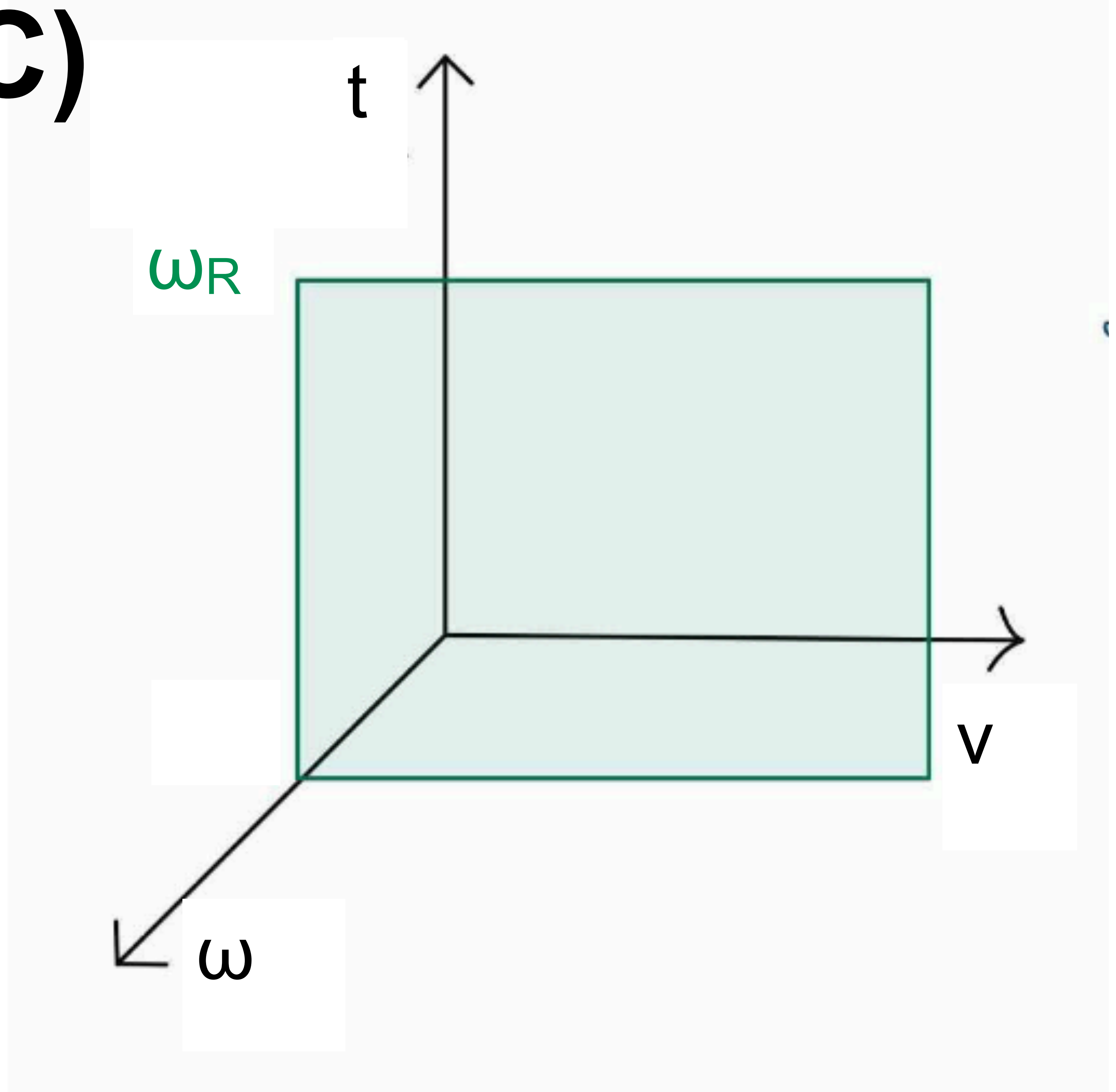

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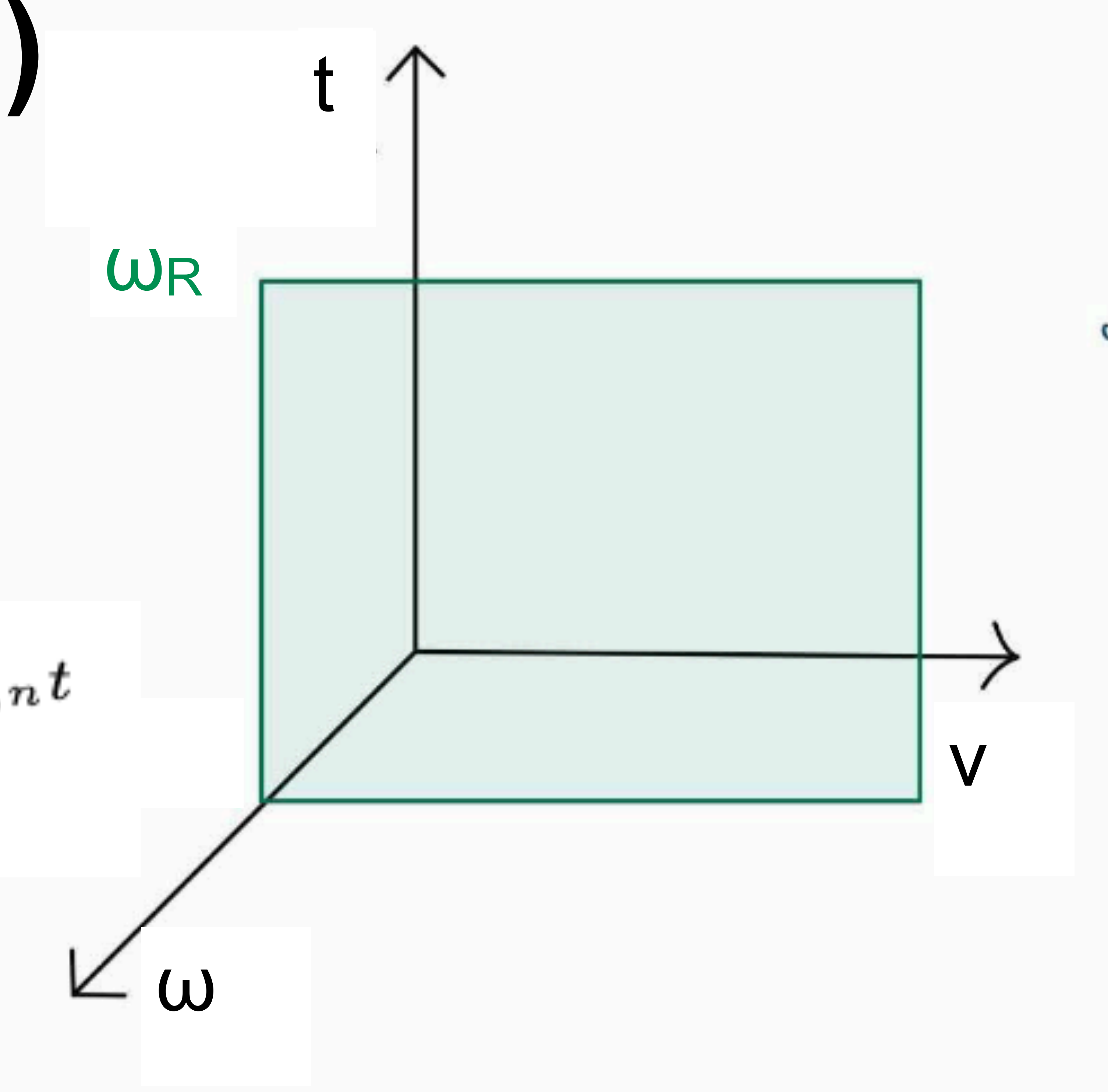

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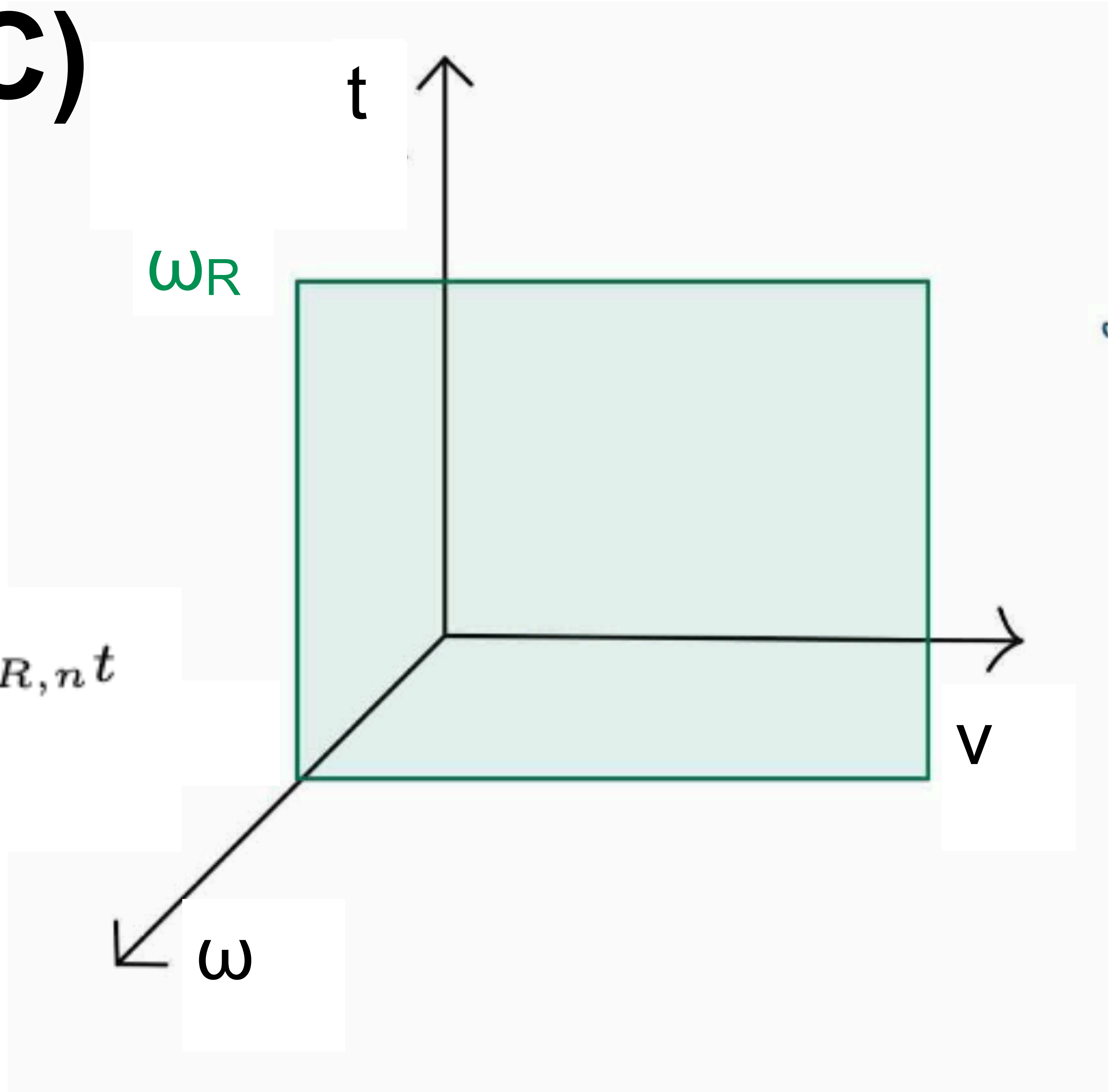

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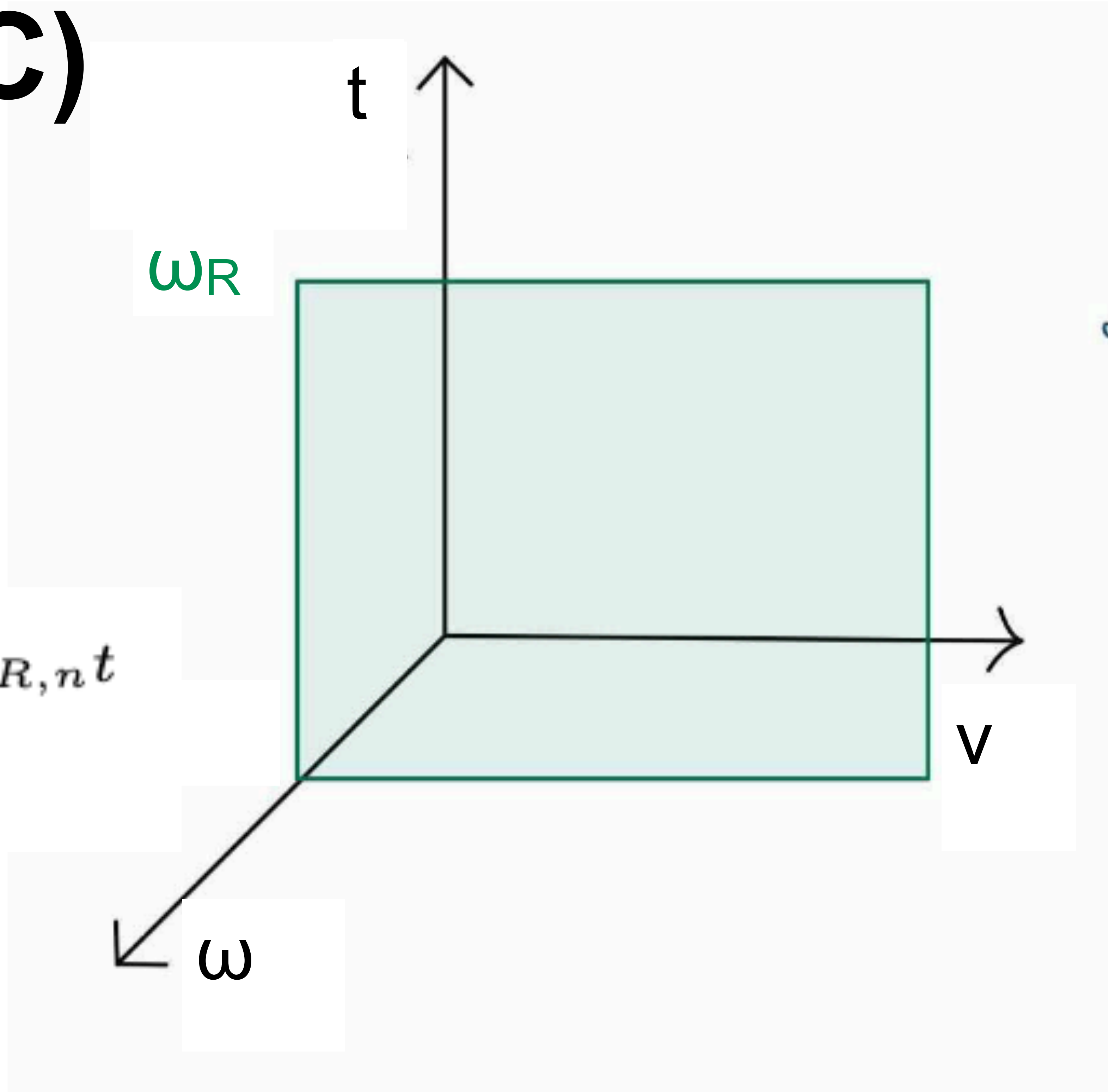

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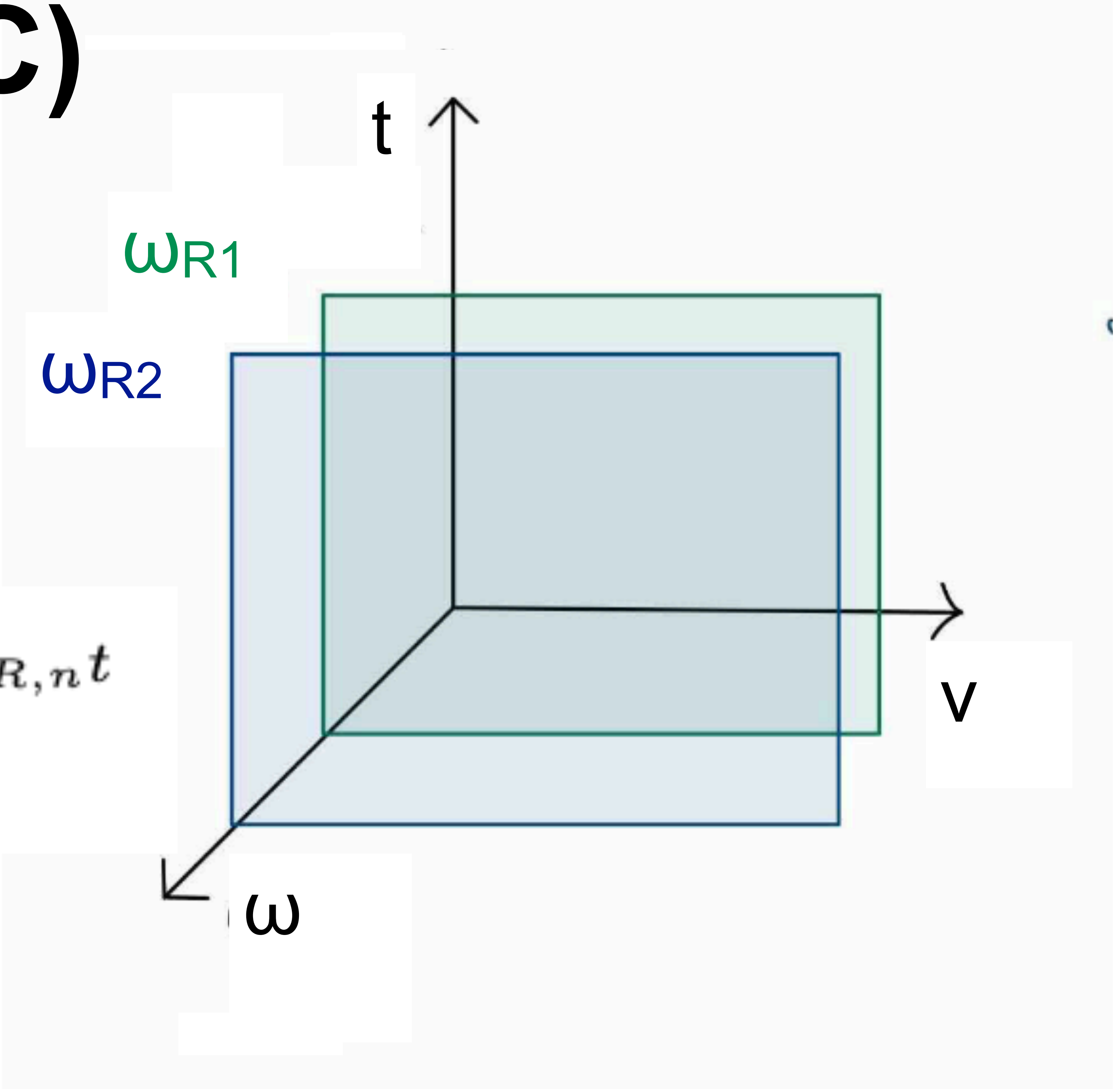

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Overview of the differences between the frequency-resolved FPC and traditional FPC

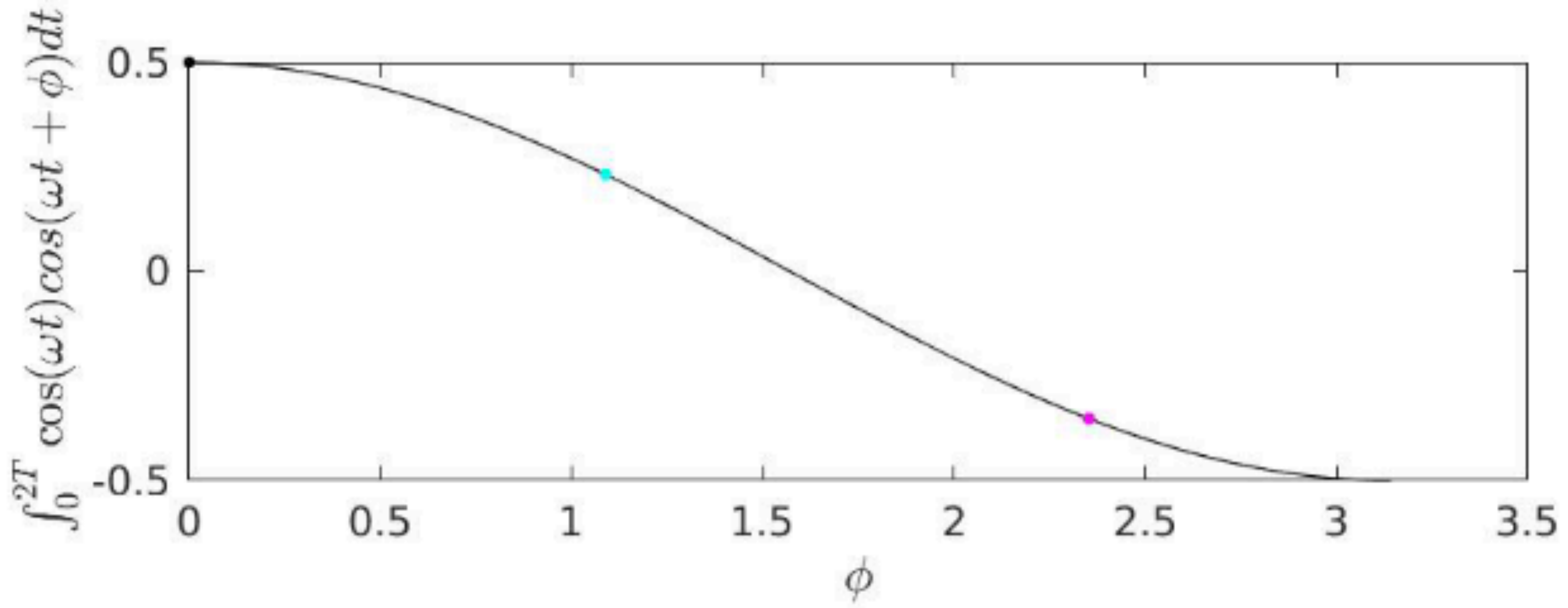
Equivalent to only looking at the real part of a Fourier transform. Both the frequency and phase information is included, but convoluted together.

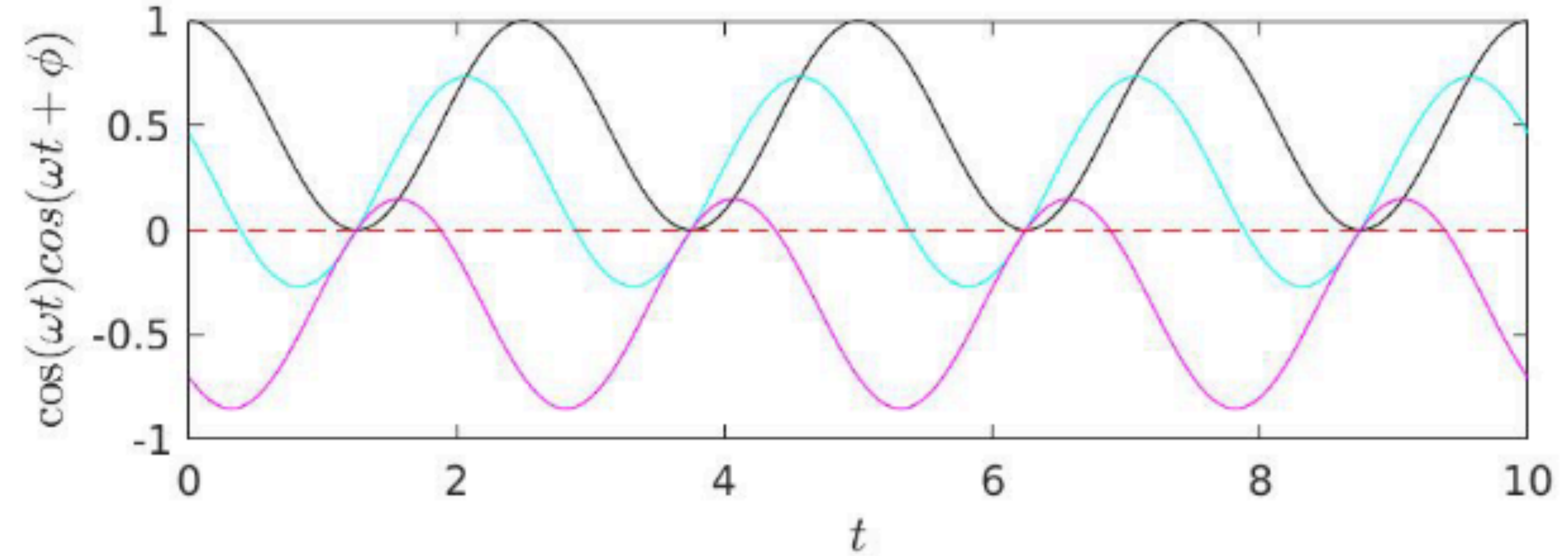
$$C_{E_\parallel} = \sum_n \int_{-\infty}^{\infty} -q_e \frac{v_\parallel^2}{2} \frac{\partial f_e}{\partial v_\parallel} E_{0,n} e^{\gamma_n t} w(t - t_0) e^{-i\omega_{R,n} t} dt$$

$$C_{E_\parallel} = \sum_n \int_{-\infty}^{\infty} x_n(t) w(t - t_0) \left[\cos(\omega_{R,n}) + i \sin(\omega_{R,n})\right] dt$$

By transforming the quantities individually first, we can recover both the magnitude and phase information as a function of time, frequency, and velocity, then evaluate the relative phase for each frequency and point in time.

$$C_E = \left| STFT\left[\Re\left(\frac{v^2}{2}\frac{\partial \delta f}{\partial v}\right)\right]\right| \left|STFT\left[\Re\left(E\right)\right]\right| F(\phi_f, \phi_E)$$





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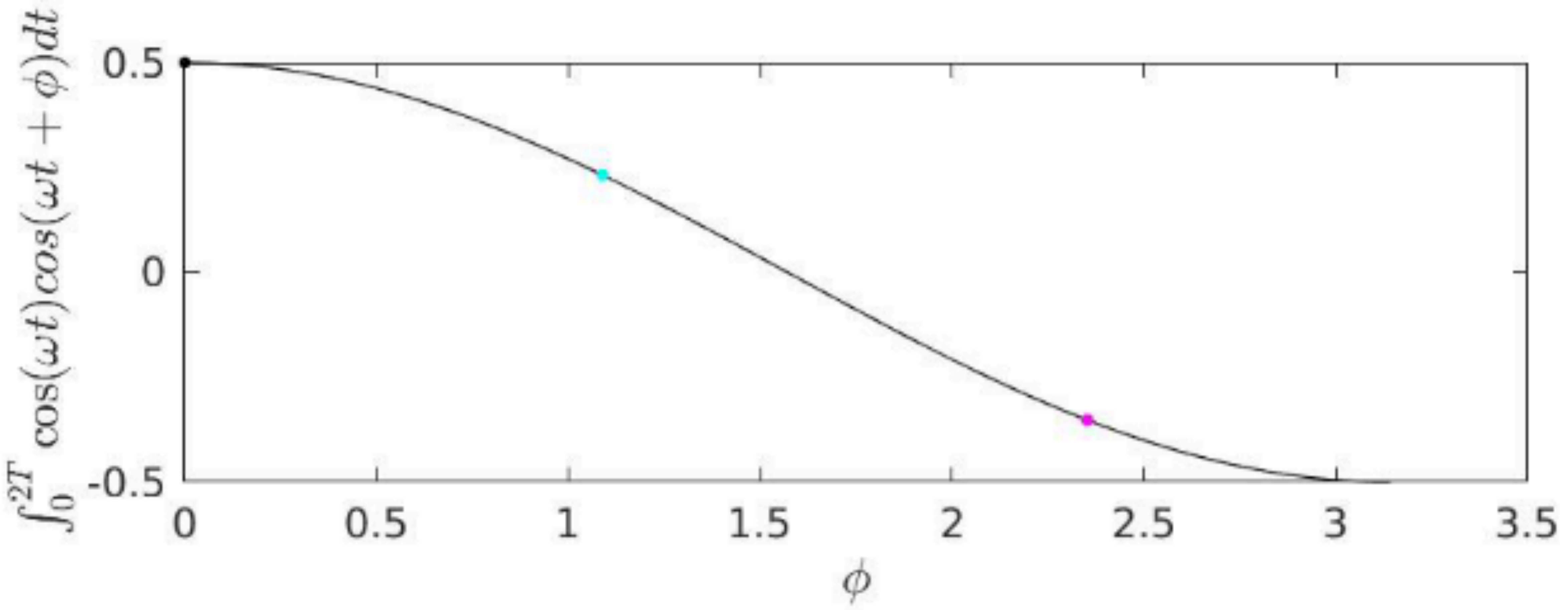
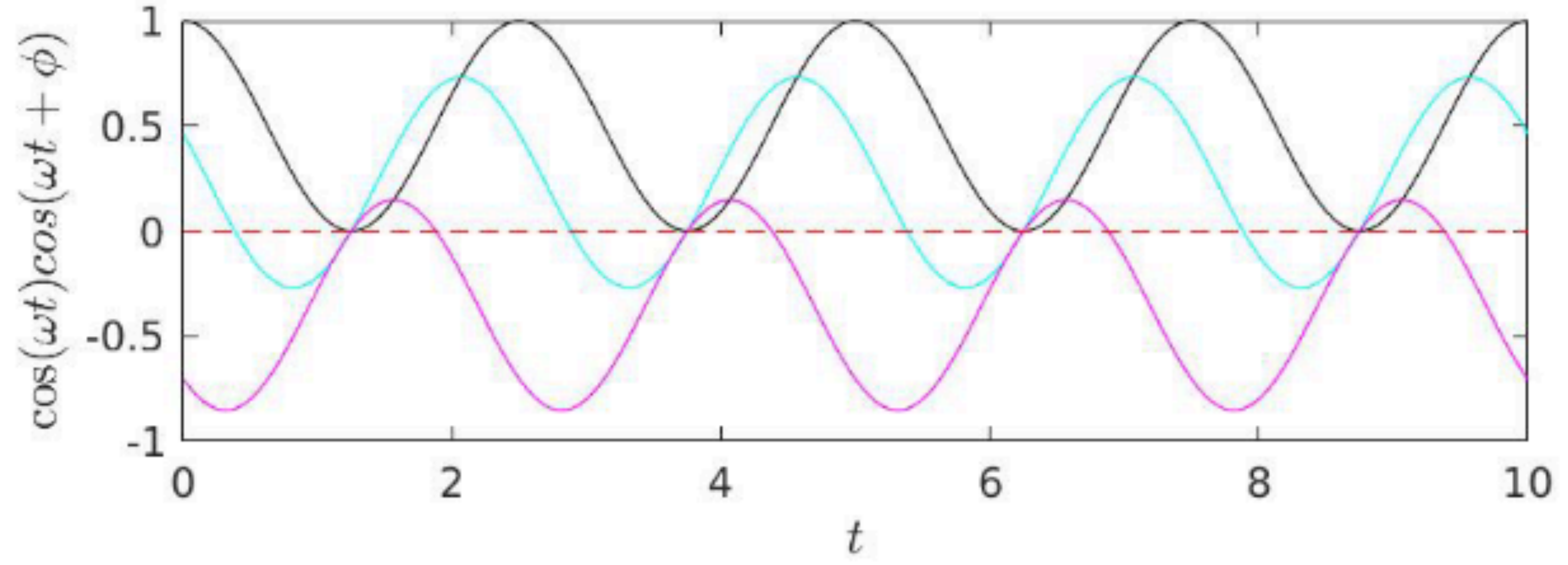
By transforming the quantities individually first, we can recover both the magnitude and phase information as a function of time, frequency, and velocity, then evaluate the relative phase for each frequency and point in time.

$$C_E = \left| STFT \left[\Re \left(\frac{v^2}{2} \frac{\partial \delta f}{\partial v} \right) \right] \right| \left| STFT \left[\Re \left(E \right) \right] \right| F(\phi_f, \phi_E)$$





Overview of the differences between the frequency-resolved FPC and traditional FPC

Equivalent to only looking at the real part of a Fourier transform. Both the frequency and phase information is included, but convoluted together.

$$C_{E_\parallel} = \sum_n \int_{-\infty}^{\infty} -q_e \frac{v_\parallel^2}{2} \frac{\partial f_e}{\partial v_\parallel} E_{0,n} e^{\gamma_n t} w(t - t_0) e^{-i\omega_{R,n} t} dt$$

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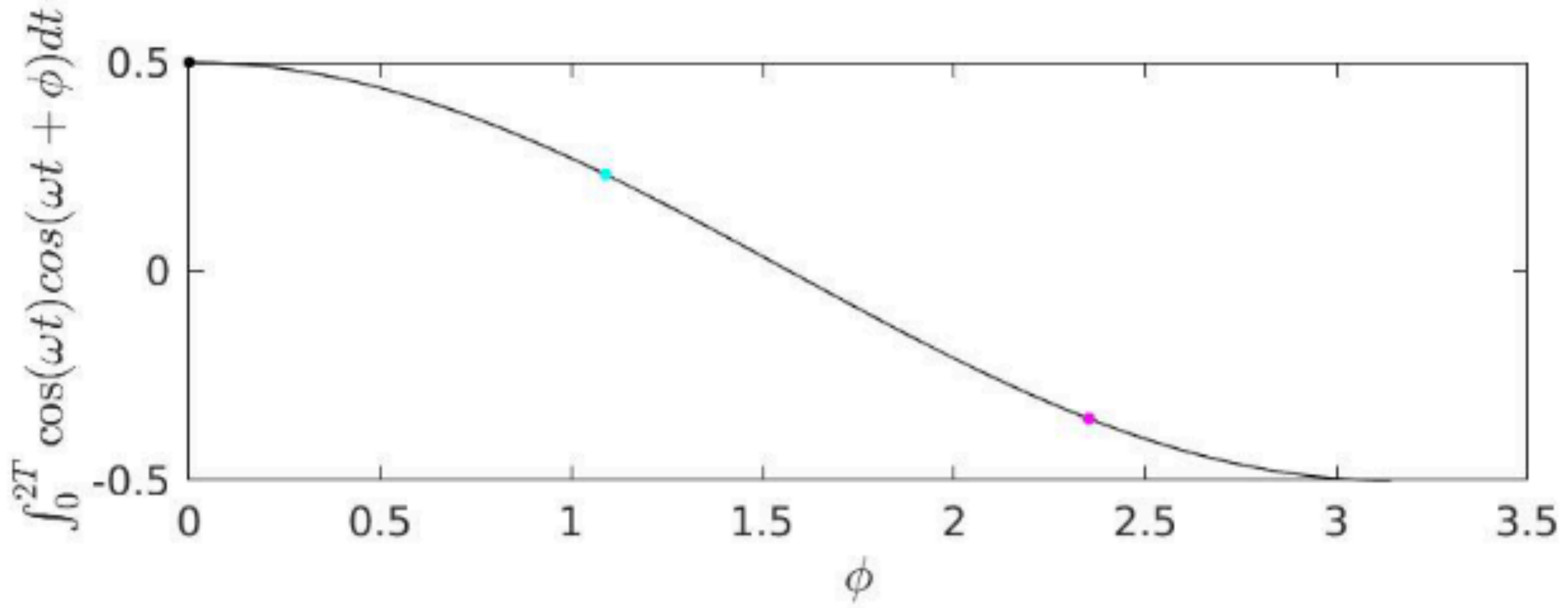

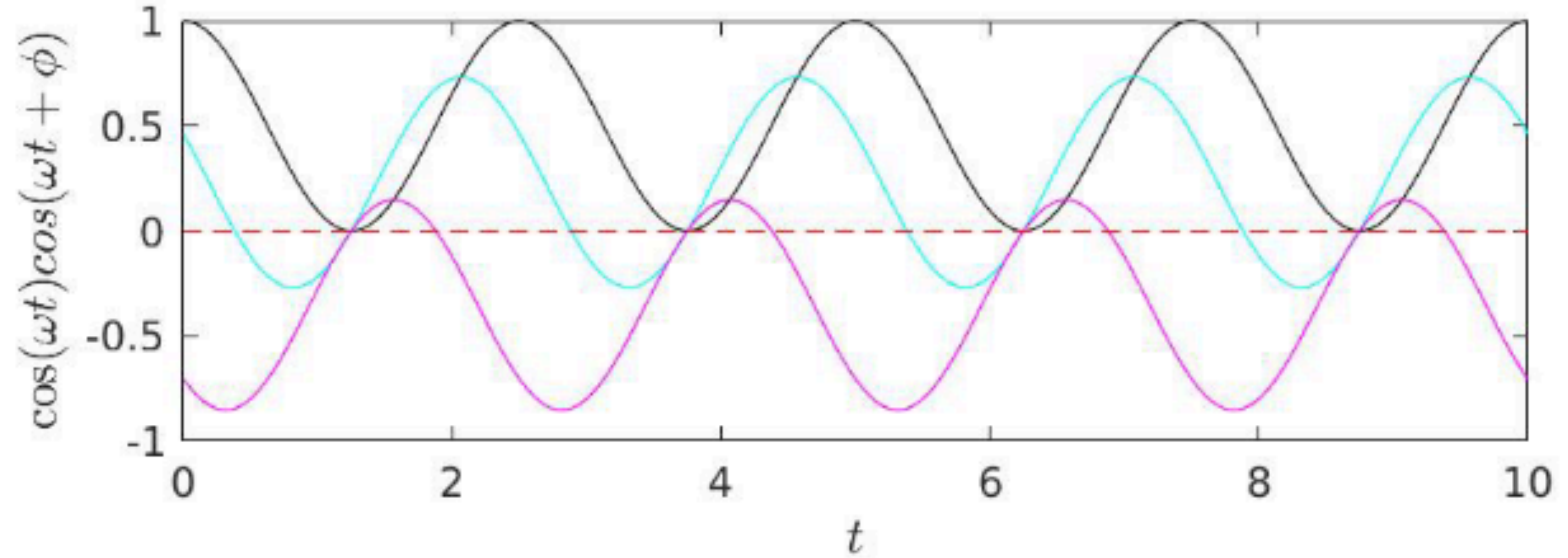

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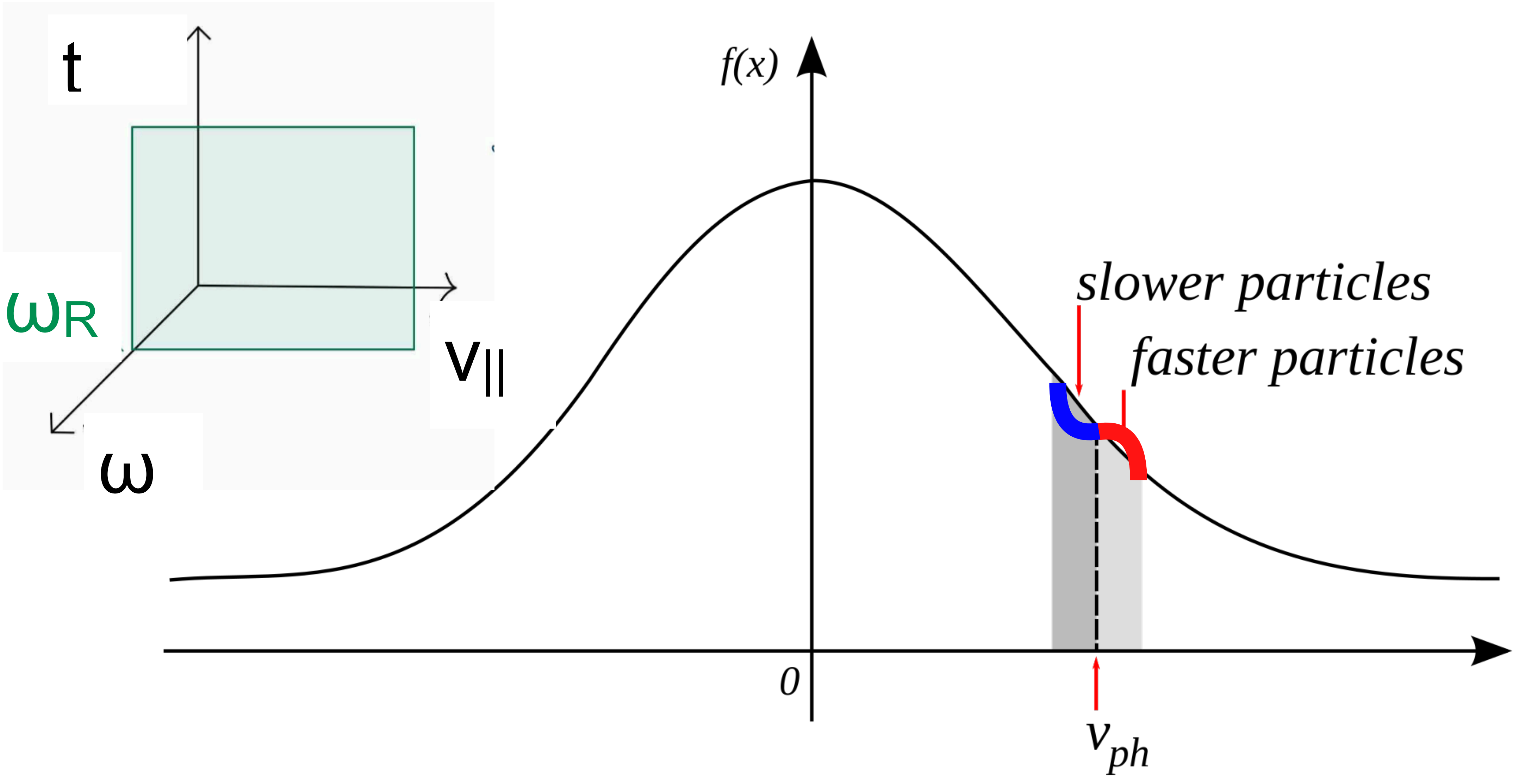
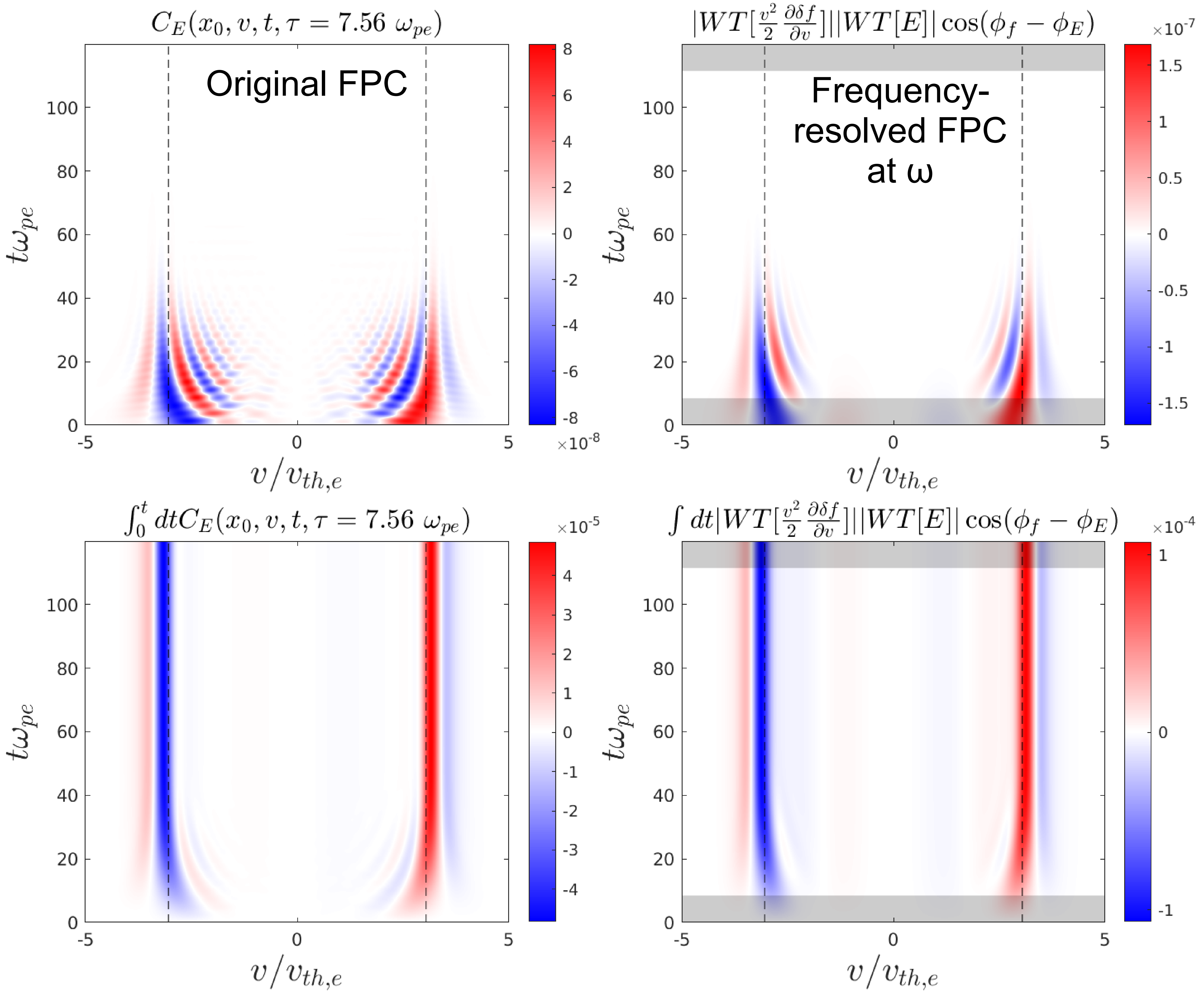
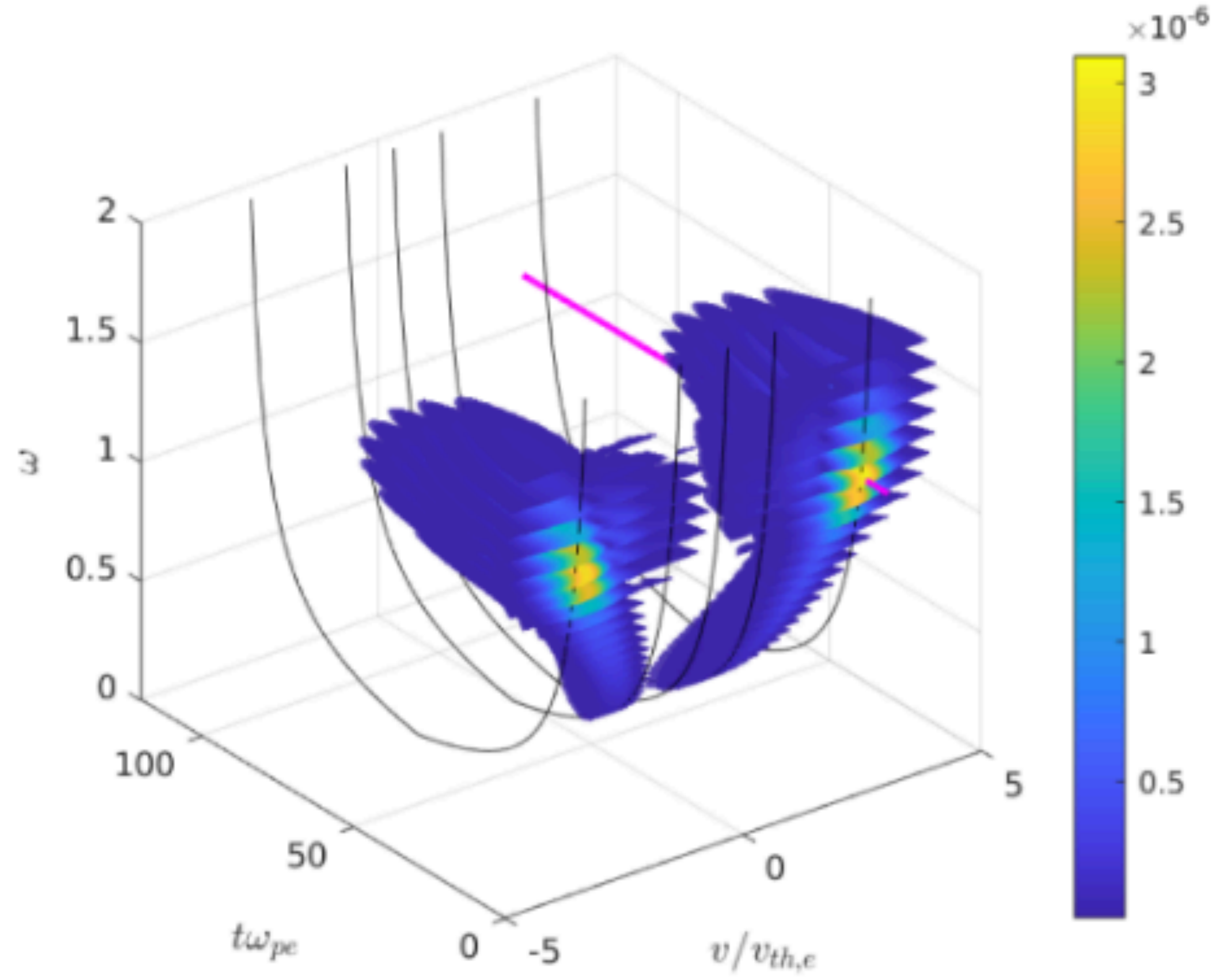
$\mathrm{Re}[e^{i(\phi_f - \phi_E)}] = \cos(\phi_f - \phi_E)$

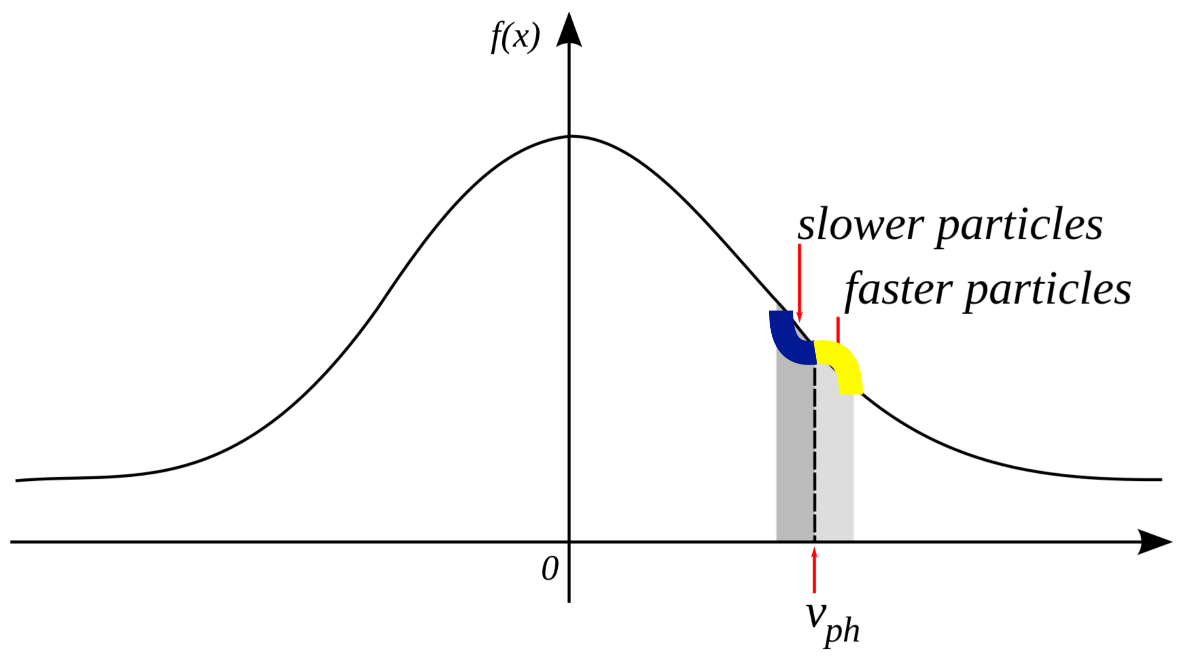
Applying the wavelet transform to electric field data from Gkeyll simulation of electron landau damping at a single frequency

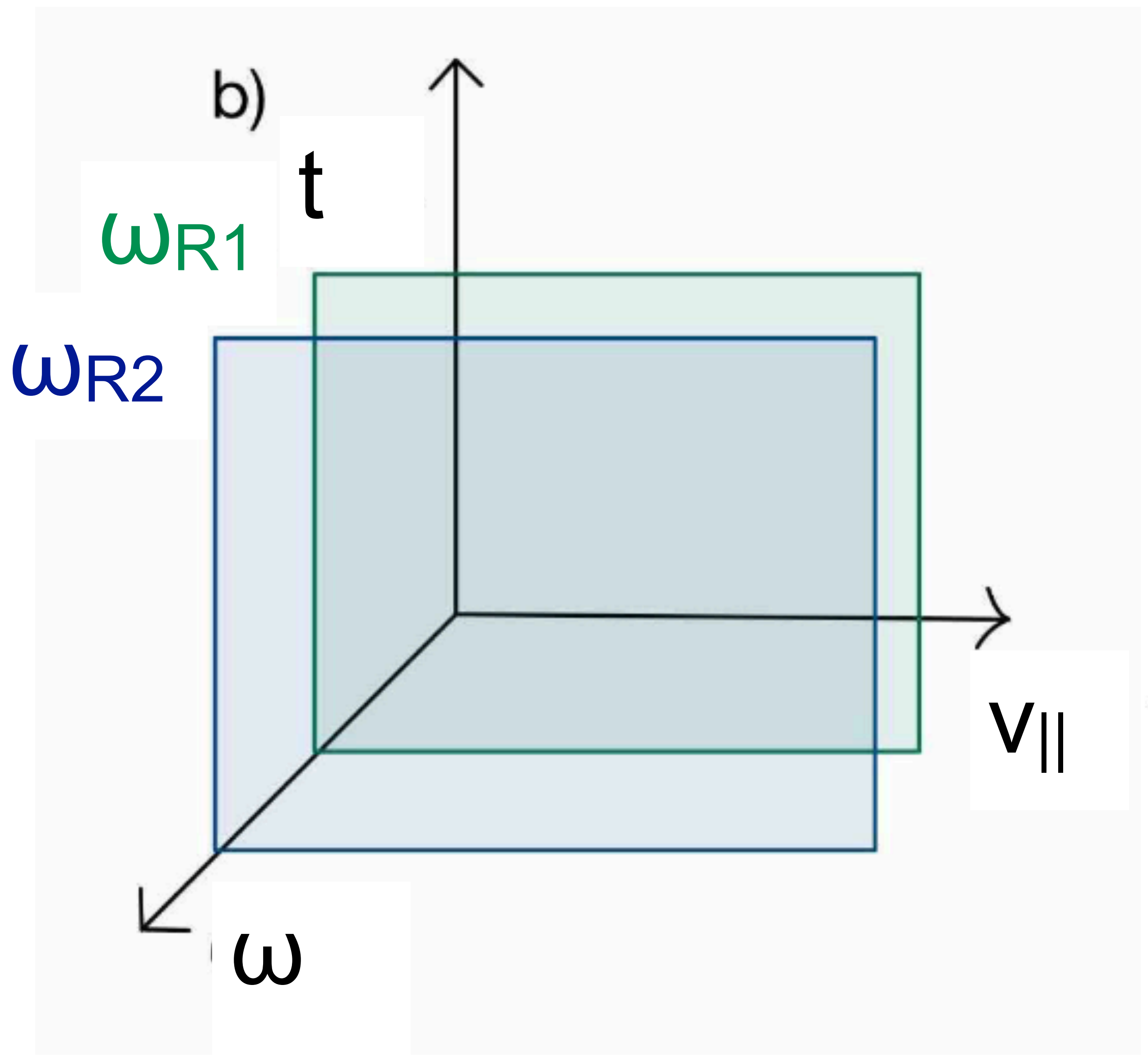


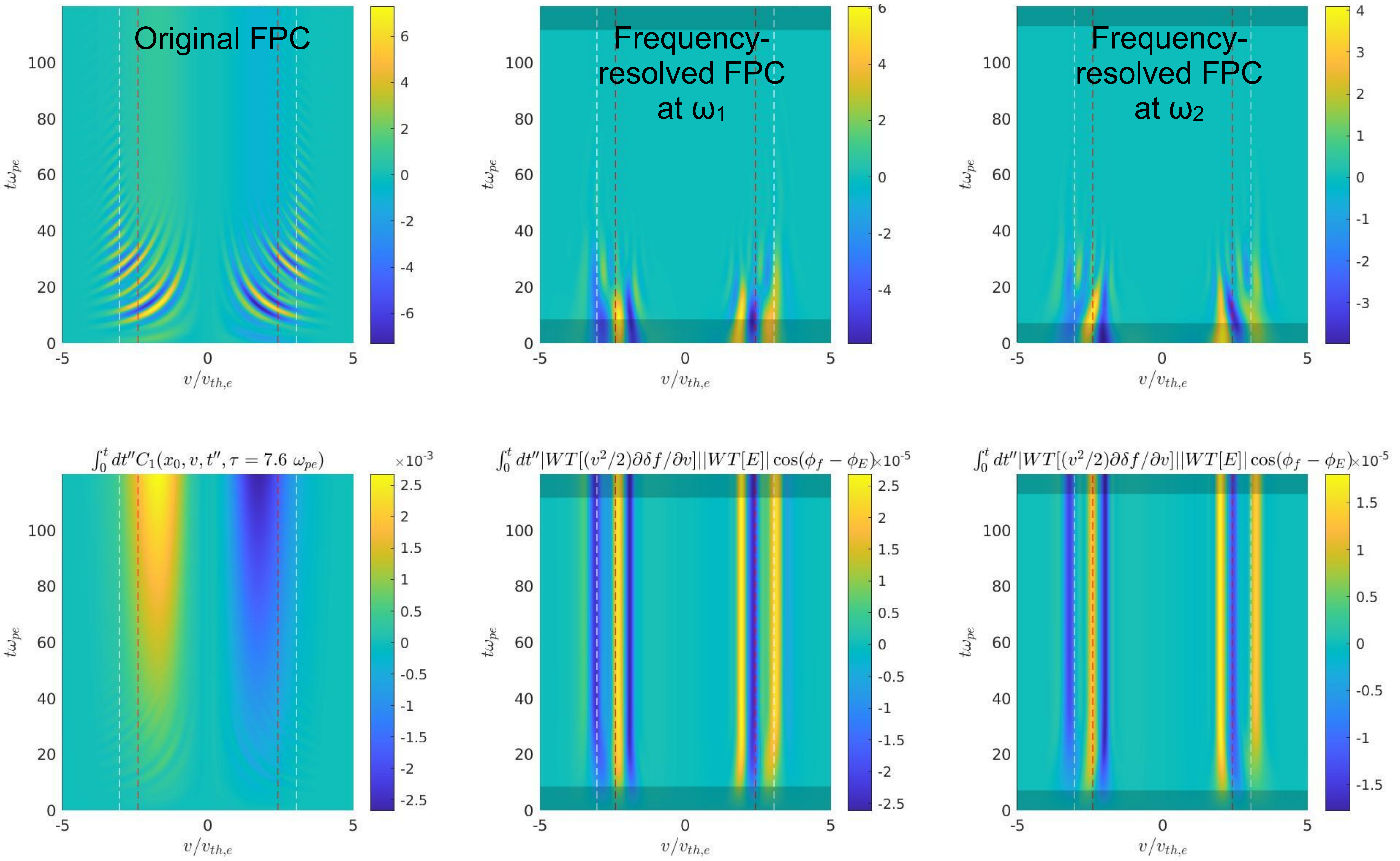




Applying the wavelet transform to electric field data from Gkeyll simulation of electron landau damping at two frequencies







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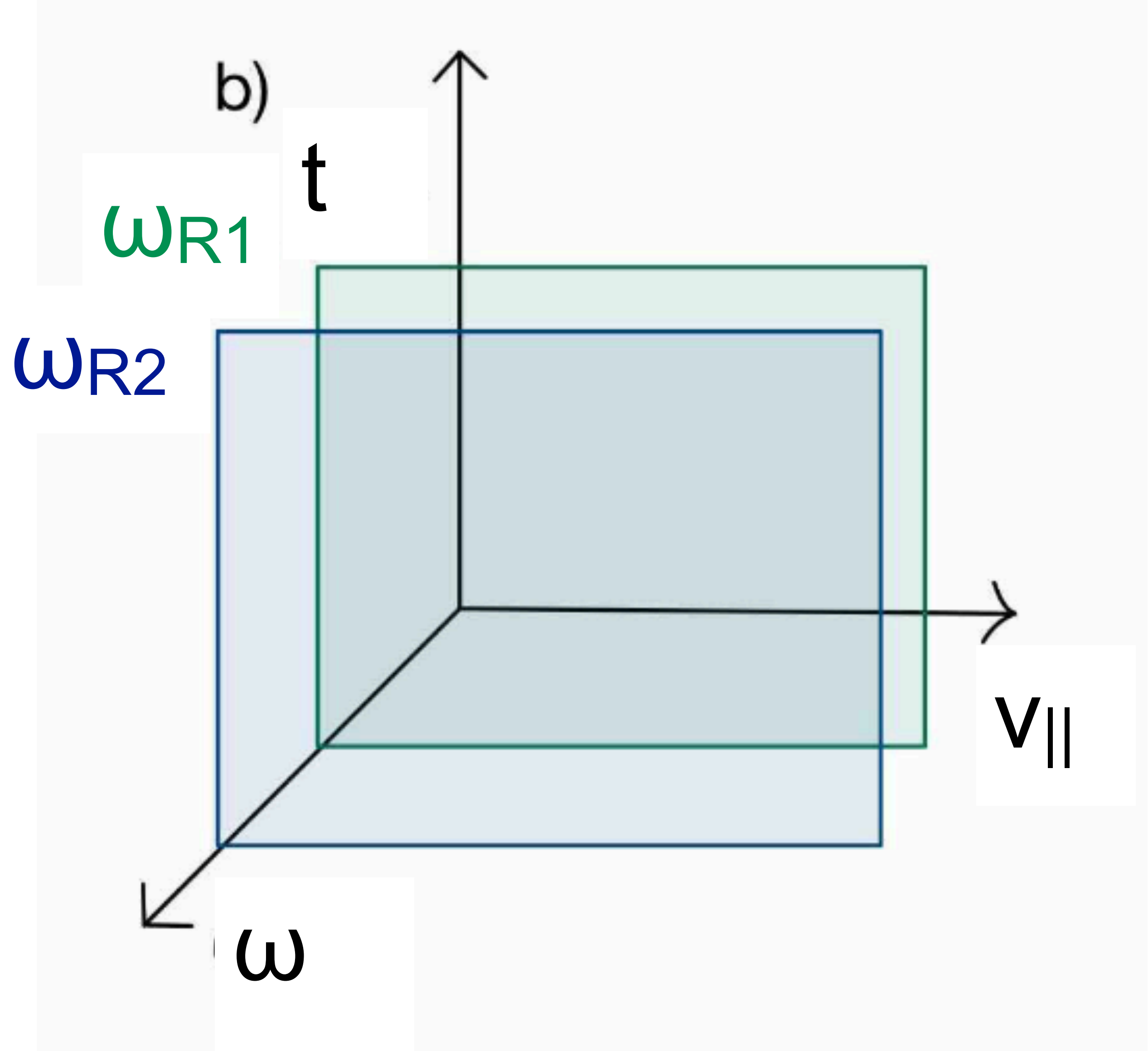
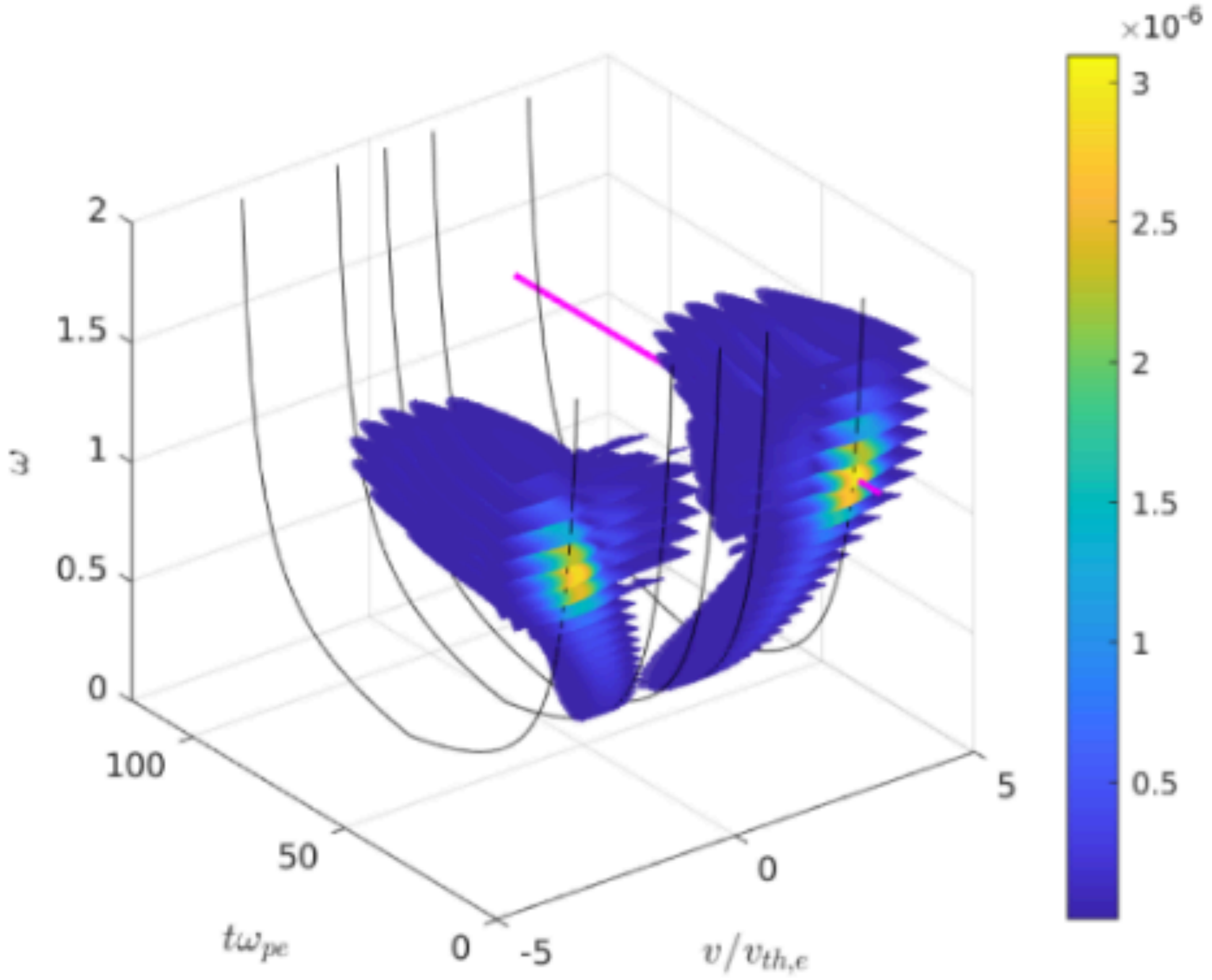
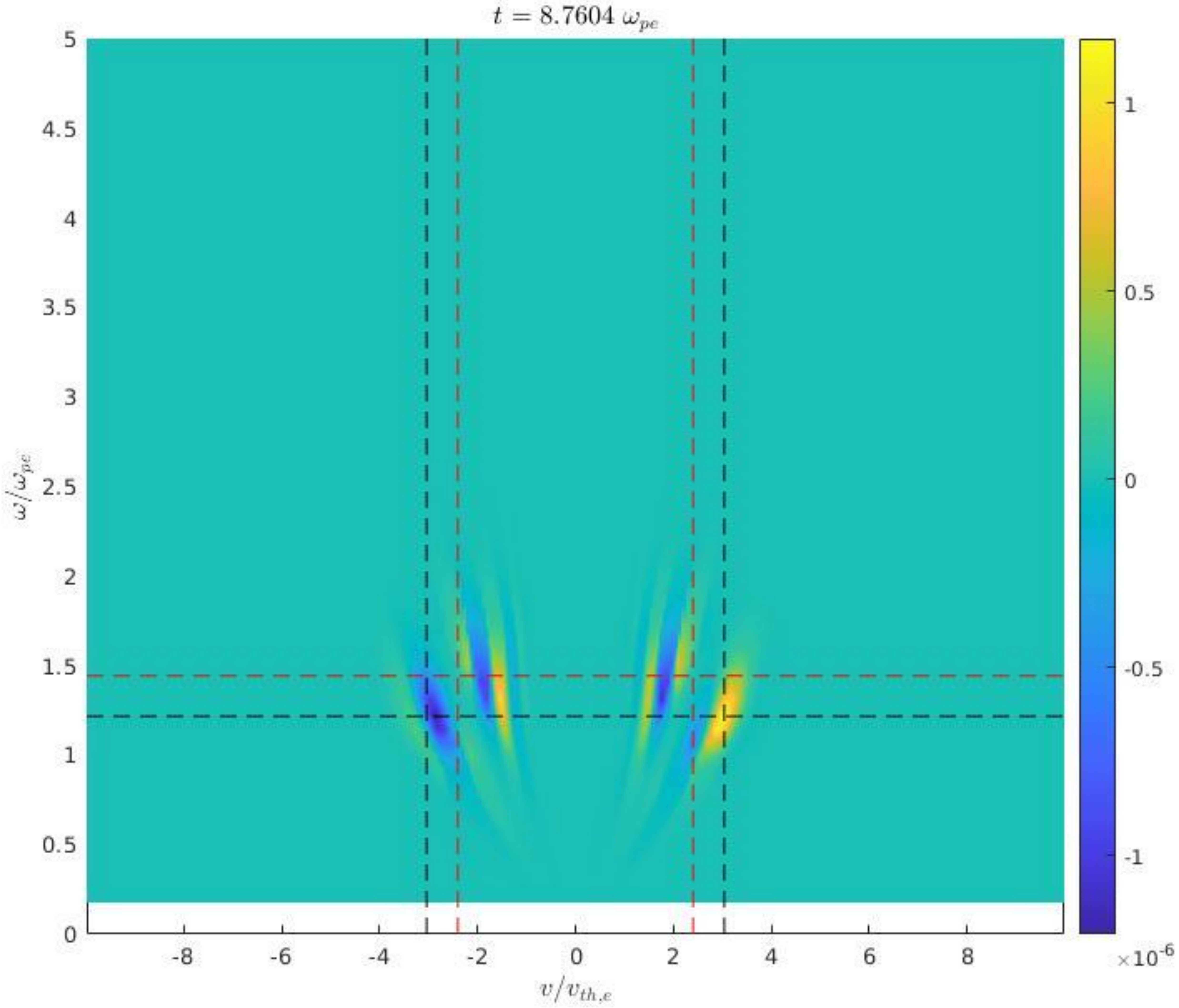
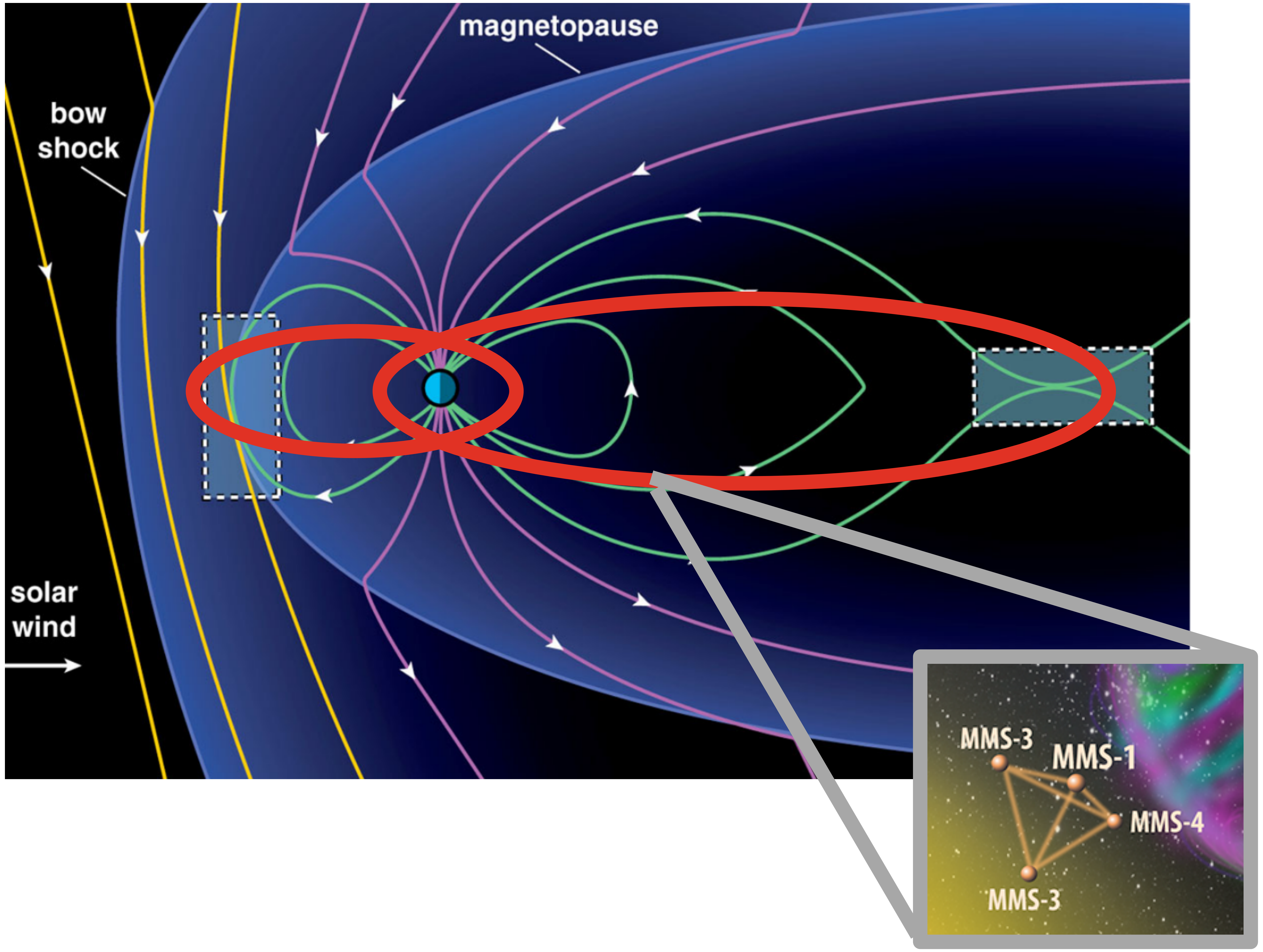
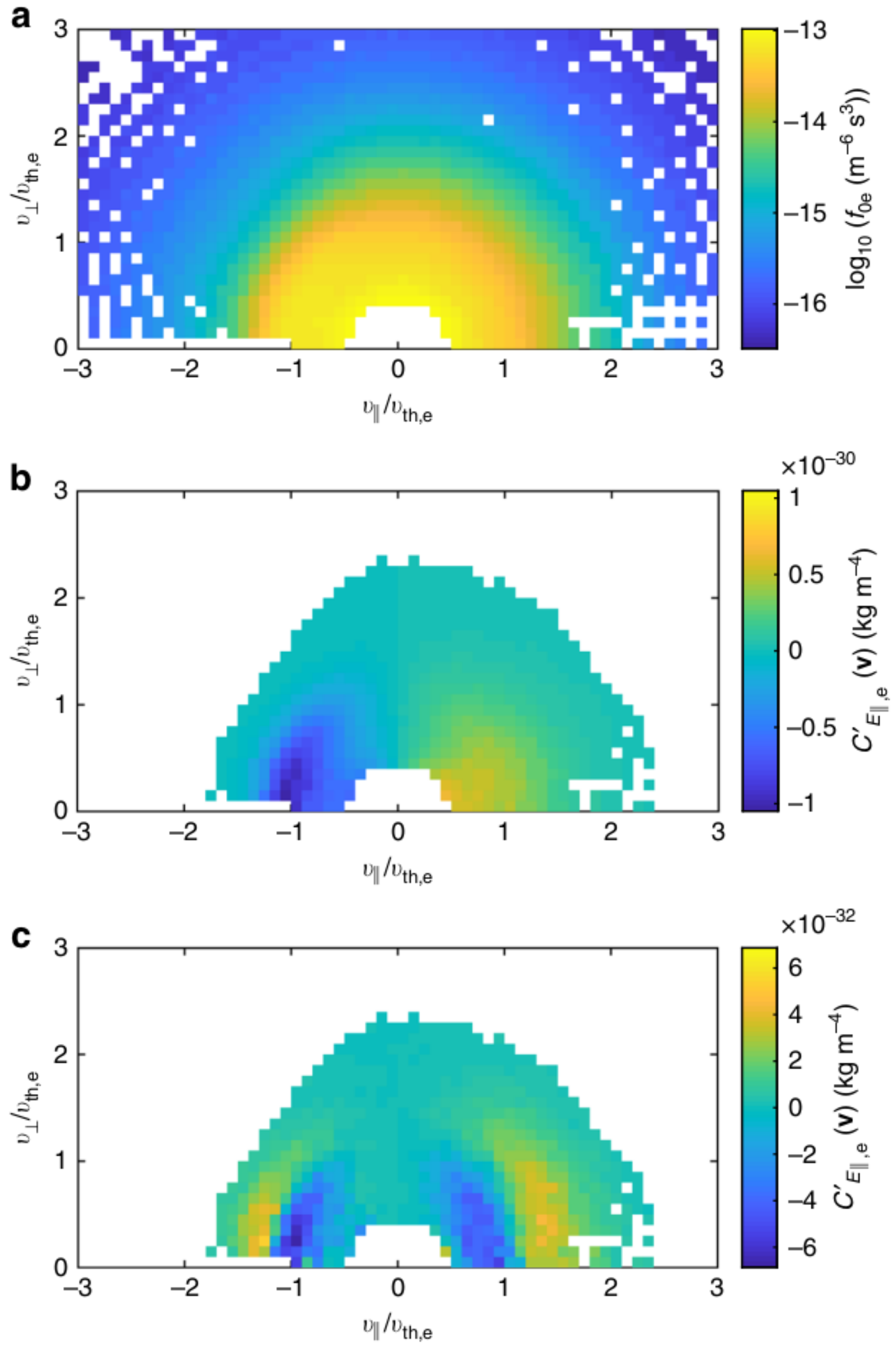






Applying the frequency-resolved FPC to data from MMS spacecraft





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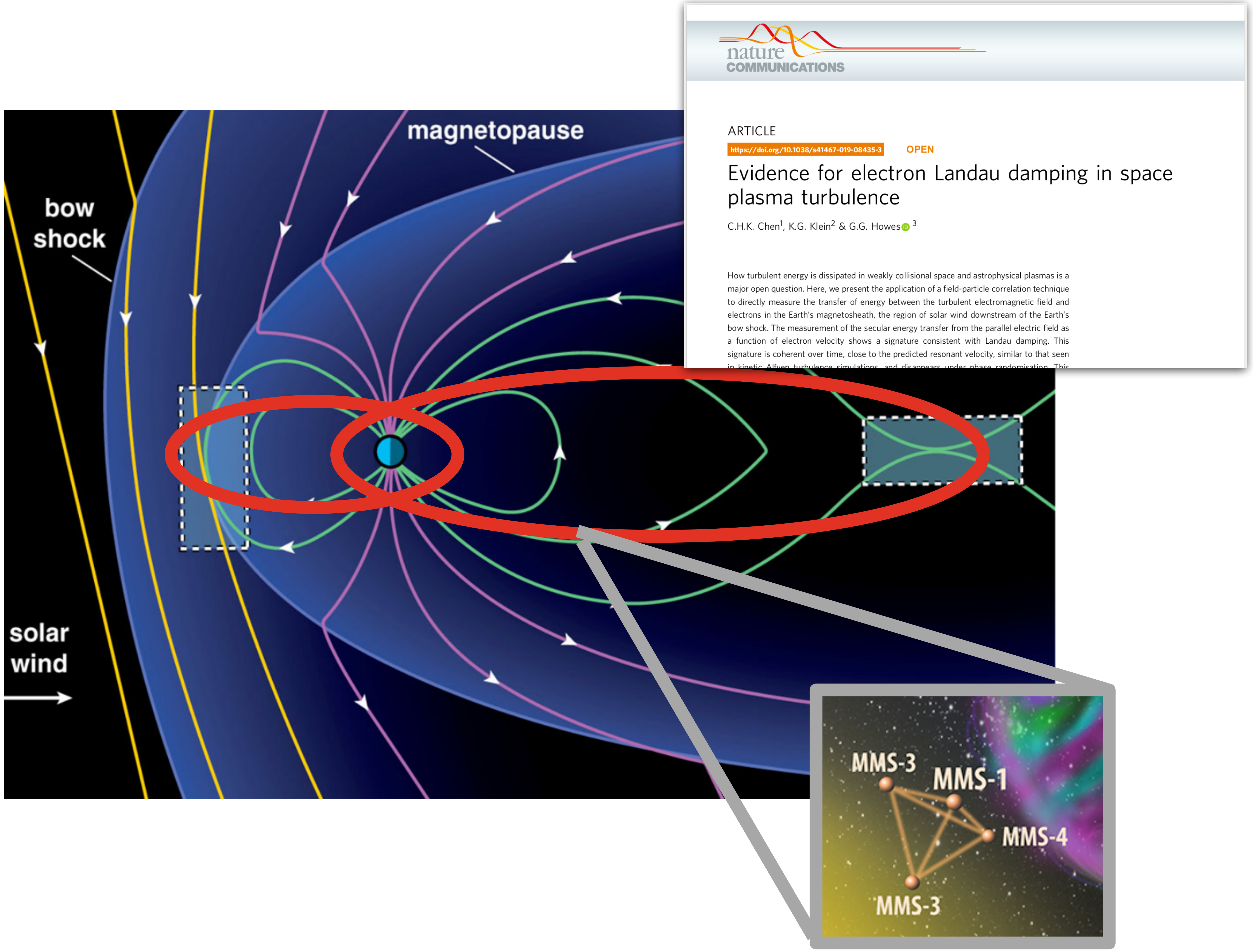

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ARTICLE

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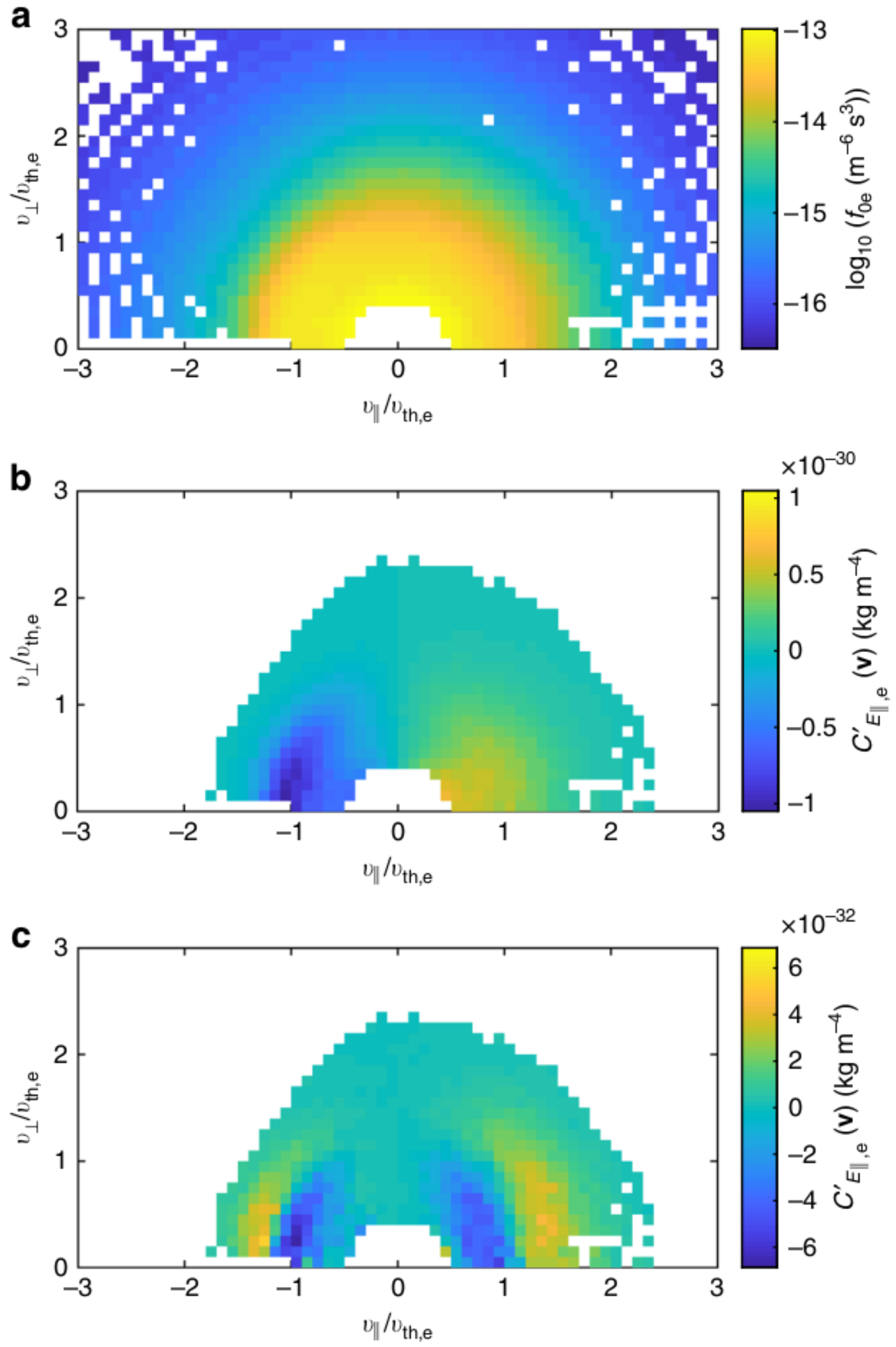
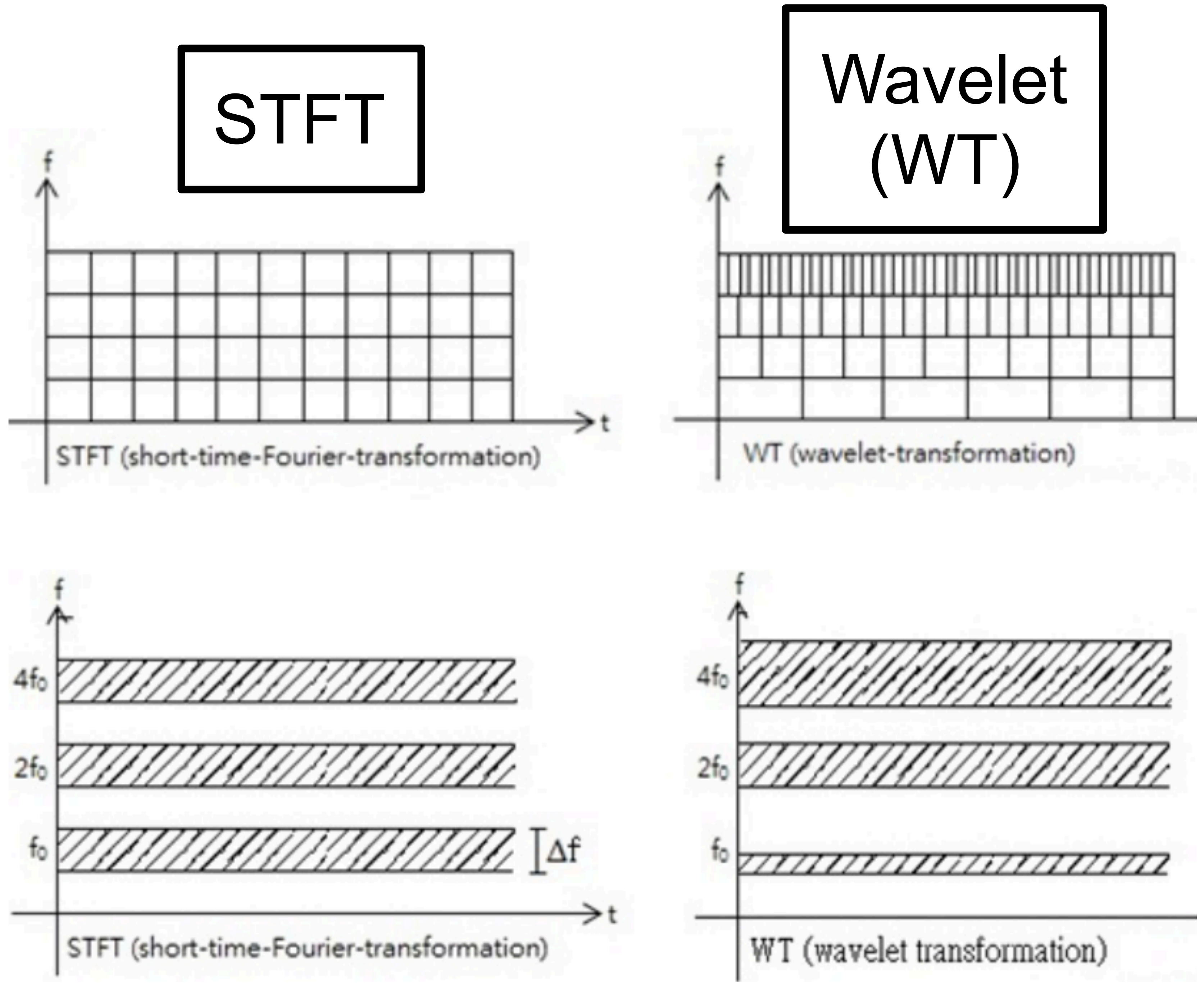
Evidence for electron Landau damping in space plasma turbulence

C.H.K. Chen[1], K.G. Klein[2] & G.G. Howes[3]

How turbulent energy is dissipated in weakly collisional space and astrophysical plasmas is a major open question. Here, we present the application of a field-particle correlation technique to directly measure the transfer of energy between the turbulent electromagnetic field and electrons in the Earth's magnetosheath, the region of solar wind downstream of the Earth's bow shock. The measurement of the secular energy transfer from the parallel electric field as a function of electron velocity shows a signature consistent with Landau damping. This signature is coherent over time, close to the predicted resonant velocity, similar to that seen

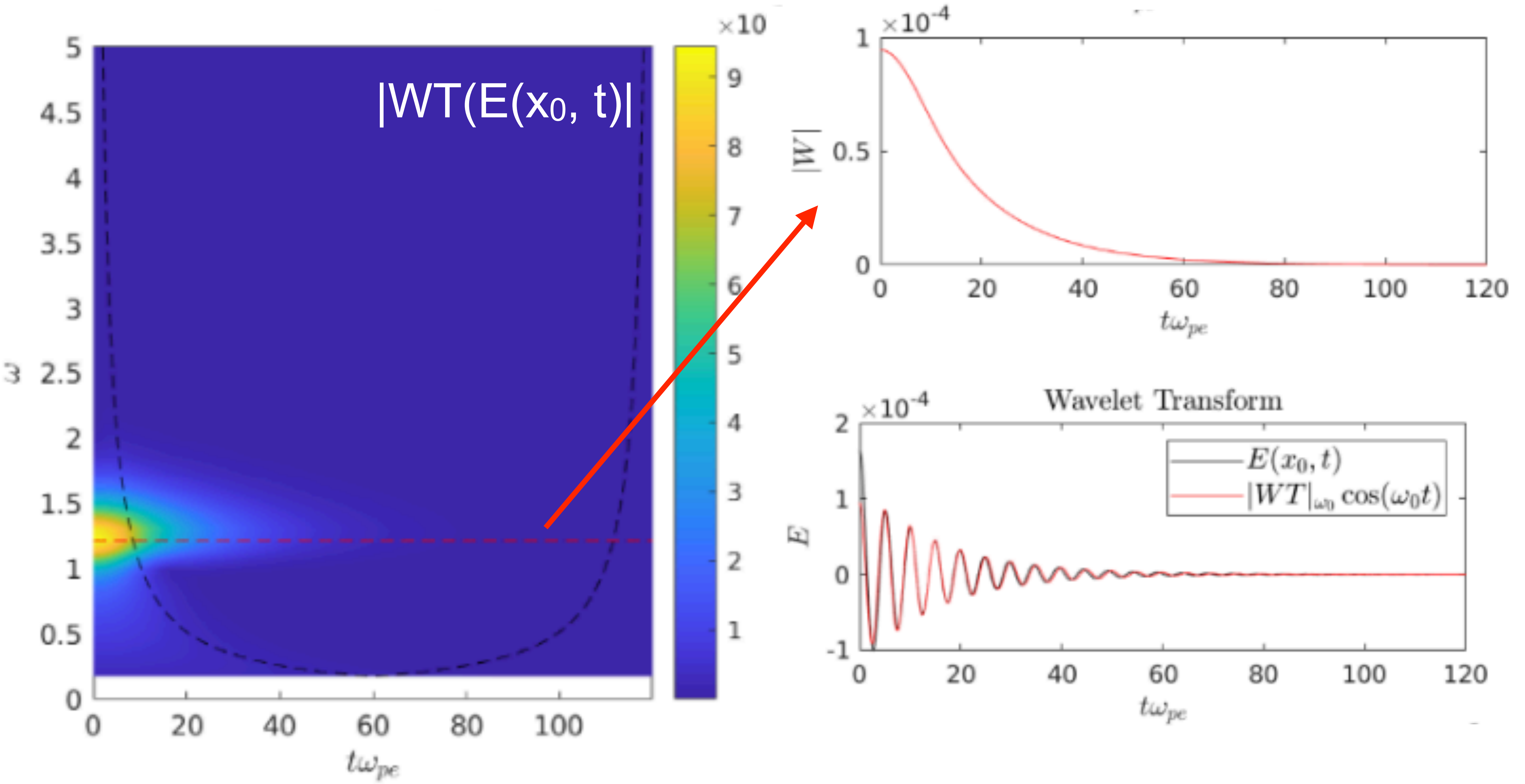


Applying the wavelet transform to electric field data from Gkeyll simulation of electron landau damping



$$\Delta f \Delta t \geq 1$$

$$C_{E_{\|}} = \int_{-\infty}^{\infty} x(t) w(t - t_0) e^{-i\omega_R t} dt$$



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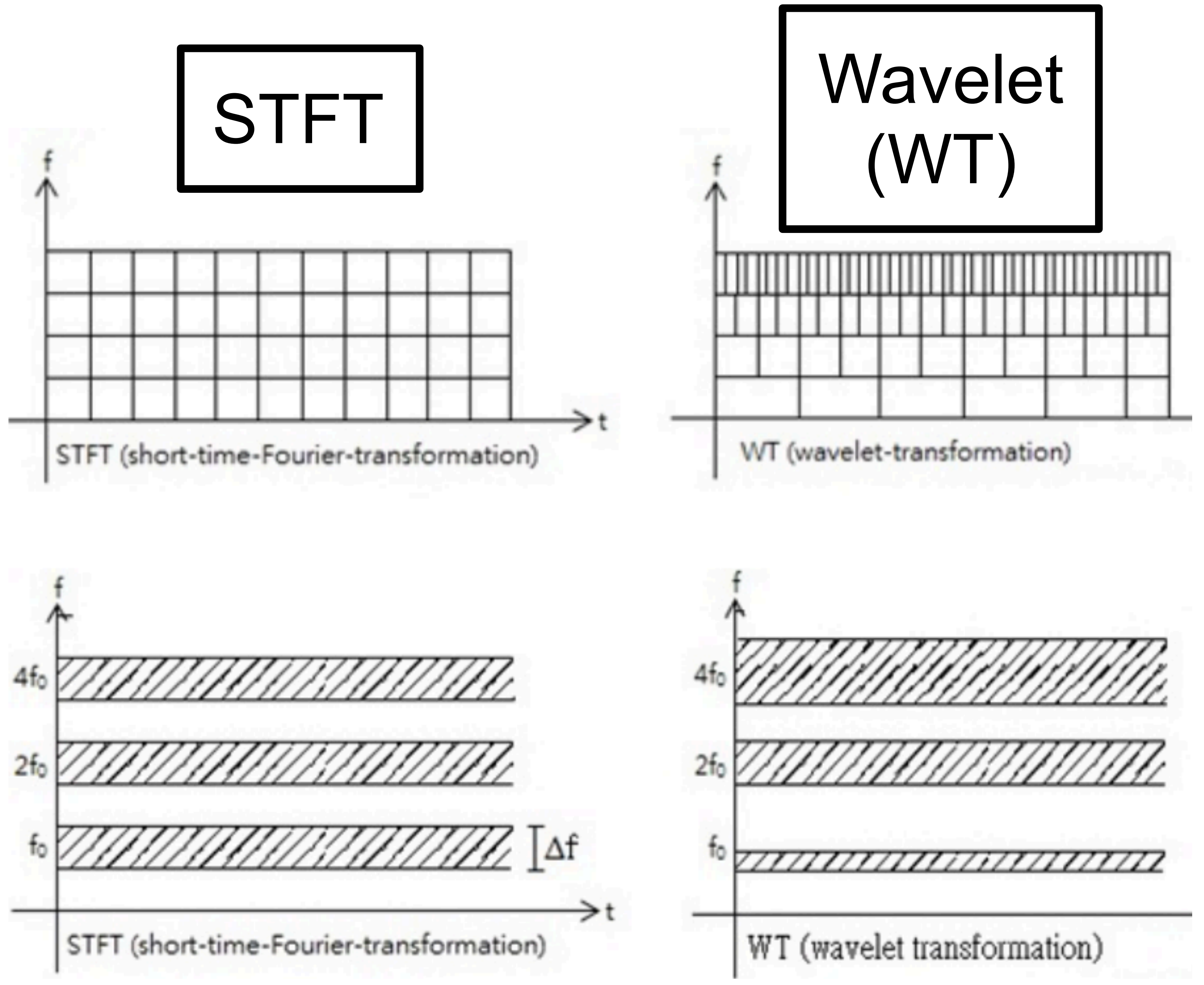

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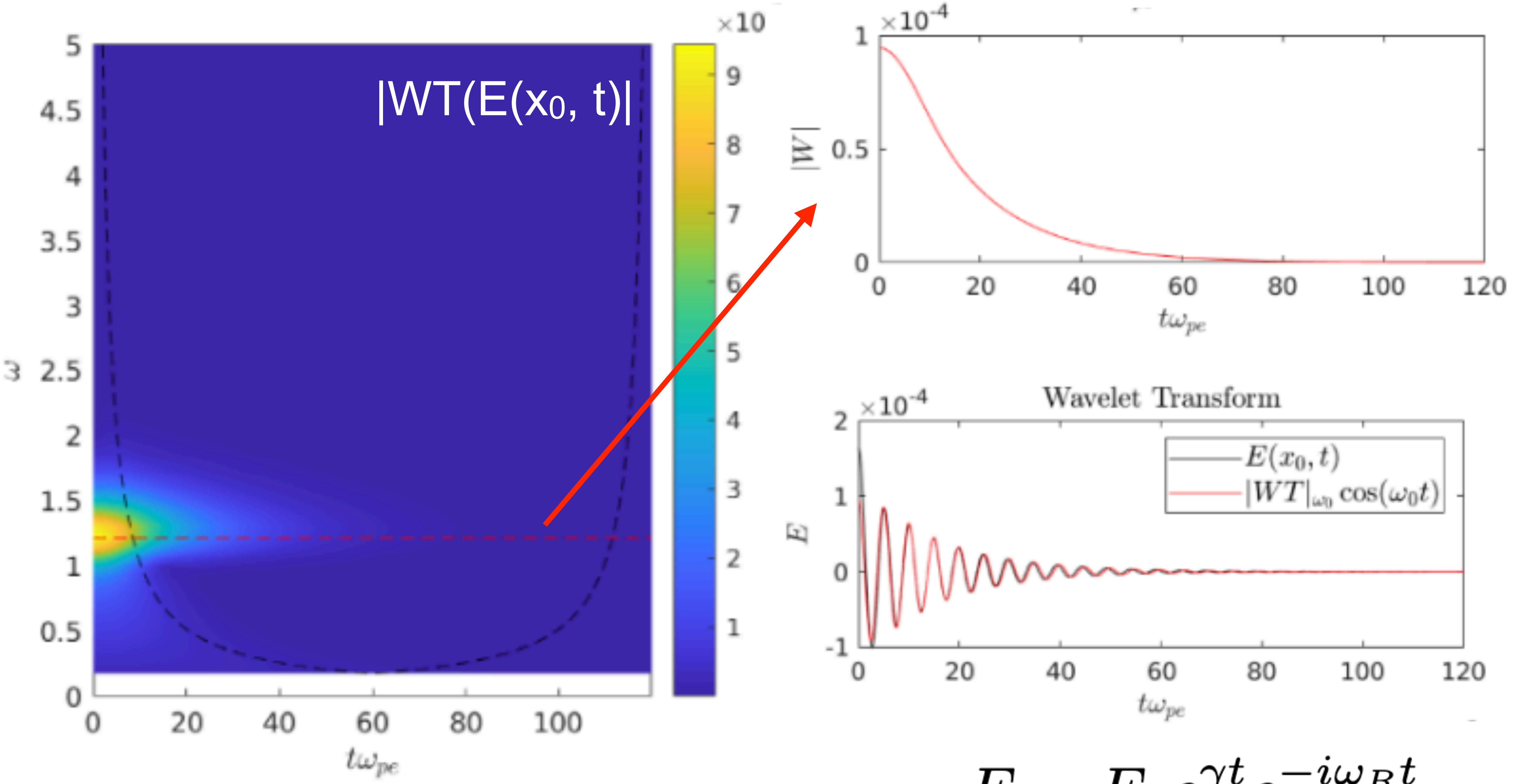


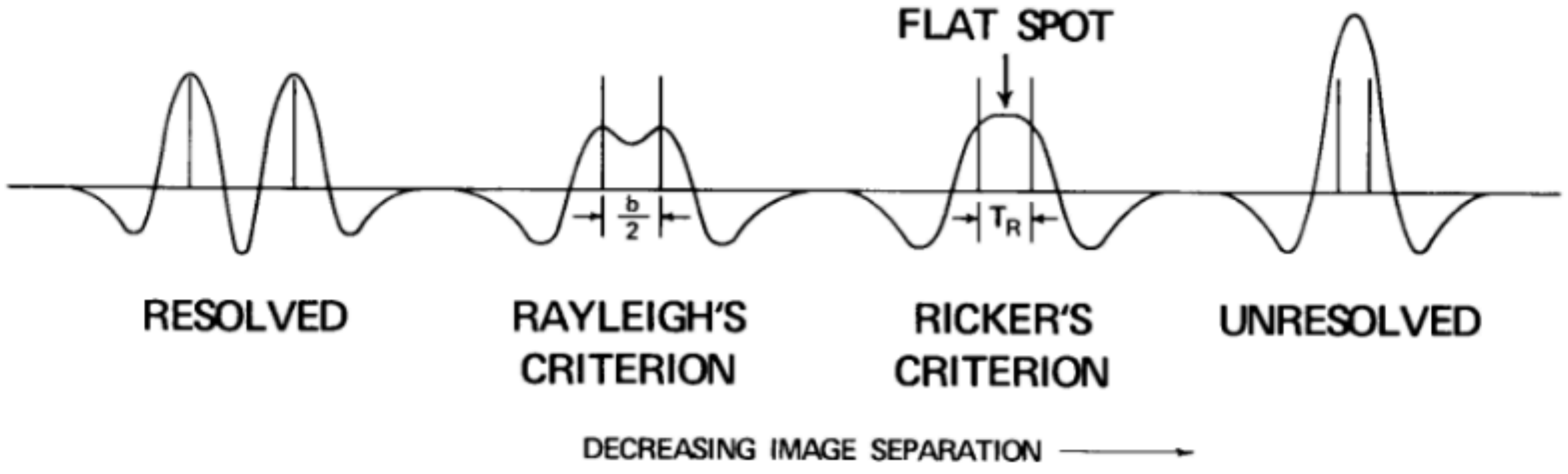
$$E = E_0 e^{\gamma t} e^{-i\omega_R t}$$

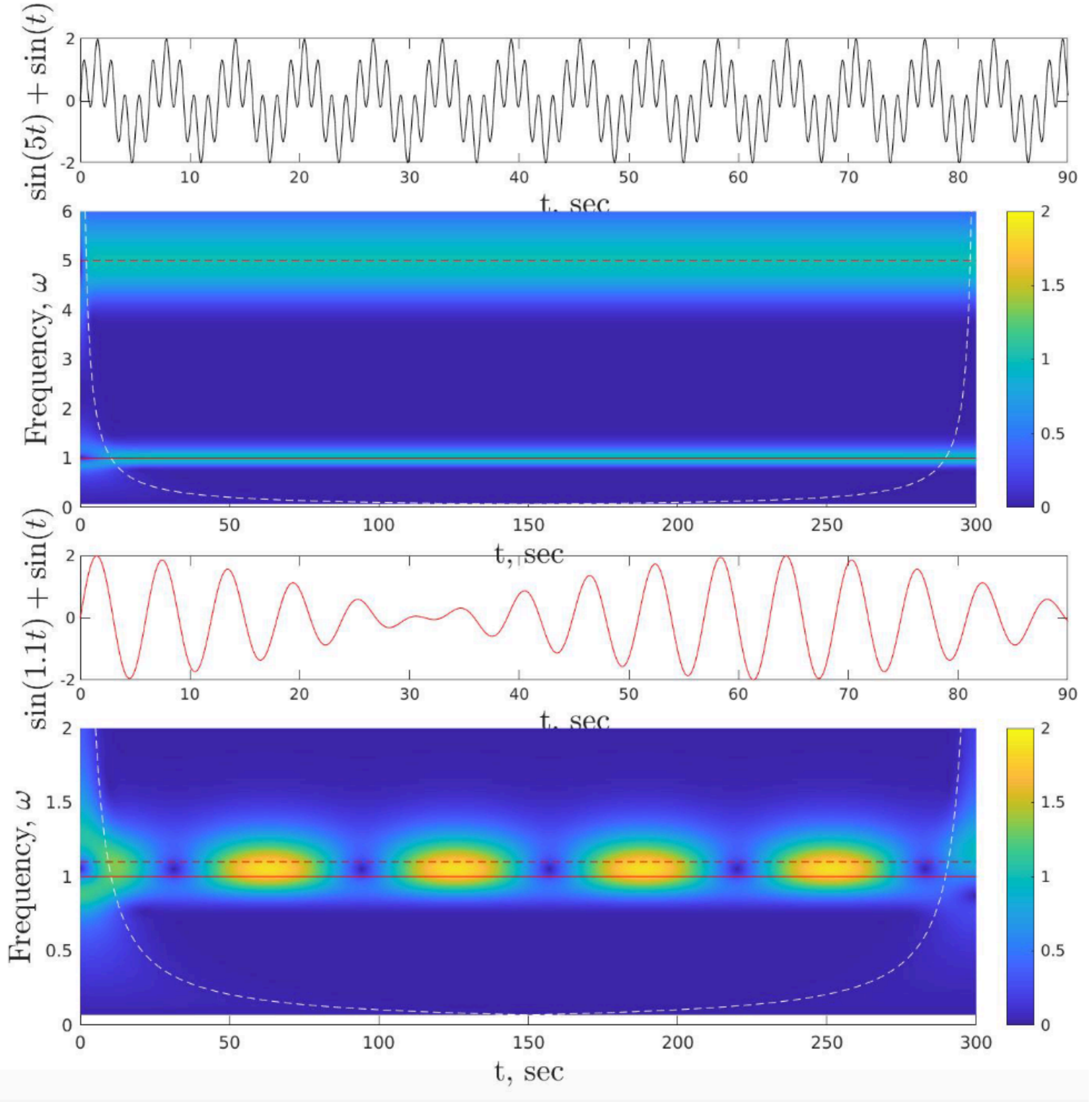
Fundamental limitation on the area in frequency-time space and on the frequency resolution

1. **Heisenberg-Gabor Limit** - because time and frequency are not independent variables, the uncertainty in a frequency over a time period is bounded by a constant, *i.e.*

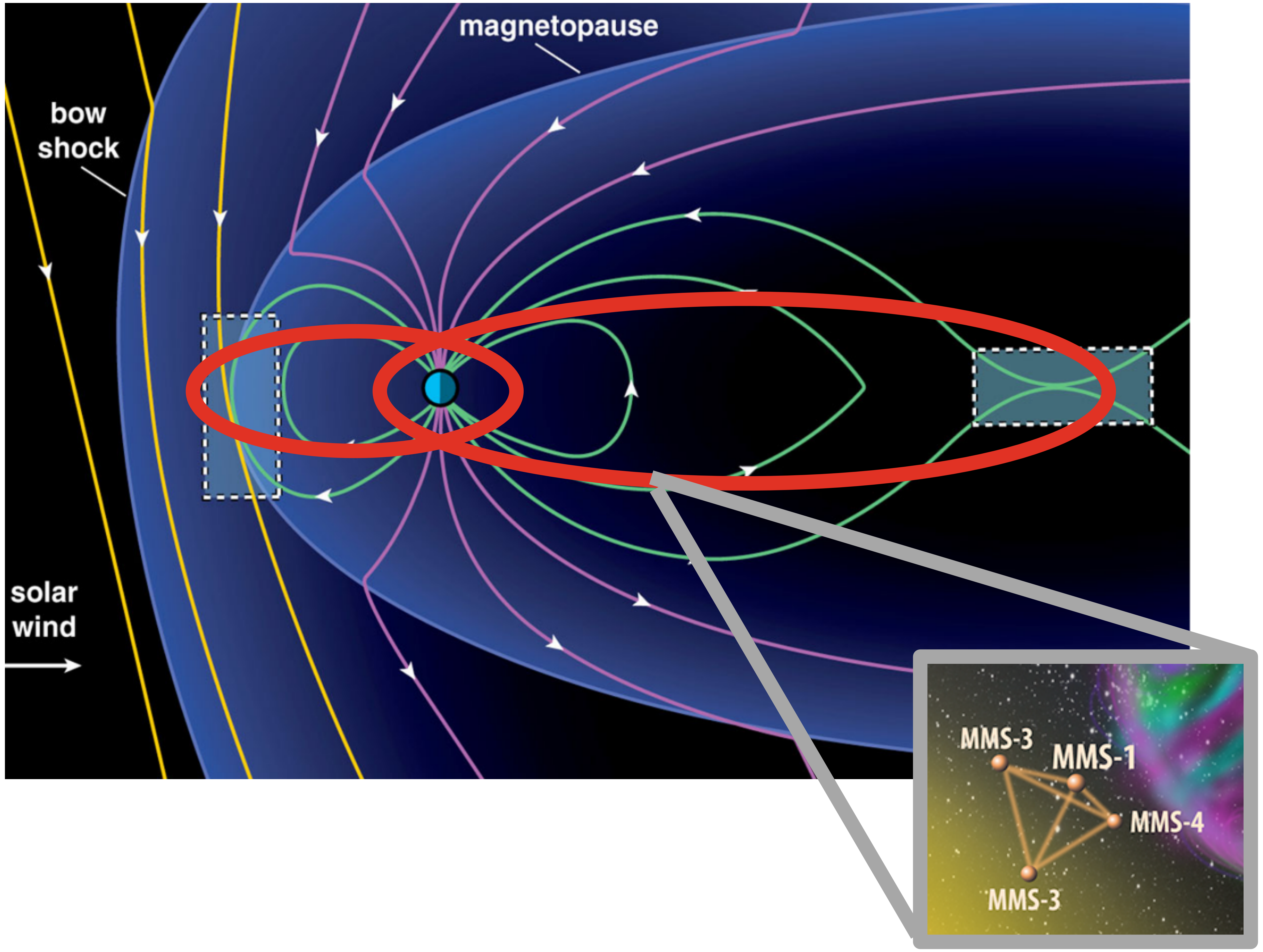
$$\Delta f \Delta t \geq 1$$

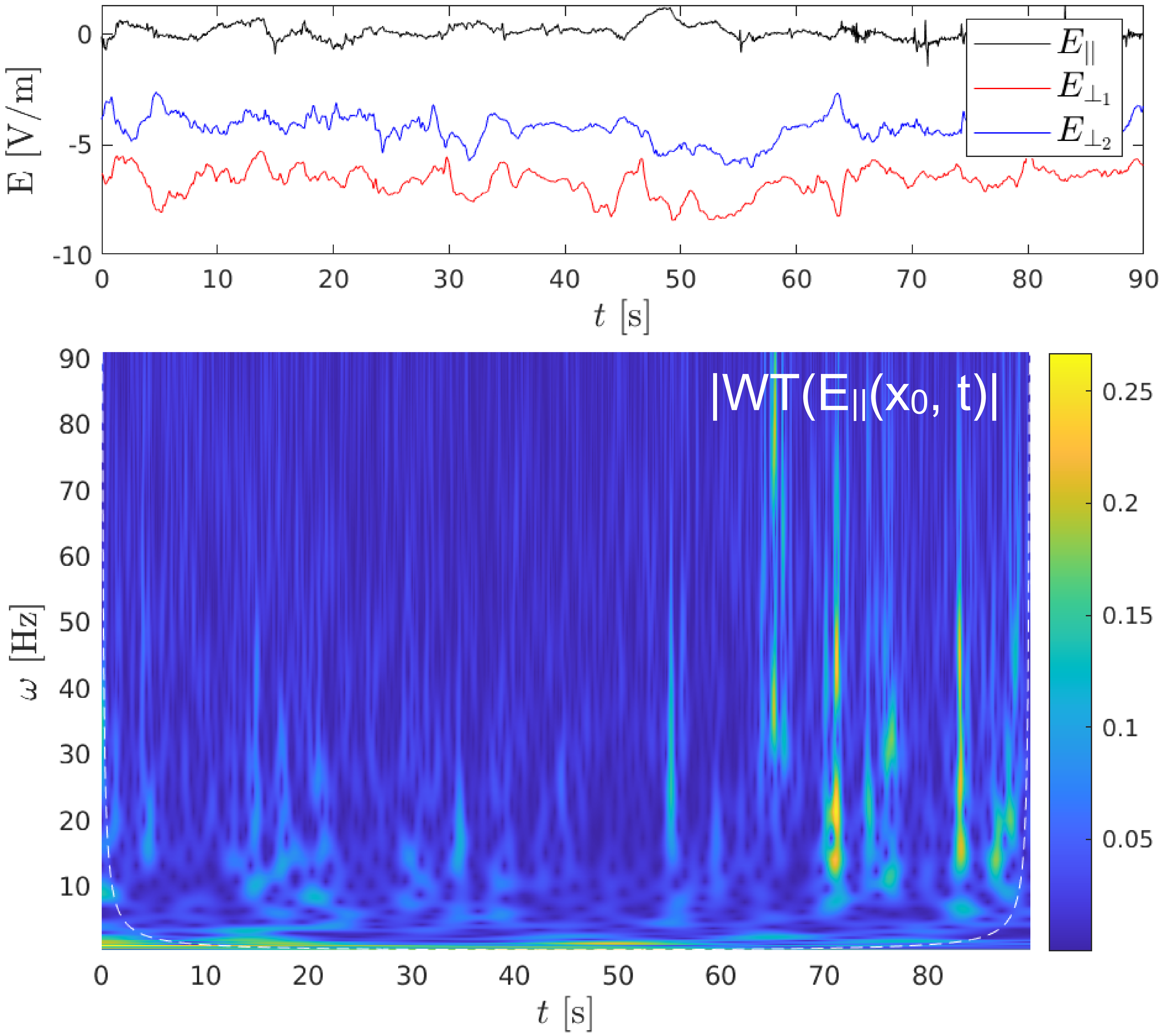
2. Limitation on the ability to distinguish peaked distributions of nearby frequencies





Applying the frequency-resolved FPC to data from MMS spacecraft





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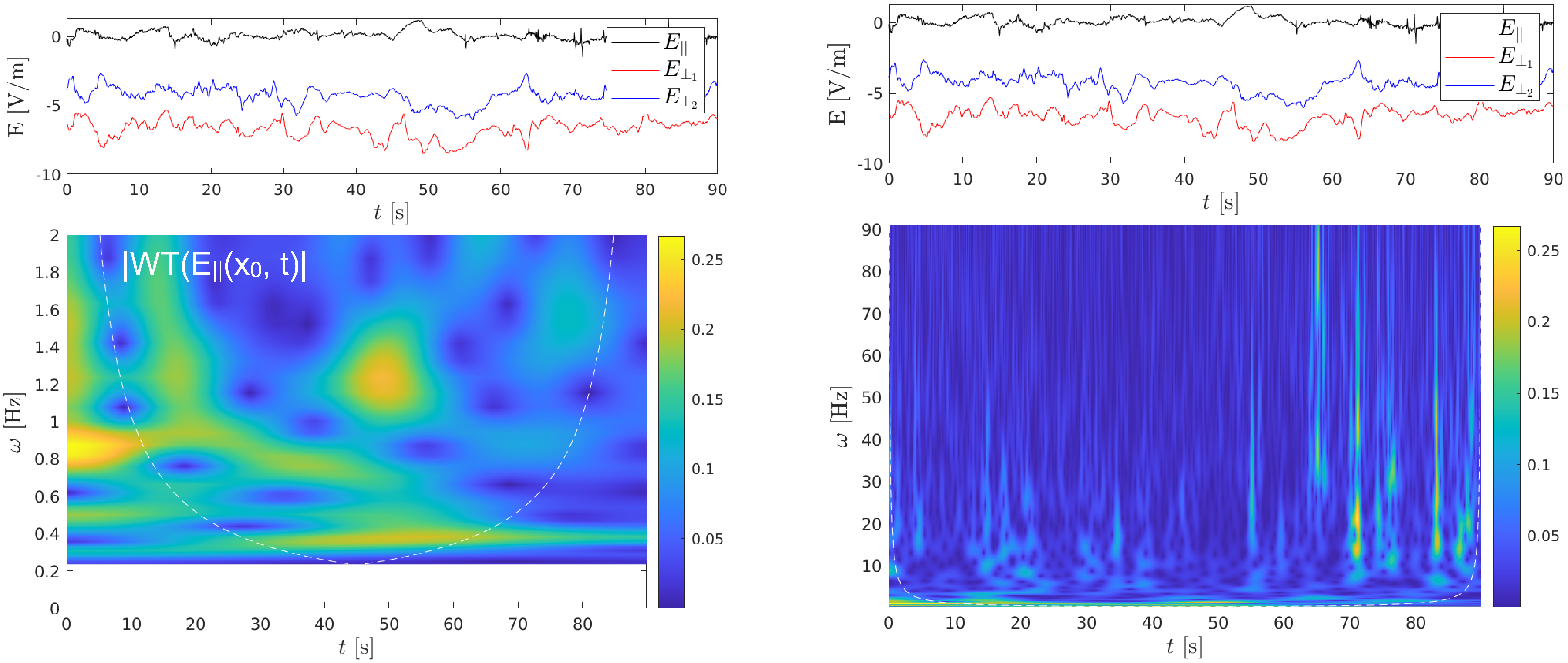

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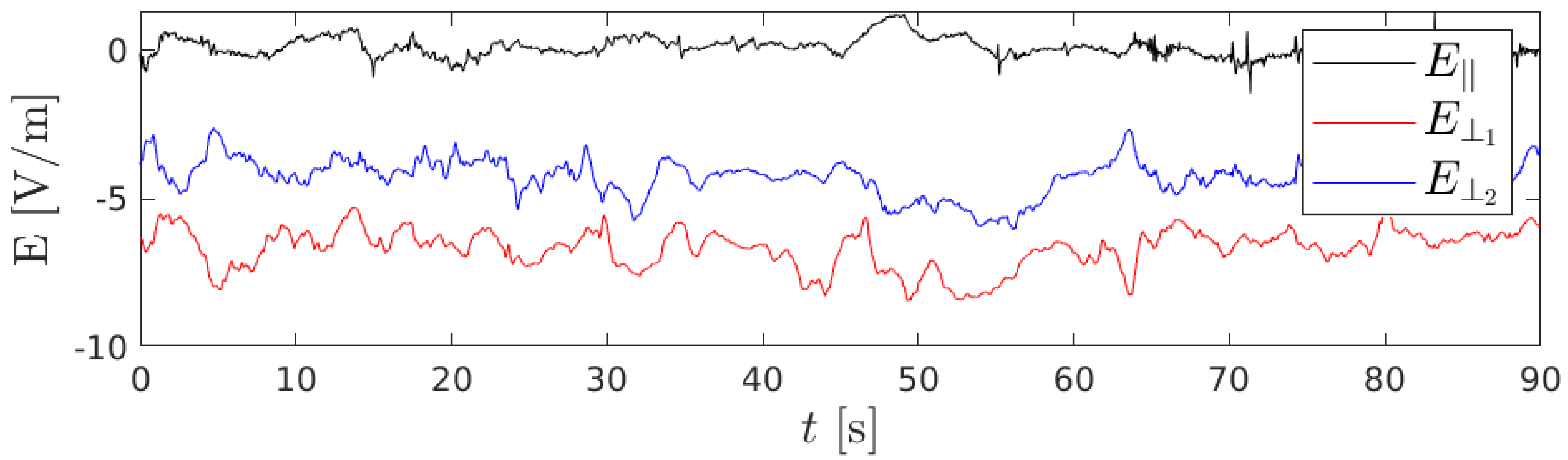

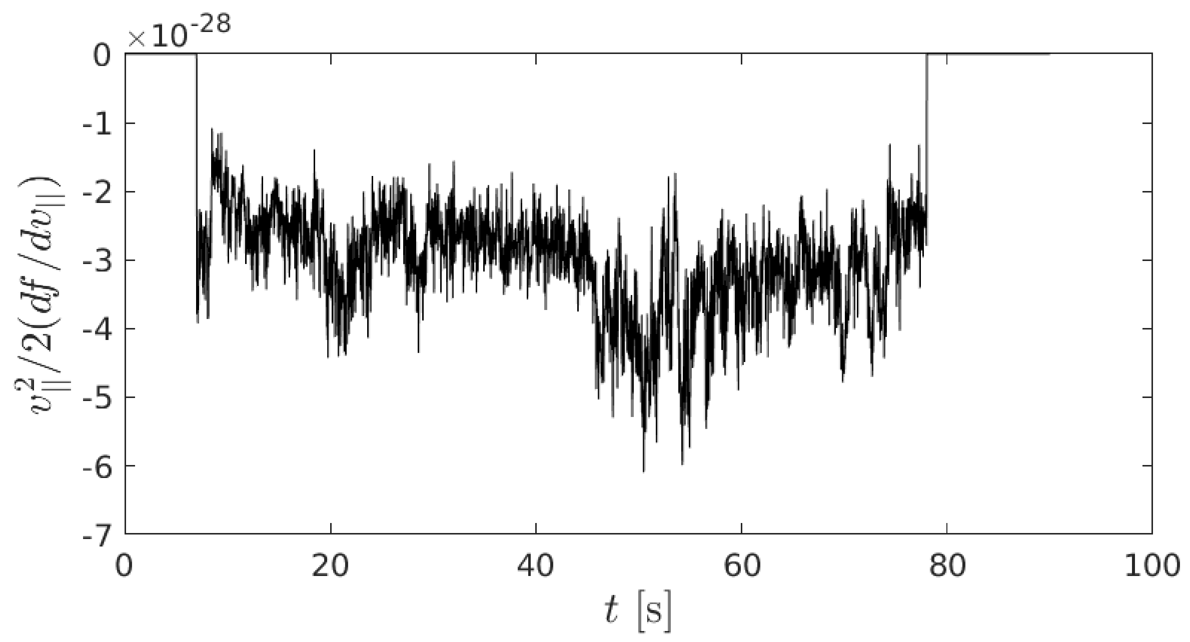

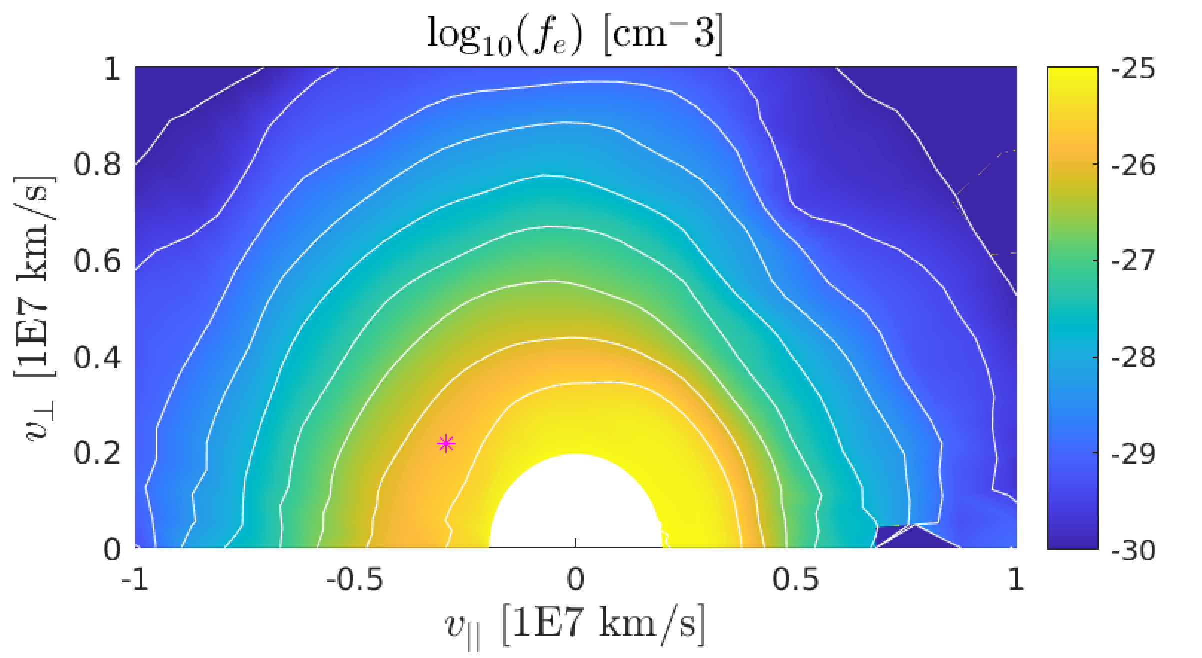

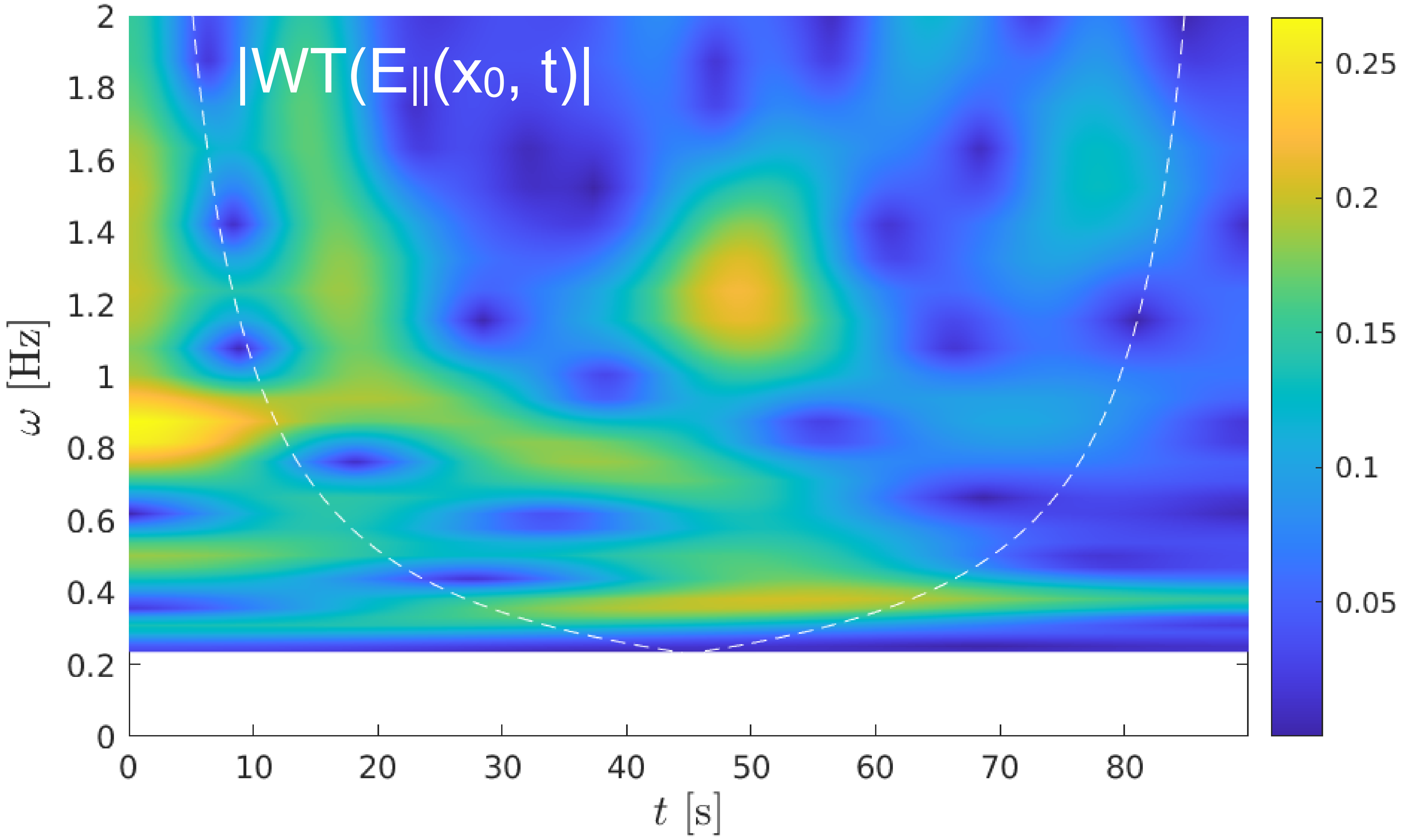

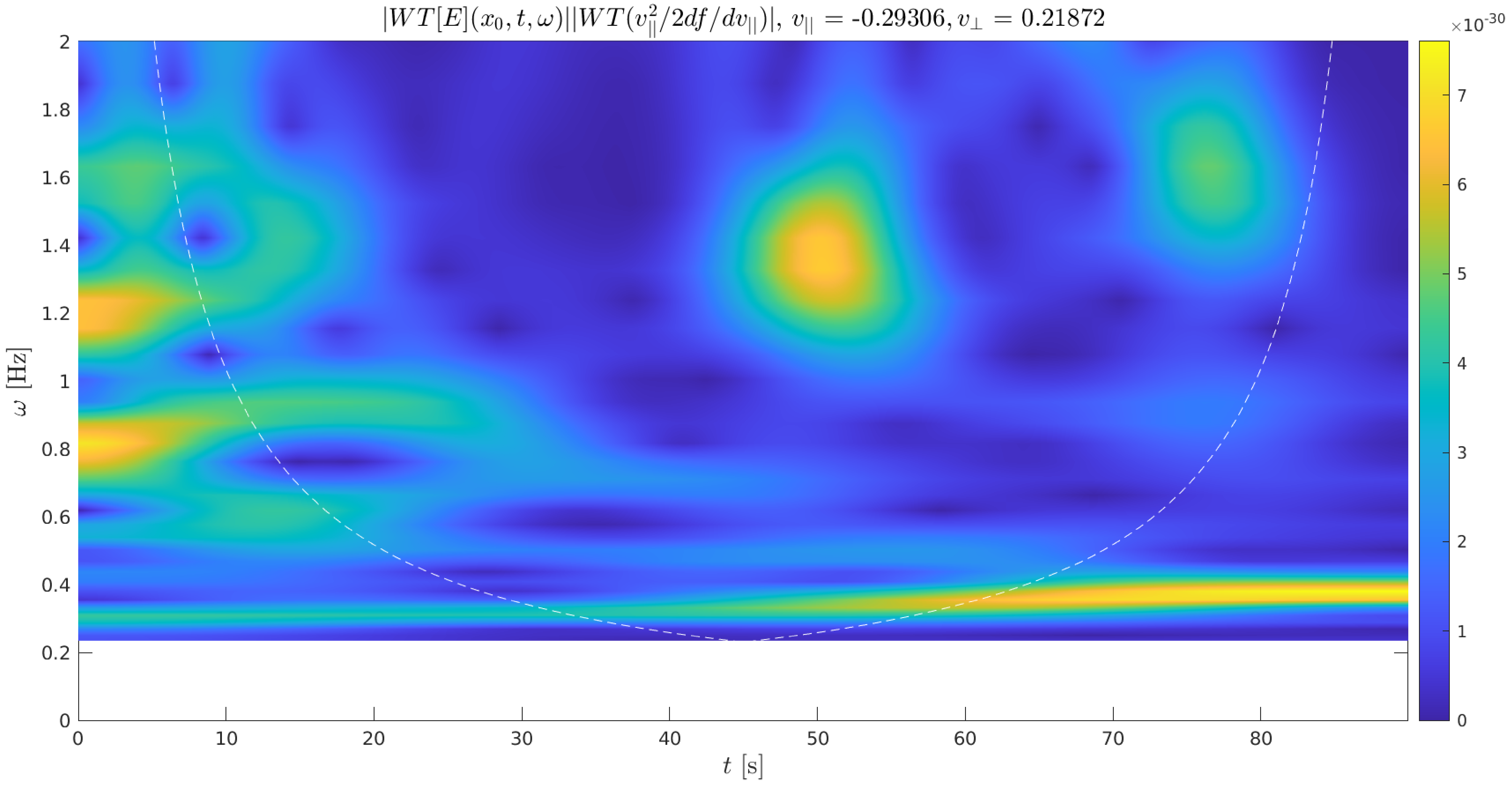

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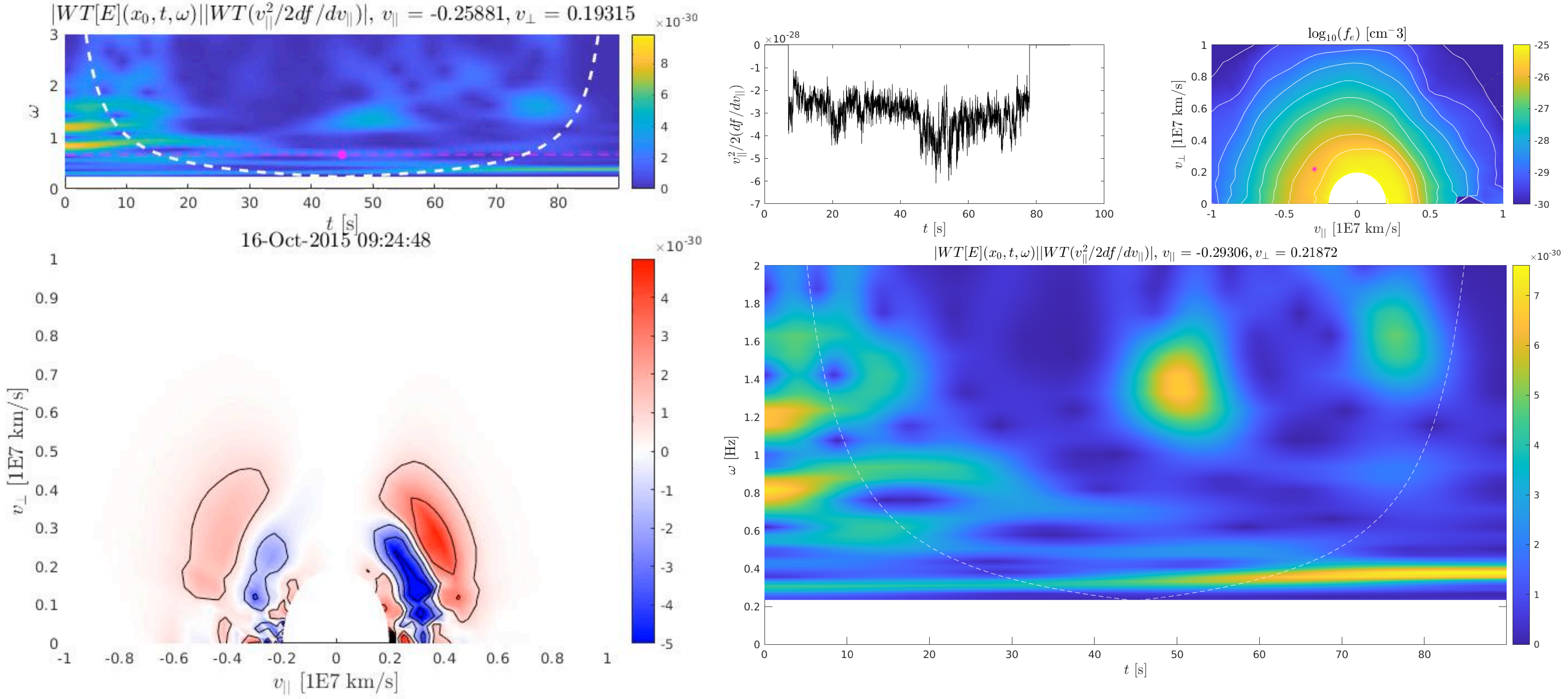

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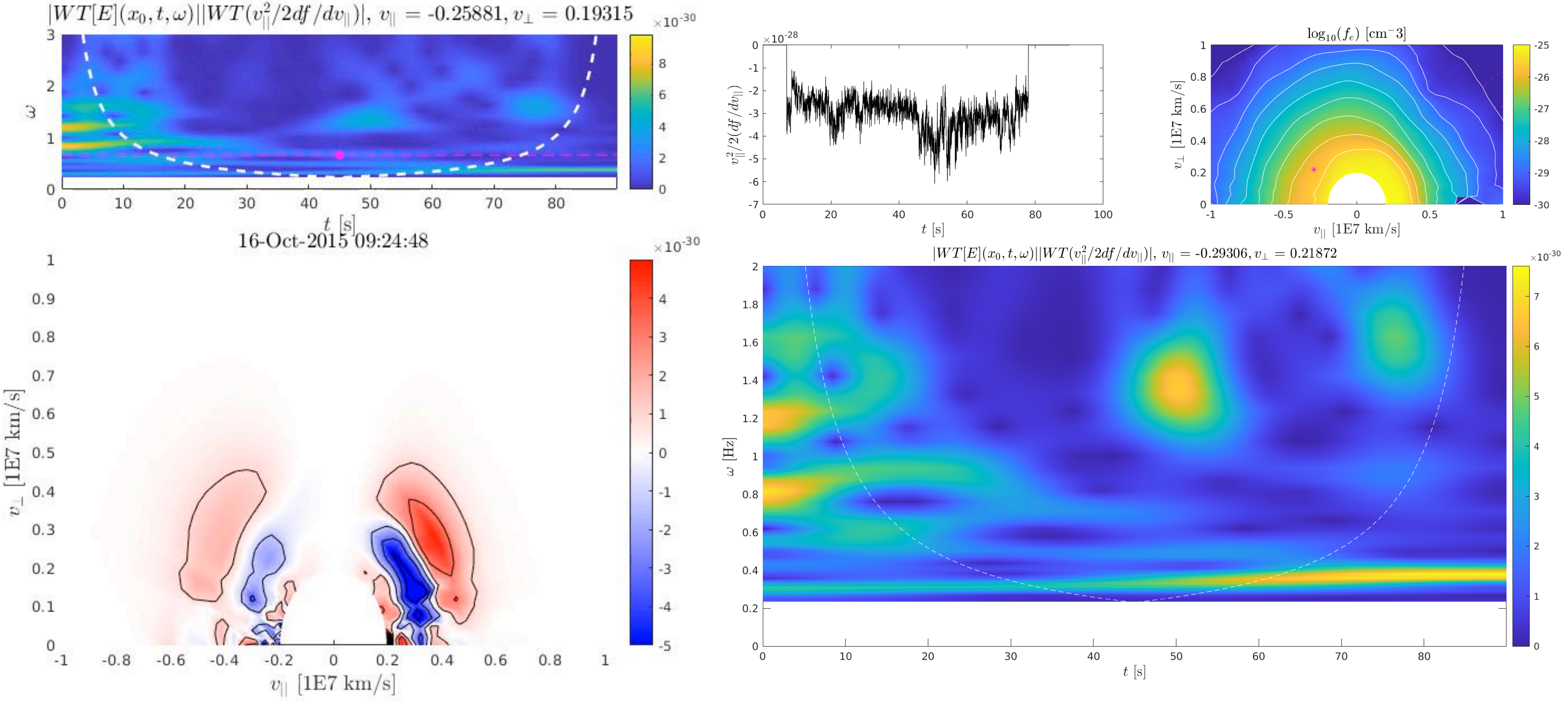

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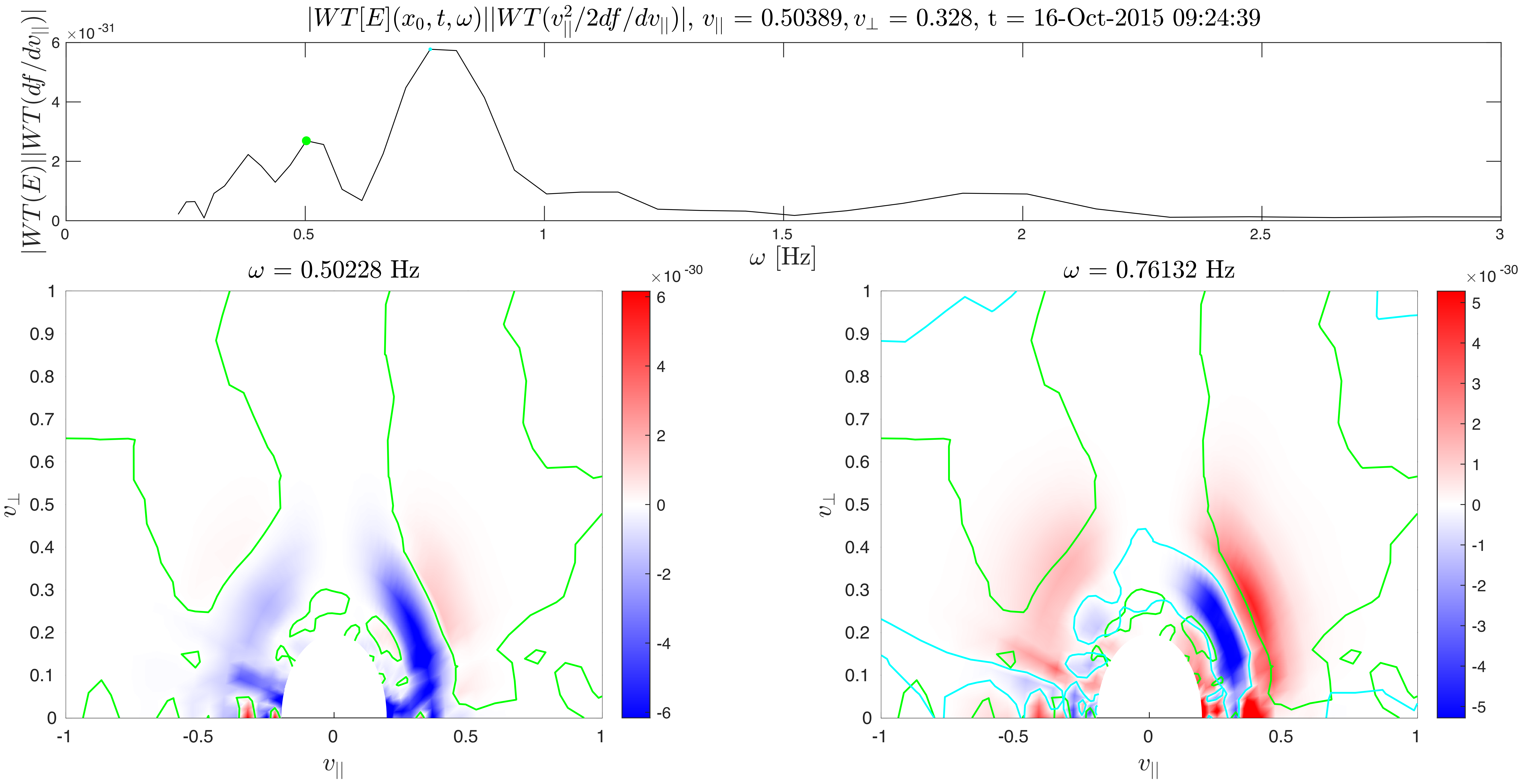
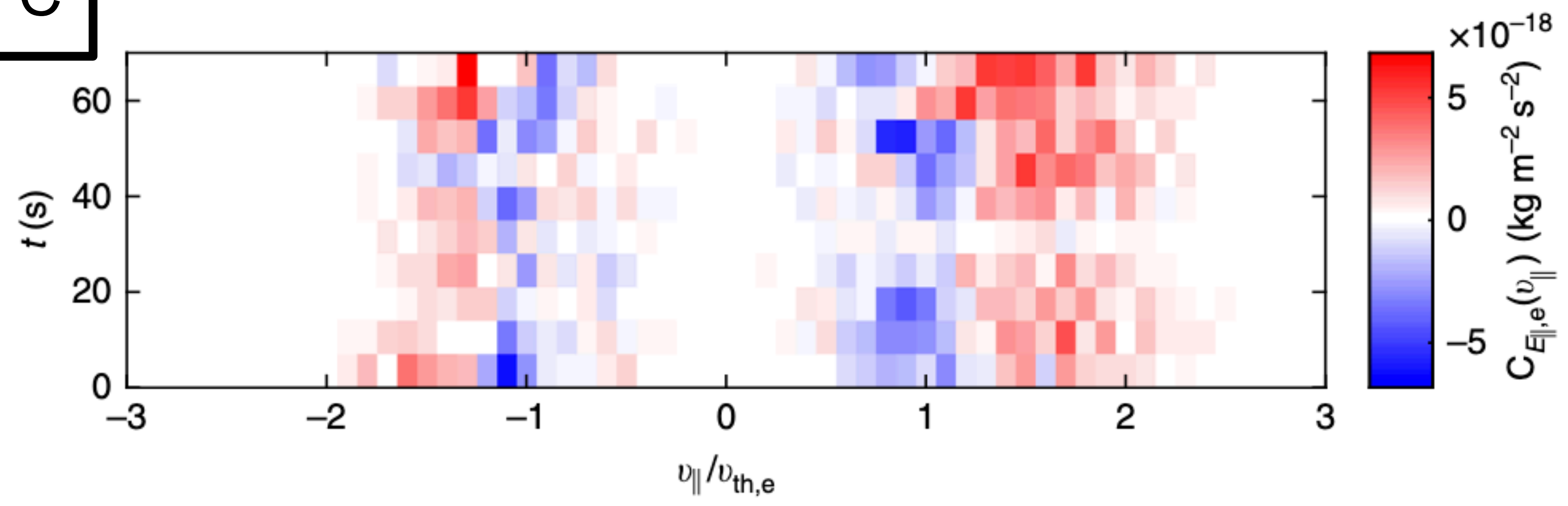
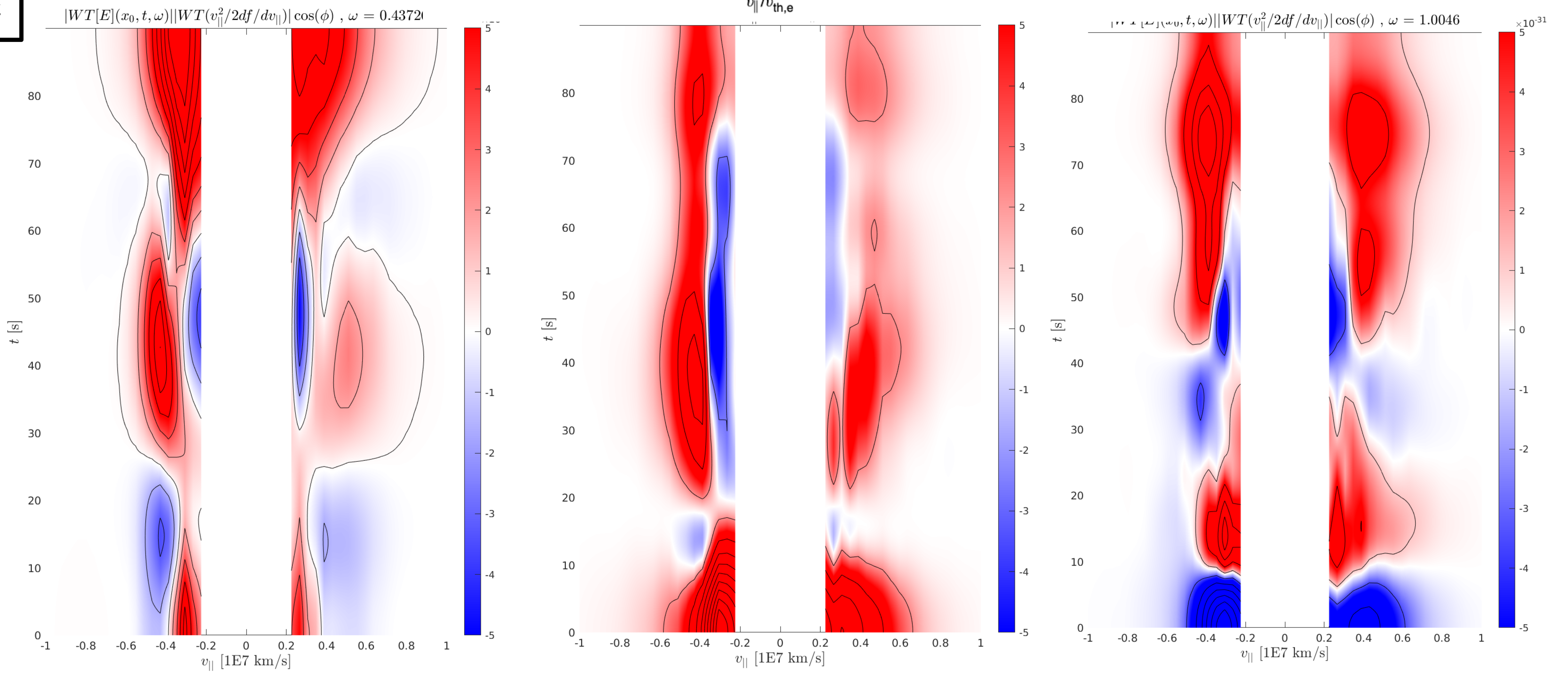


Applying the frequency-resolved FPC to data from MMS spacecraft

OG FPC

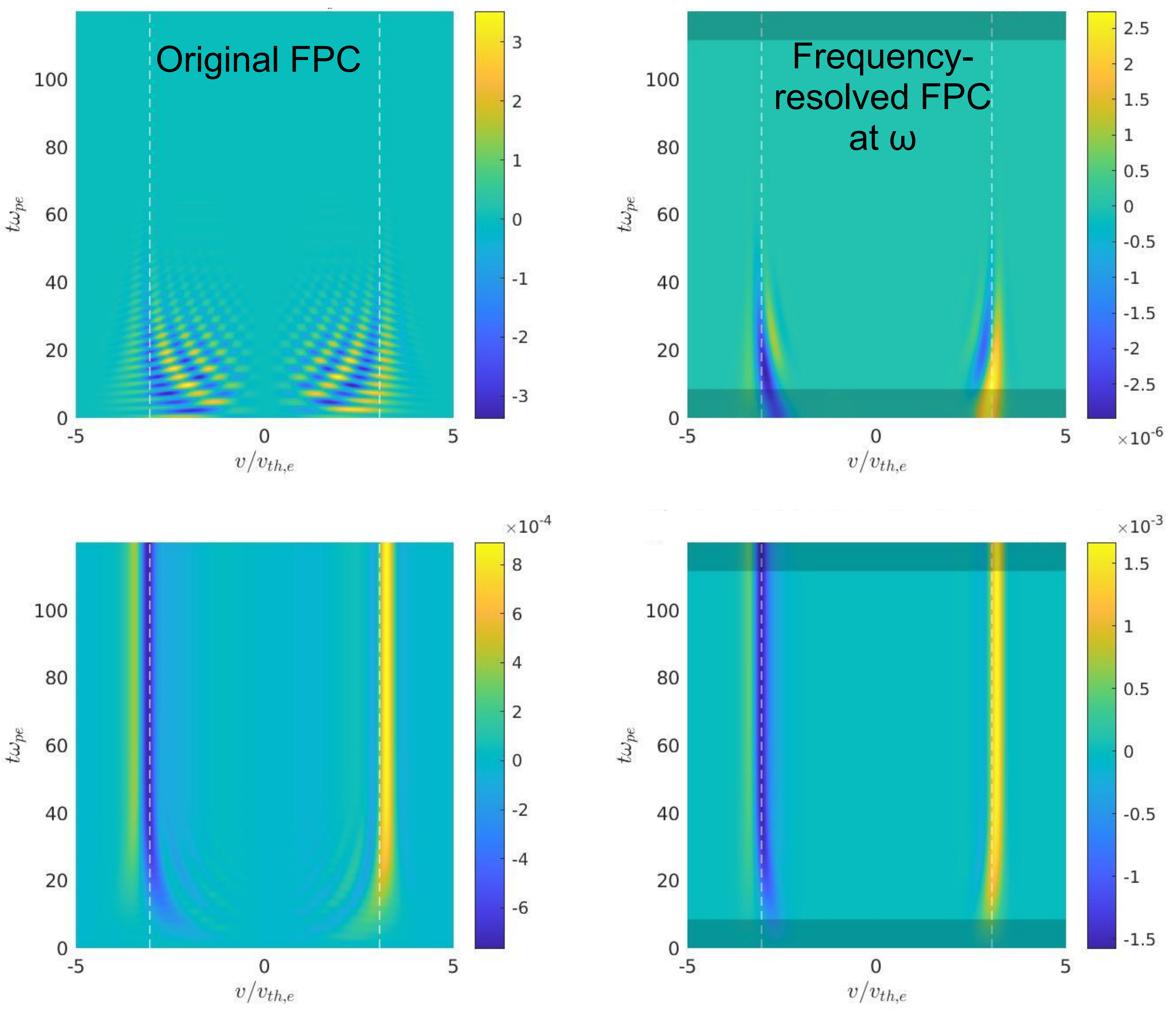


FR-FPC



Summary

- **Updated method of determining the energy transfer between the fields and particles from kinetic processes**
- **Not only does this method add an additional dimension to the traditional field-particle correlation method, but it provides better resolution when there are competing dissipation processes**



Summary

- Updated method of determining the energy transfer between the fields and particles from kinetic processes
- Not only does this method add an additional dimension to the traditional field-particle correlation method, but it provides better resolution when there are competing dissipation processes

Thank you!
Questions?

Obtaining the Frequency-Resolved Field-Particle Correlation (FR-FPC)

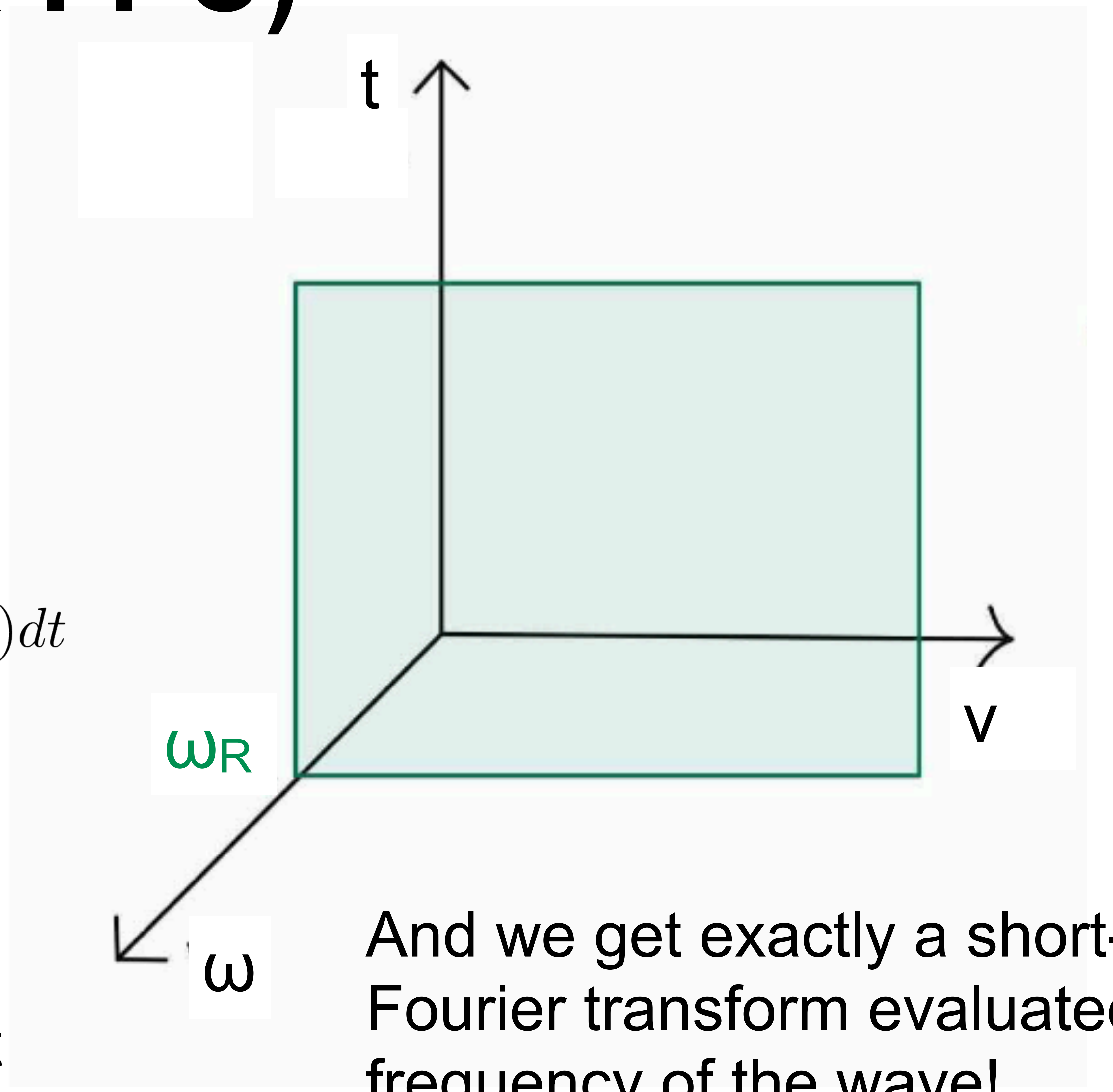
Start with the field-particle correlation

$$C_E(\mathbf{v},t,\tau) = C\left(-q_e\frac{v^2}{2}\frac{\partial f_e}{\partial v}, E\right) = -\frac{1}{N}\sum_{j=1}^{N} q_e\frac{v^2}{2}\frac{\partial f_e}{\partial v}E$$

Because

$$C_E(v,t_0,\tau) = \int_{t_0-\tau/2}^{t_0+\tau/2}\left(-q_e\frac{v^2}{2}\frac{\partial f_e}{\partial v}E\right)dt = \int_{-\infty}^{\infty}\left(-q_e\frac{v^2}{2}\frac{\partial f_e}{\partial v}E\right)w(t-t_0;\tau)dt$$

where $w(t-t_0;\tau) = \begin{cases} 1 & \text{if } t_0-\tau/2 \le t \le t_0+\tau/2 \\ 0 & \text{otherwise} \end{cases}$



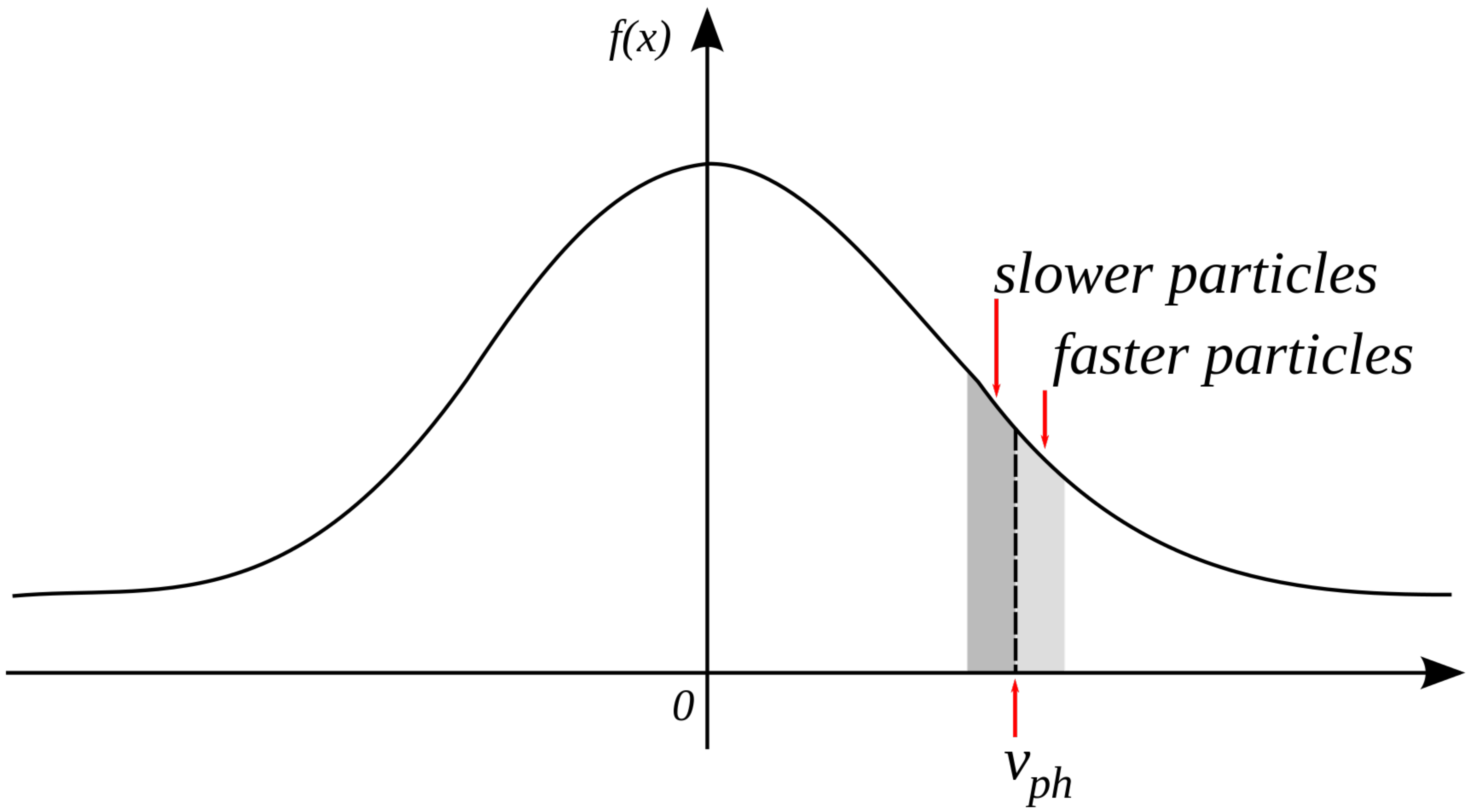
Imagine that we have a single wave with frequency ω_R, damped at some rate γ, and plug that into the FPC expression

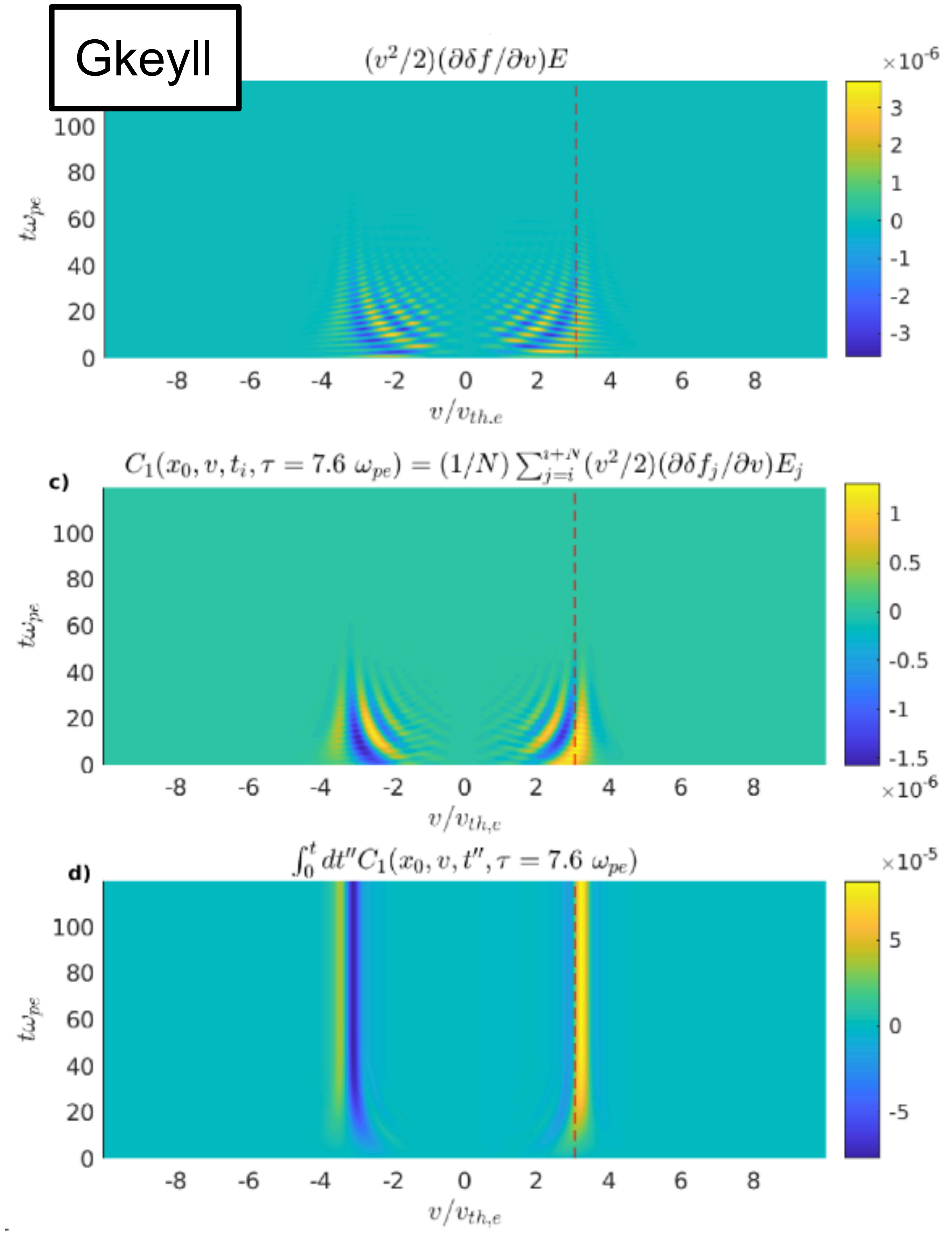
And we get exactly a short-time Fourier transform evaluated at the frequency of the wave!

$$C_E = \int_{-\infty}^{\infty}\left(-q_e\frac{v^2}{2}\frac{\partial f_e}{\partial v}E_0e^{\gamma t}\right)w(t-t_0;\tau)e^{-i\omega_R t}dt \longrightarrow C_E = \int_{-\infty}^{\infty}x(t)w(t-t_0;\tau)e^{-i\omega_R t}dt$$

Field-particle correlation (FPC) of electron Landau damping

$$C_{E_\parallel}(\mathbf{v},t,\tau) = C\left(-q_e \frac{v_\parallel^2}{2}\frac{\partial f_e}{\partial v_\parallel}, E_\parallel\right) = -\frac{1}{N}\sum_{j=1}^{N} q_e \frac{v_\parallel^2}{2}\frac{\partial f_e}{\partial v_\parallel} E_\parallel$$





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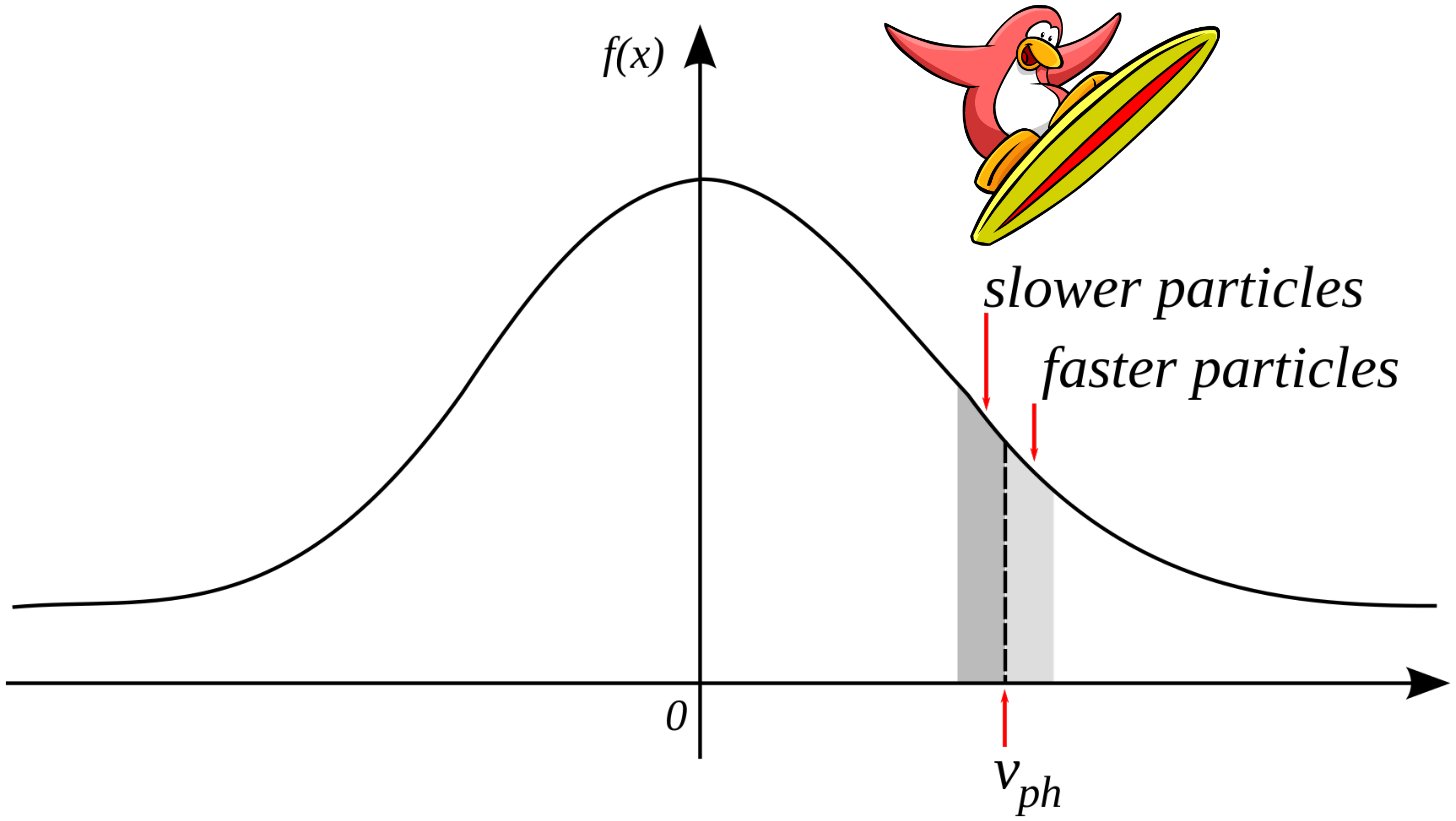

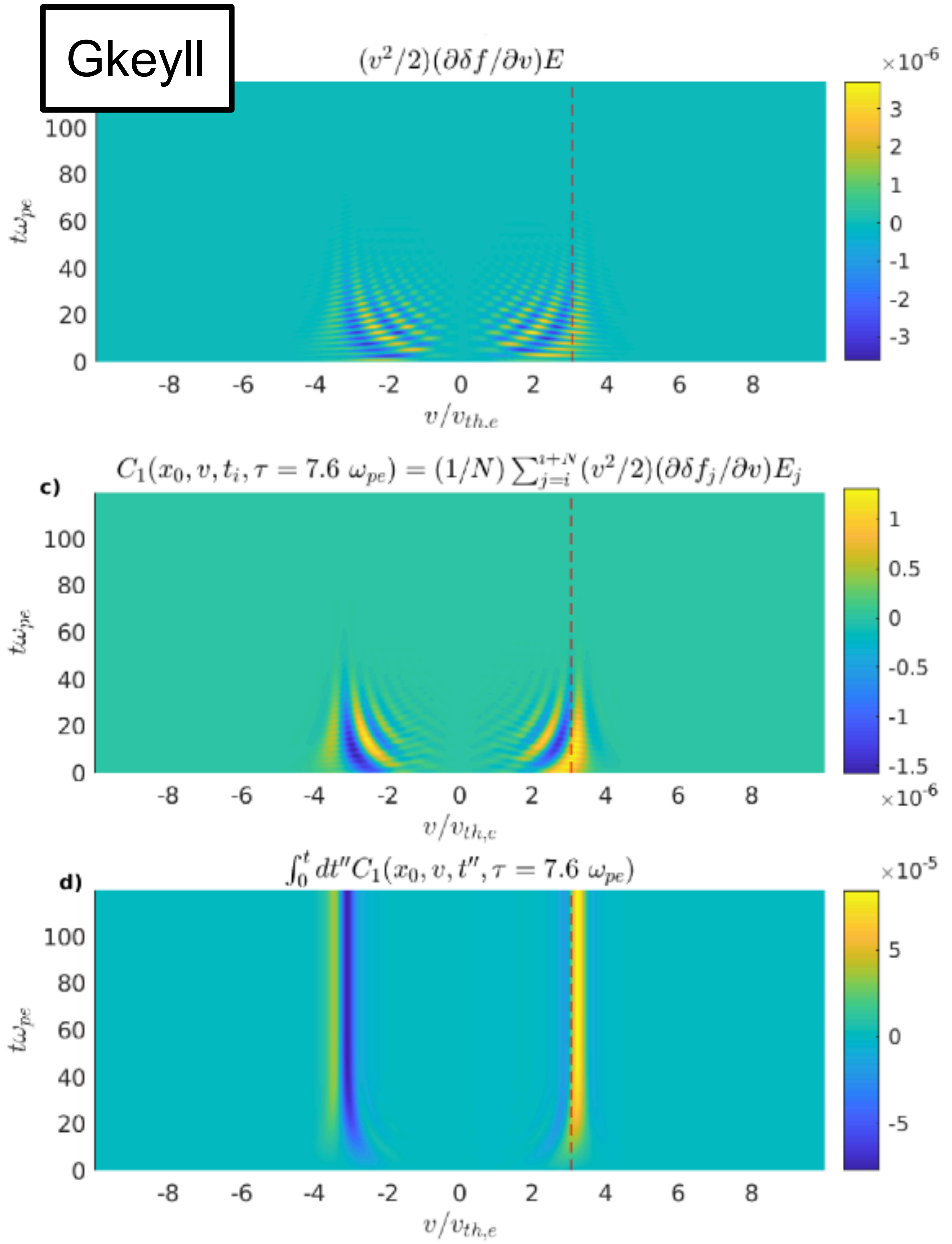

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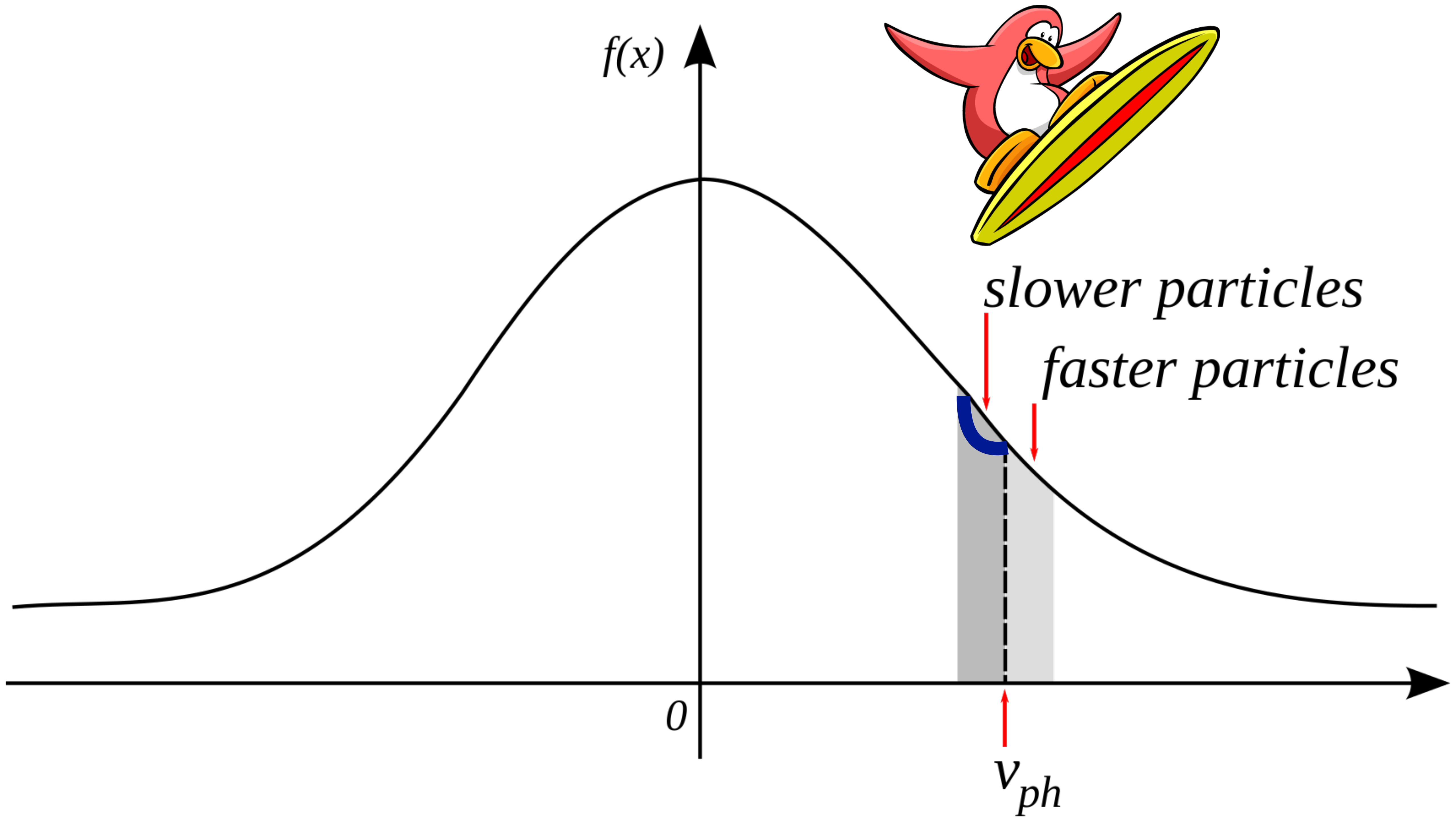

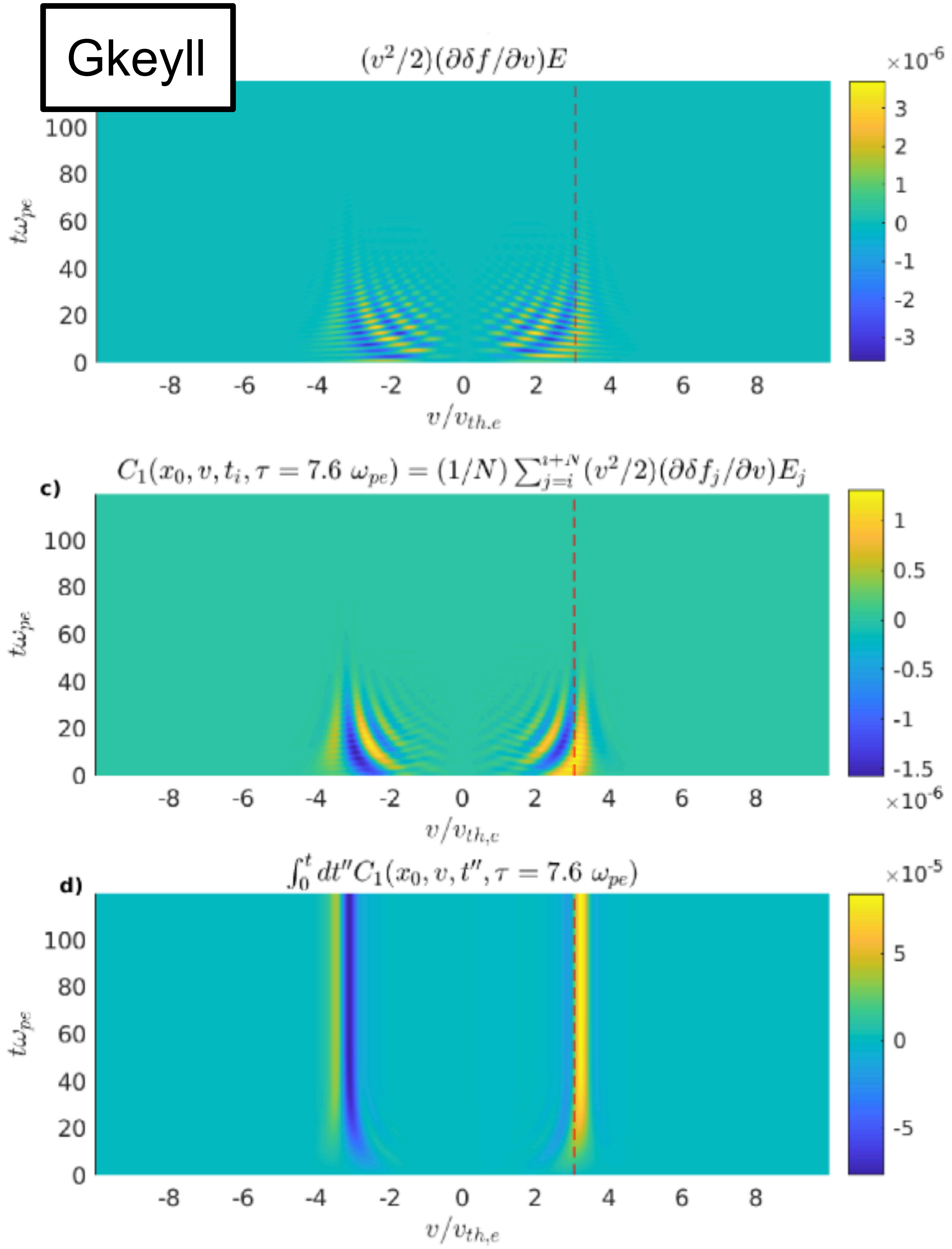

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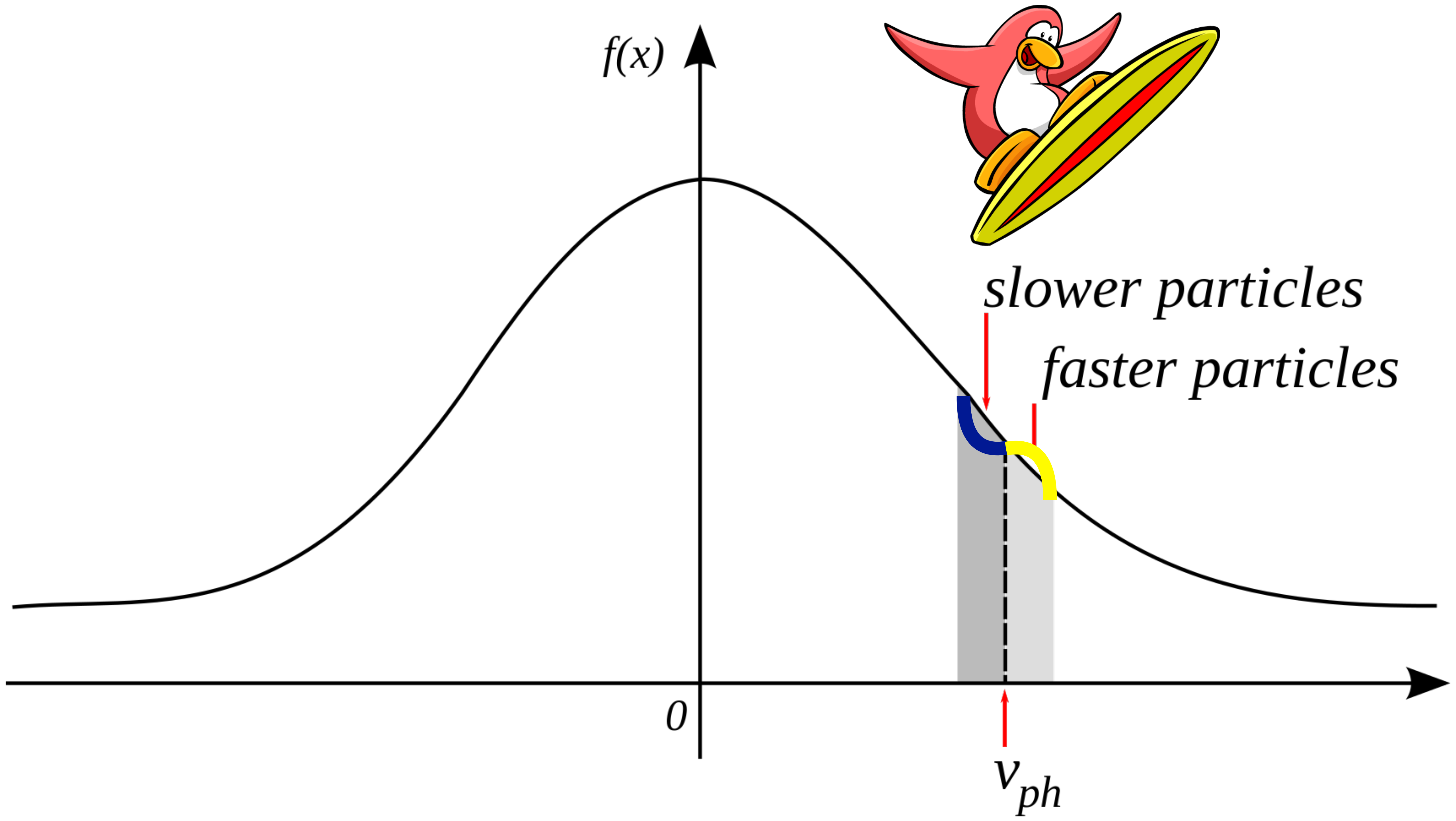
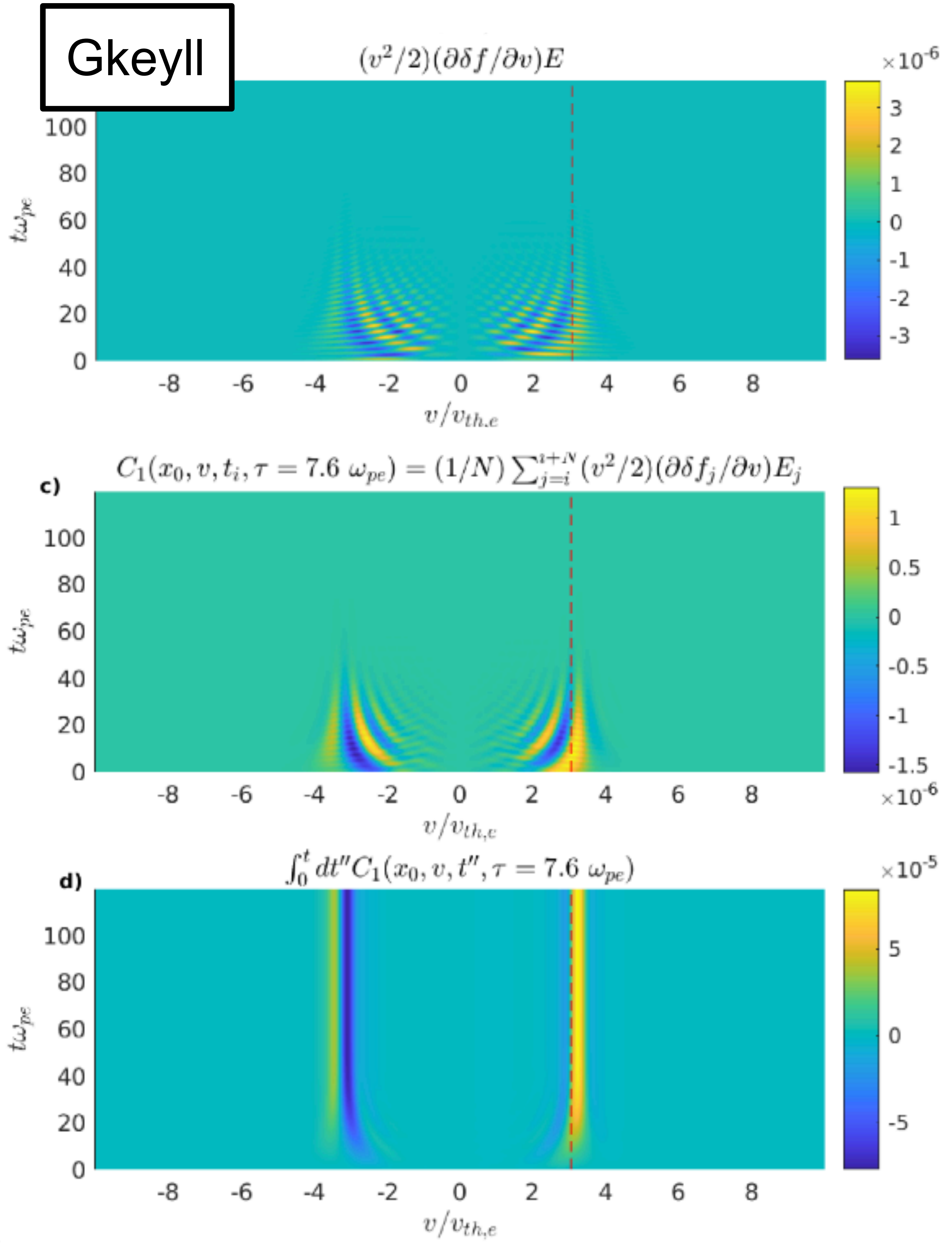


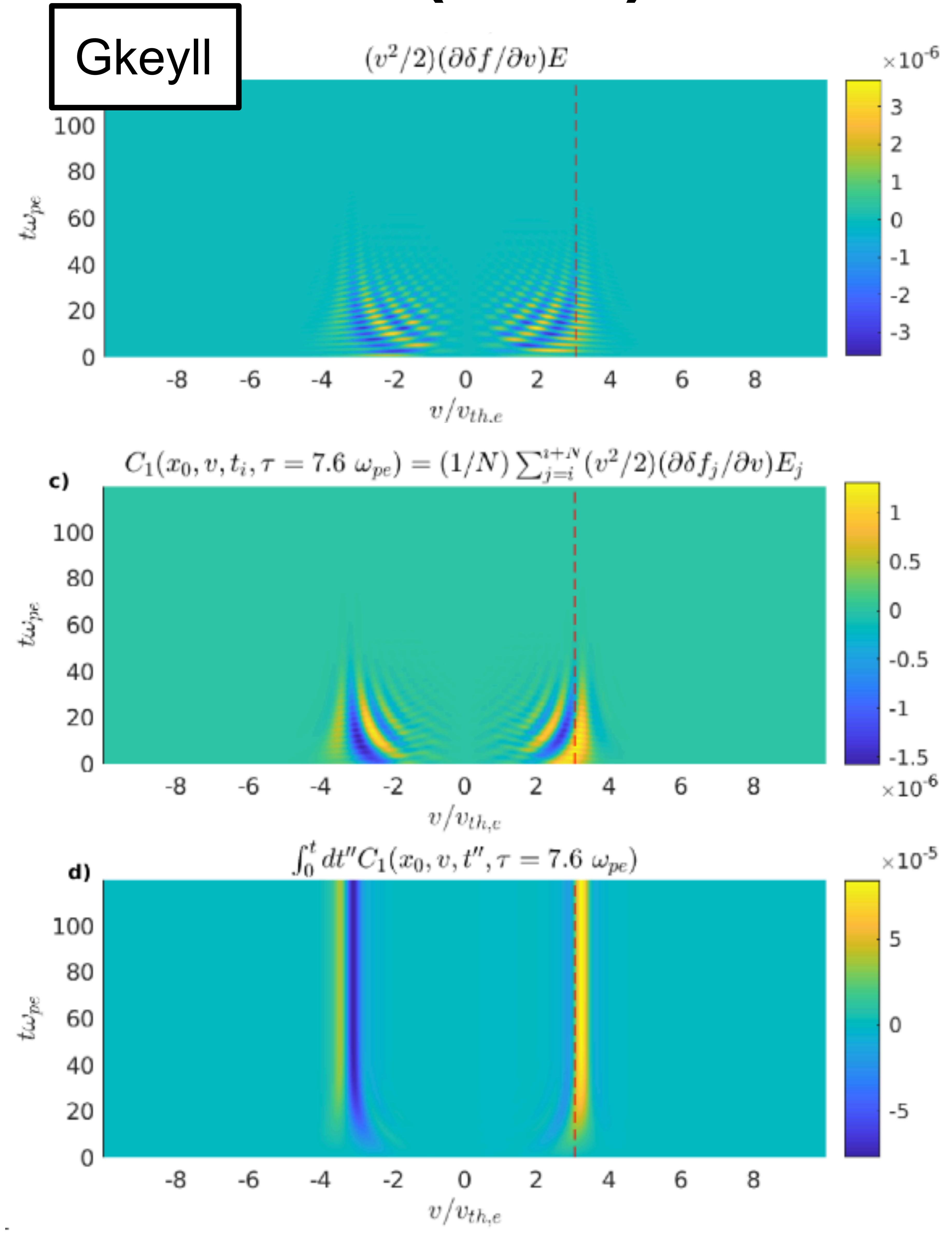


What is the Field-Particle Correlation (FPC)?

Starting with the Vlasov equation, we multiply by the energy for the given species ($m_s v^2/2$), to obtain an expression for the evolution of the phase space energy density

$$\frac{\partial w_s}{\partial t} = -\mathbf{v}\cdot\nabla\omega_s - q_s\frac{v^2}{2}\mathbf{E}\cdot\frac{\partial f_s}{\partial v} - q_s\frac{v^2}{2}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_s}{\partial v}$$

where $w_s(\mathbf{r},\mathbf{v},t) = m_s v^2 f_s(\mathbf{r},\mathbf{v},t)/2$



(Klein+ 2016, Howes+2017)

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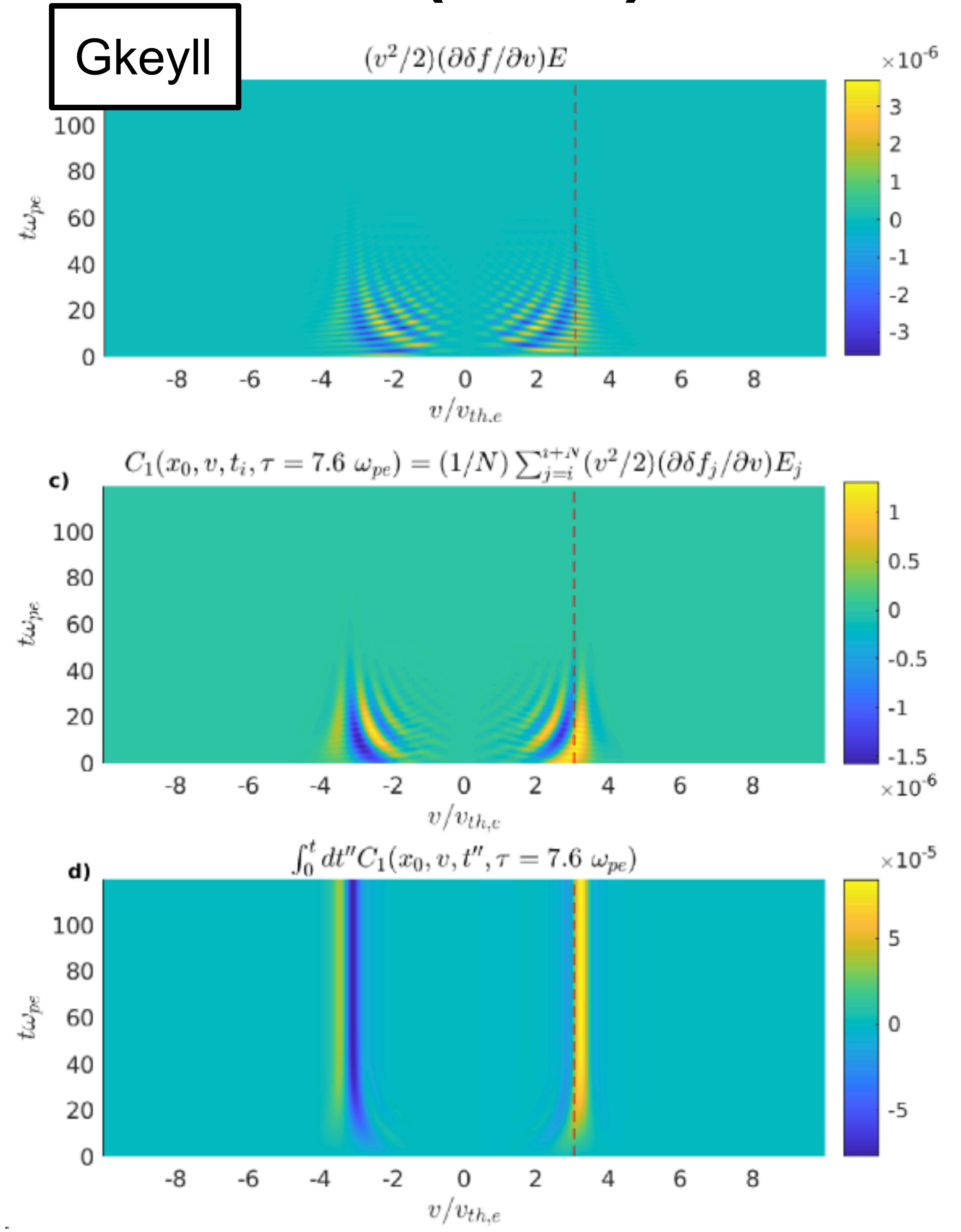
where $w_s(\mathbf{r}, \mathbf{v}, t) = m_s v^2 f_s(\mathbf{r}, \mathbf{v}, t)/2$

- Contributes zero net energy - only oscillations in the energy as a function of space and time



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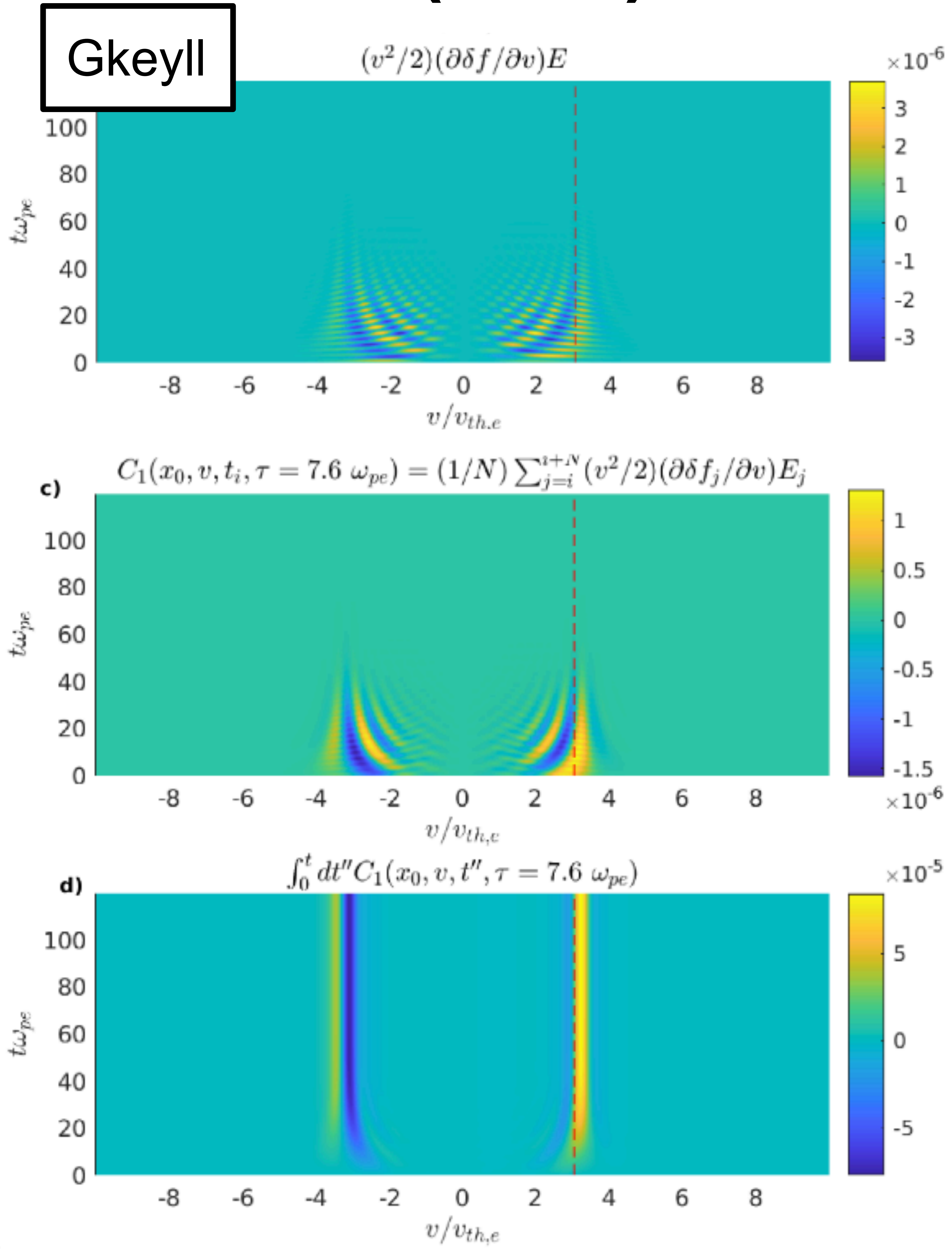
where $w_s(\mathbf{r}, \mathbf{v}, t) = m_s v^2 f_s(\mathbf{r}, \mathbf{v}, t)/2$

- Contributes zero net energy - only oscillations in the energy as a function of space and time
- Magnetic fields do no work



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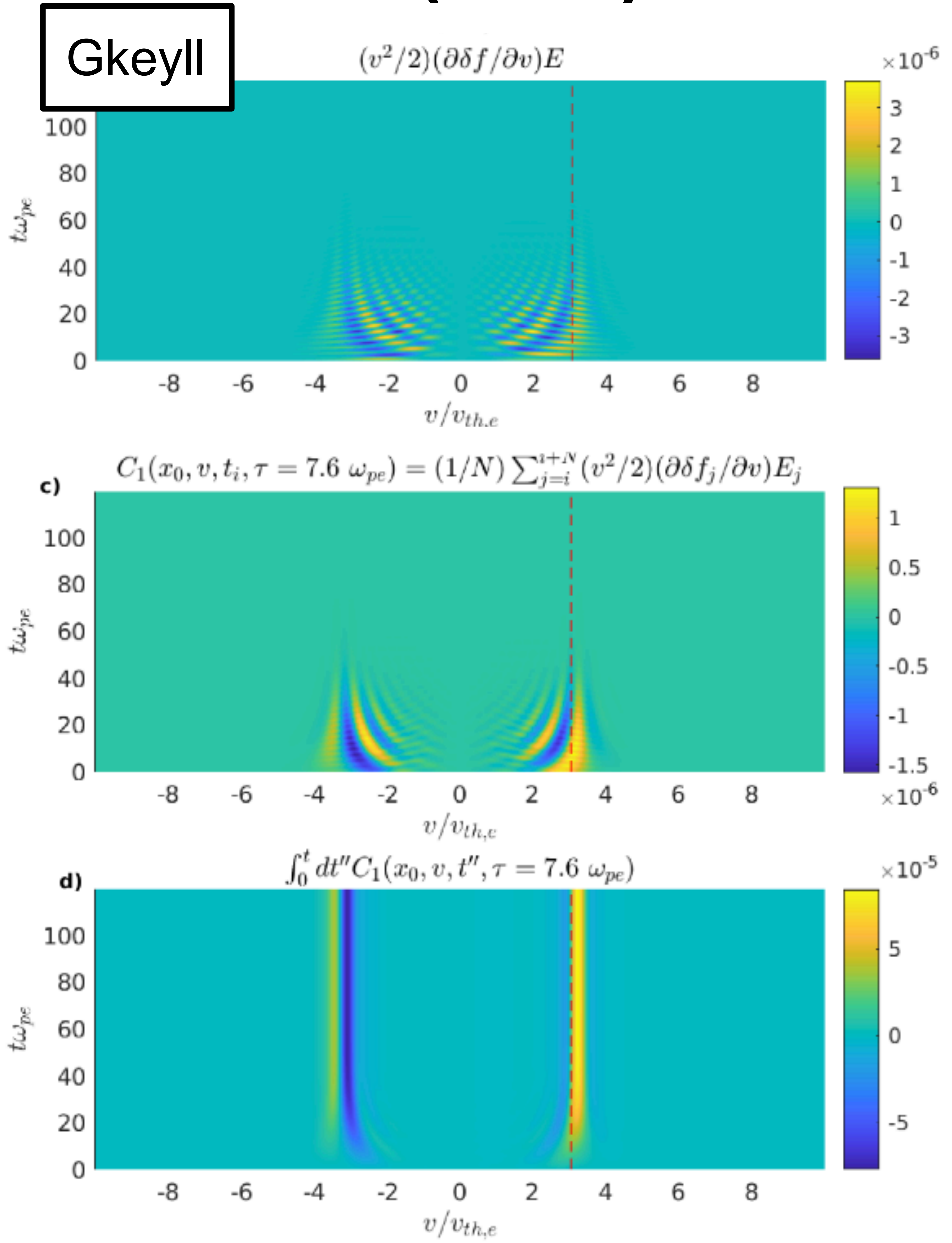
What is the Field-Particle Correlation (FPC)?

Starting with the Vlasov equation, we multiply by the energy for the given species ($m_s v^2/2$), to obtain an expression for the evolution of the phase space energy density

$$\frac{\partial w_s}{\partial t} = -\mathbf{v}\cdot\nabla\omega_s - q_s\frac{v^2}{2}\mathbf{E}\cdot\frac{\partial f_s}{\partial v} - q_s\frac{v^2}{2}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_s}{\partial v}$$

where $w_s(\mathbf{r},\mathbf{v},t) = m_s v^2 f_s(\mathbf{r},\mathbf{v},t)/2$

- Contributes zero net energy - only oscillations in the energy as a function of space and time
- Magnetic fields do no work



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It can be shown that when integrated over physical space and velocity space, this term is proportional to j·E

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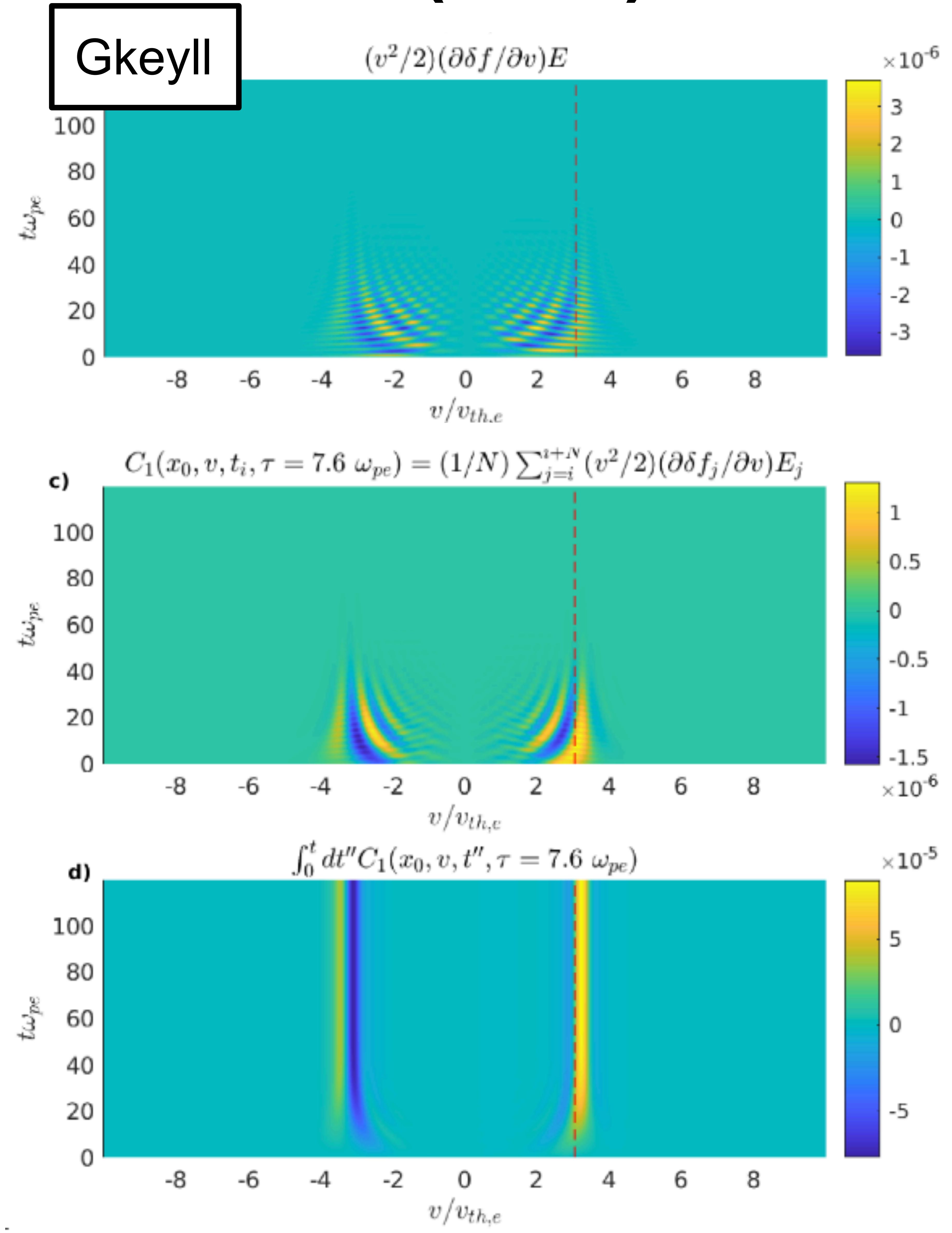

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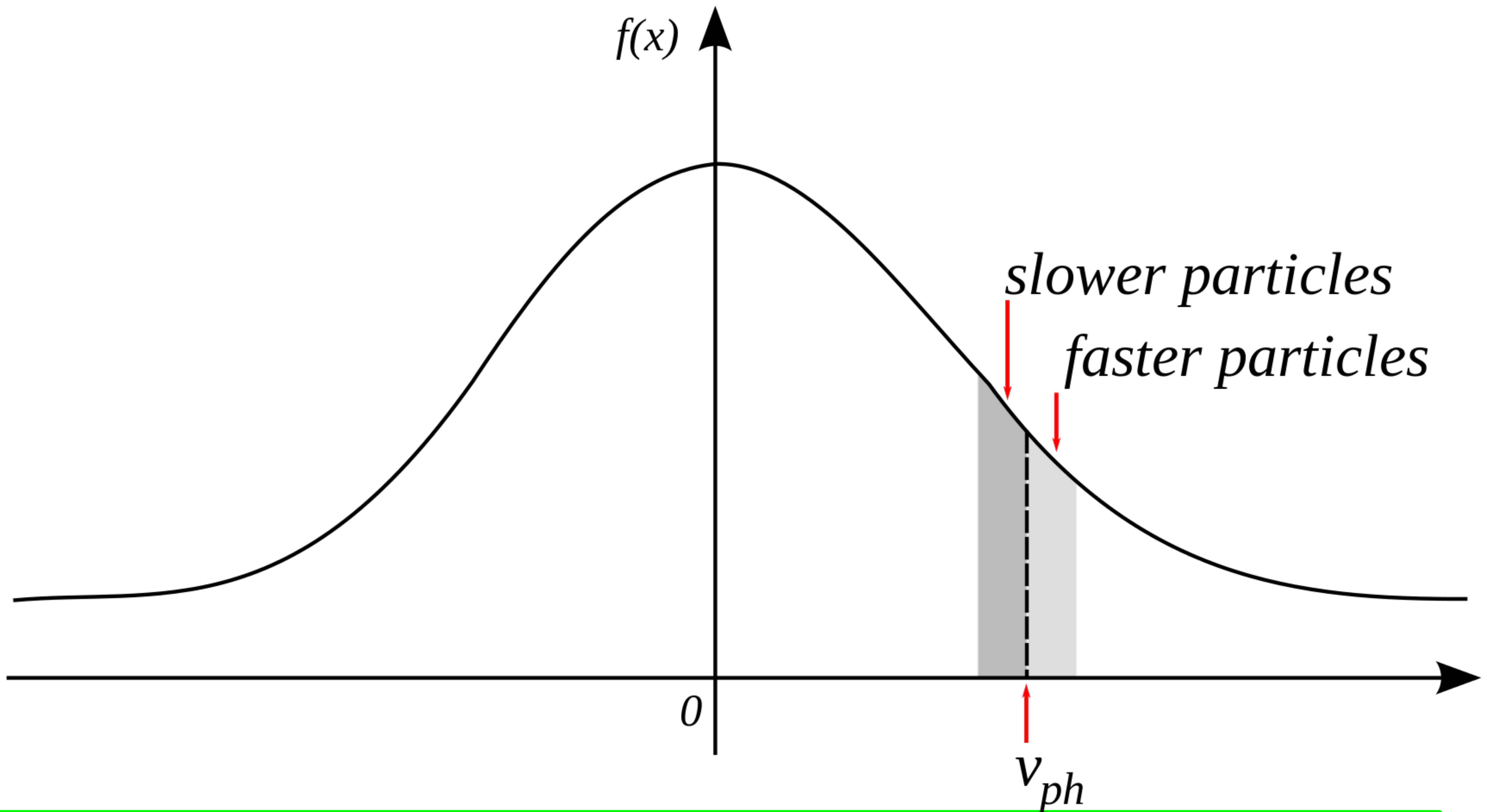

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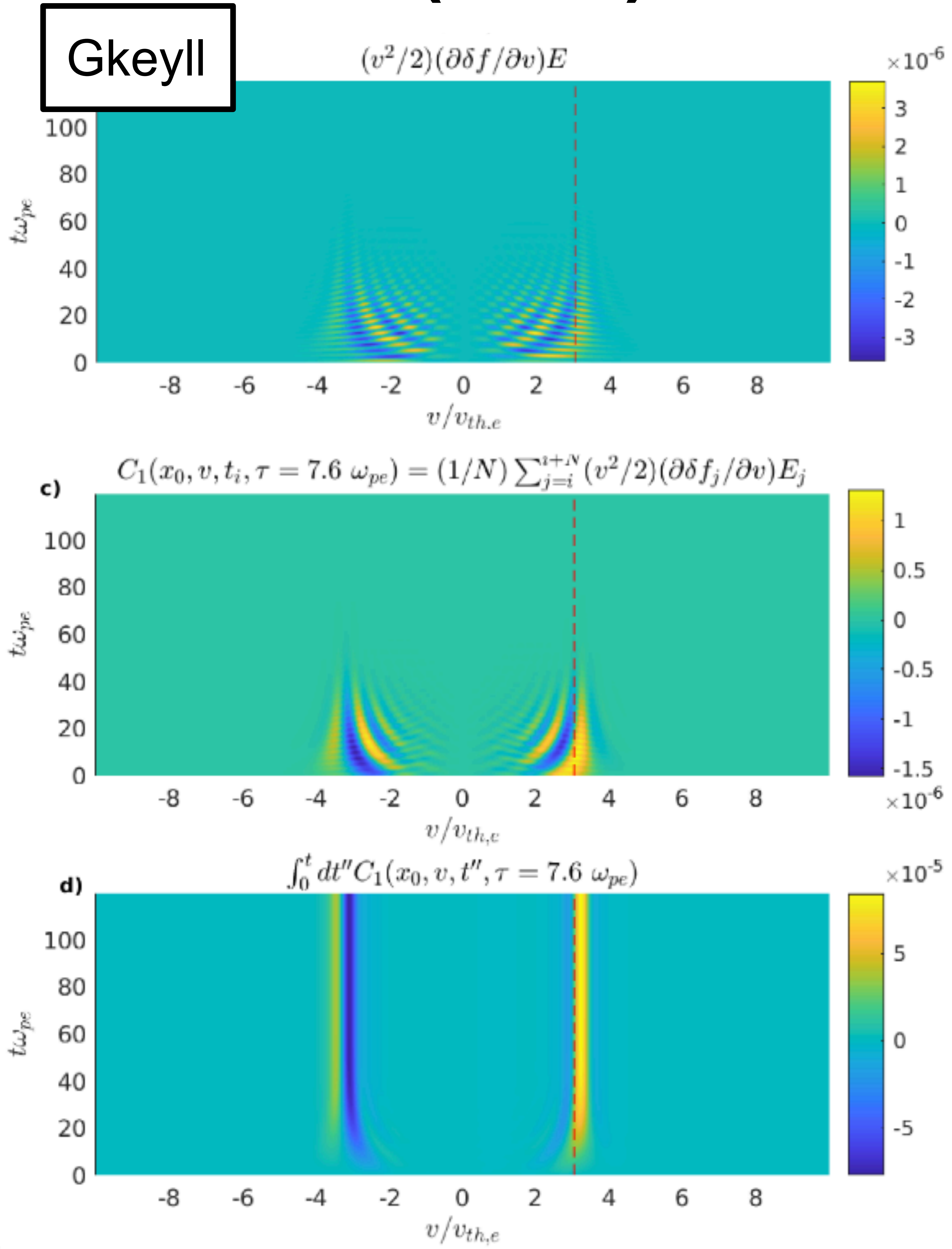

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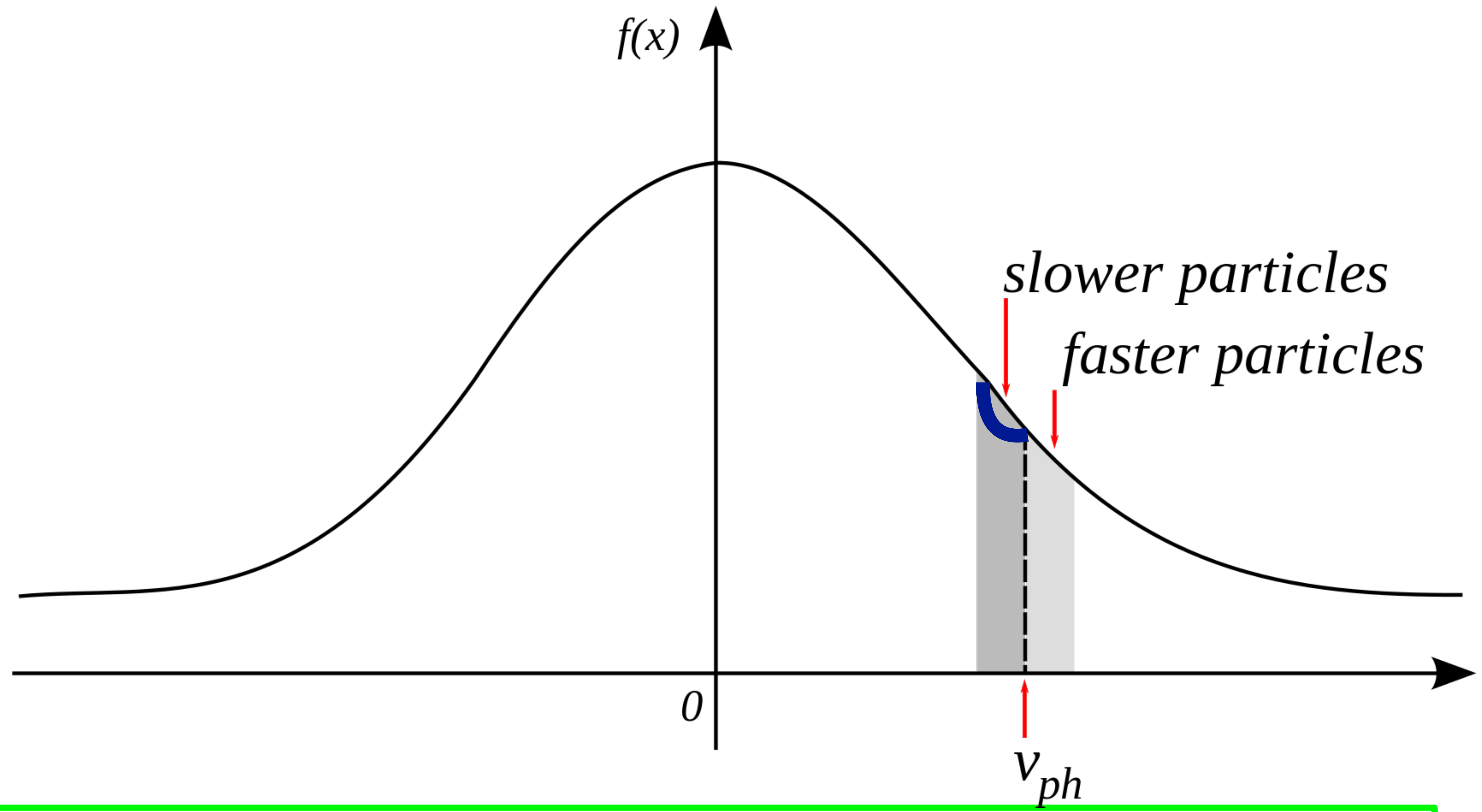

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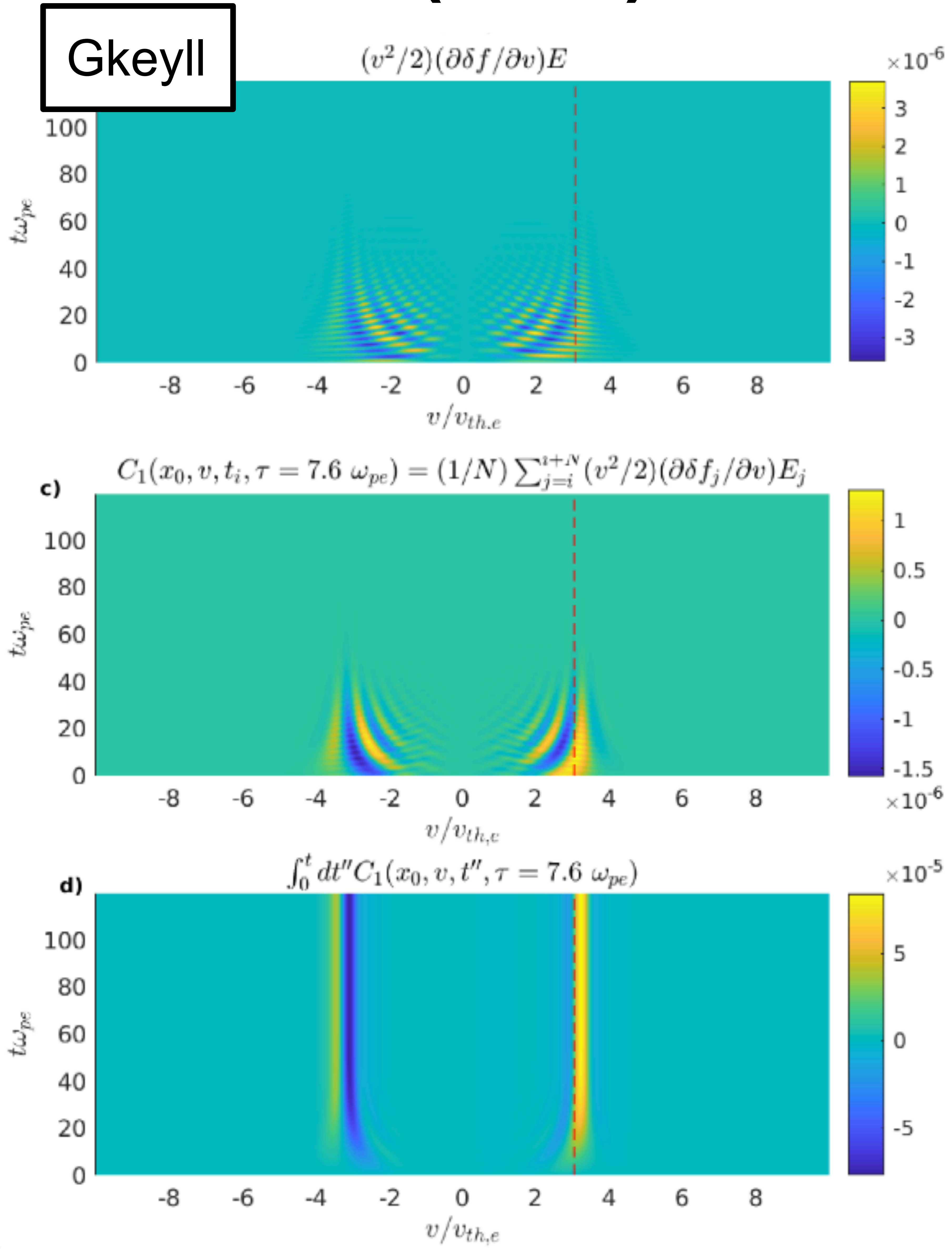

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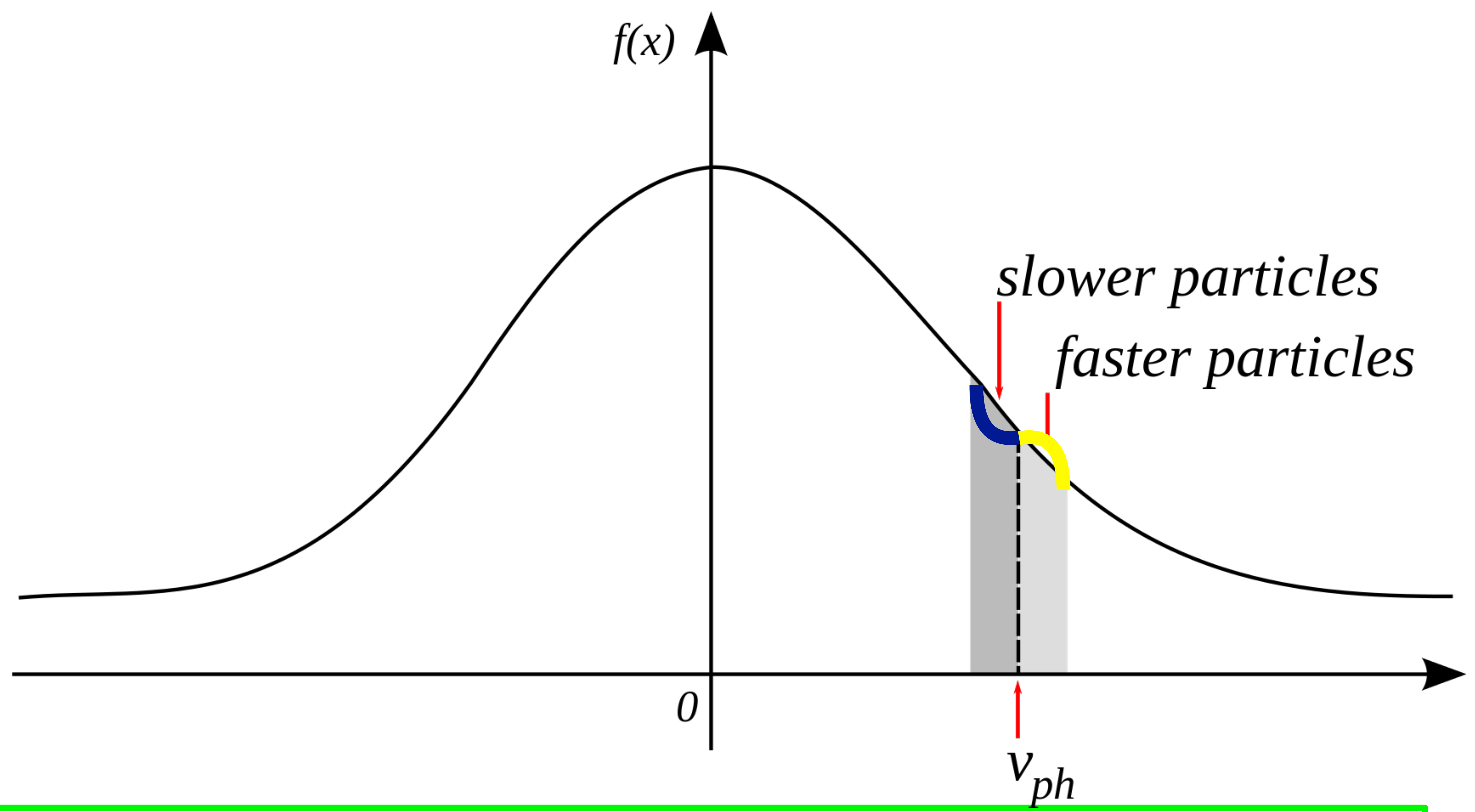

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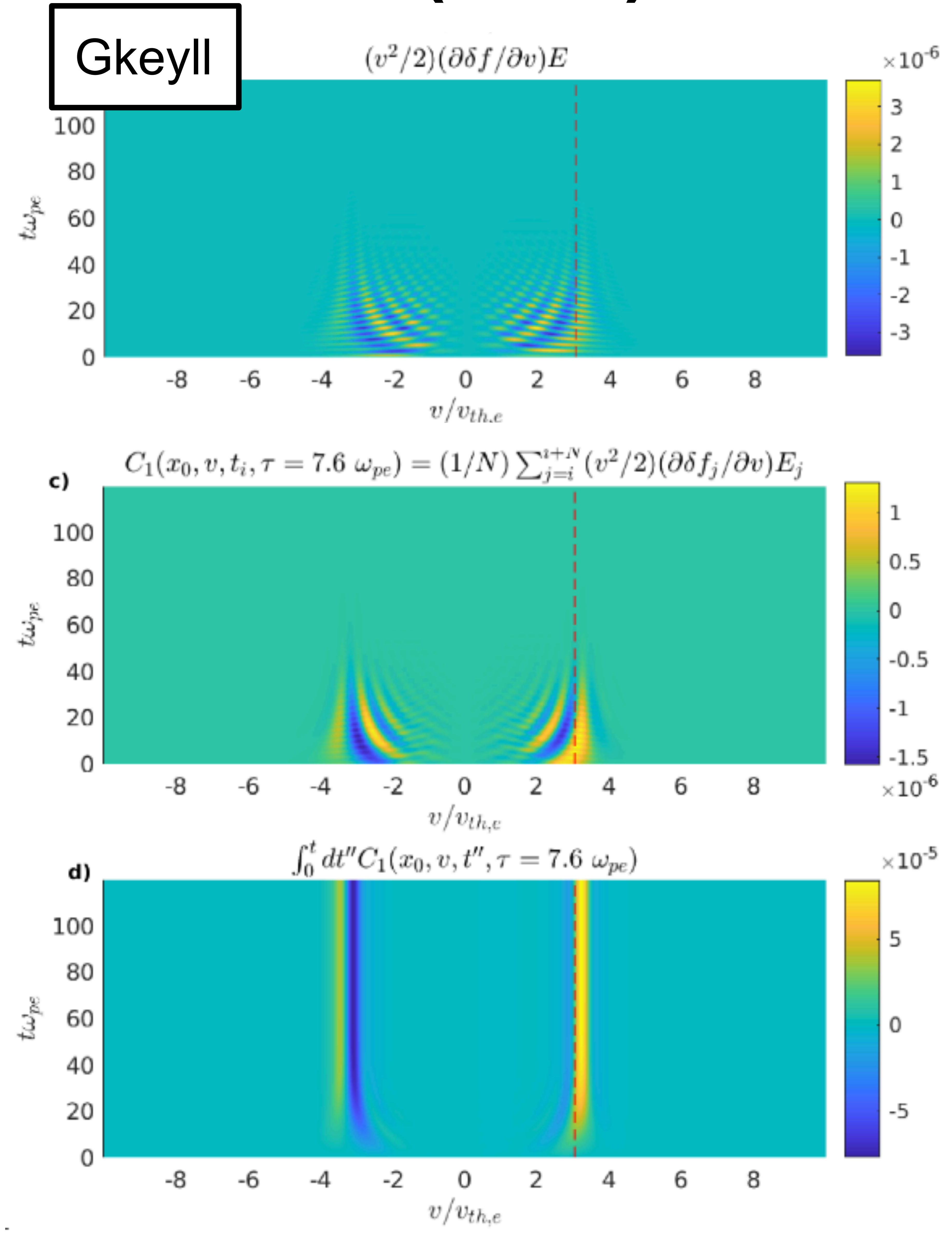
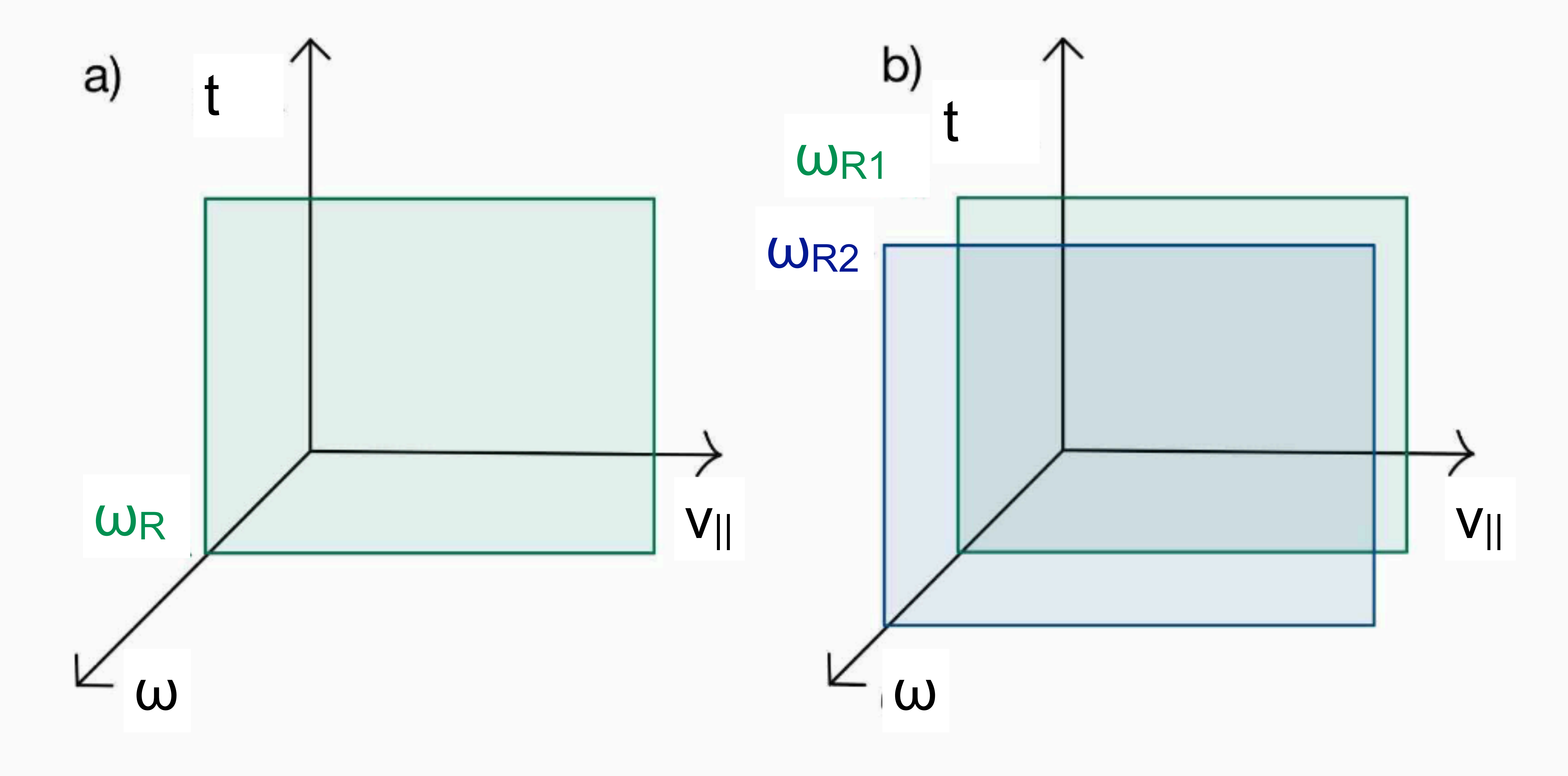


Obtaining the Frequency-Resolved Field-Particle Correlation (FR-FPC)

The difficult aspect comes when we have an electric field that's composed of more than one wave

$$E = \sum_n E_{0,n} e^{\gamma_n t} e^{-i\omega_{R,n} t}$$

In that case, when we insert the electric field into our form of the field-particle correlation we get that the FPC is the sum of the short-time Fourier transform at each constituent frequency, summed together



$$C_{E_{\|}} = \sum_n \int_{-\infty}^{\infty} -q_e \frac{v_{\|}^2}{2} \frac{\partial f_e}{\partial v_{\|}} E_{0,n} e^{\gamma_n t} w(t-t_0) e^{-i\omega_{R,n} t} dt \longrightarrow C_{E_{\|}} = \sum_n \int_{-\infty}^{\infty} x_n(t) w(t-t_0) e^{-i\omega_{R,n} t} dt$$

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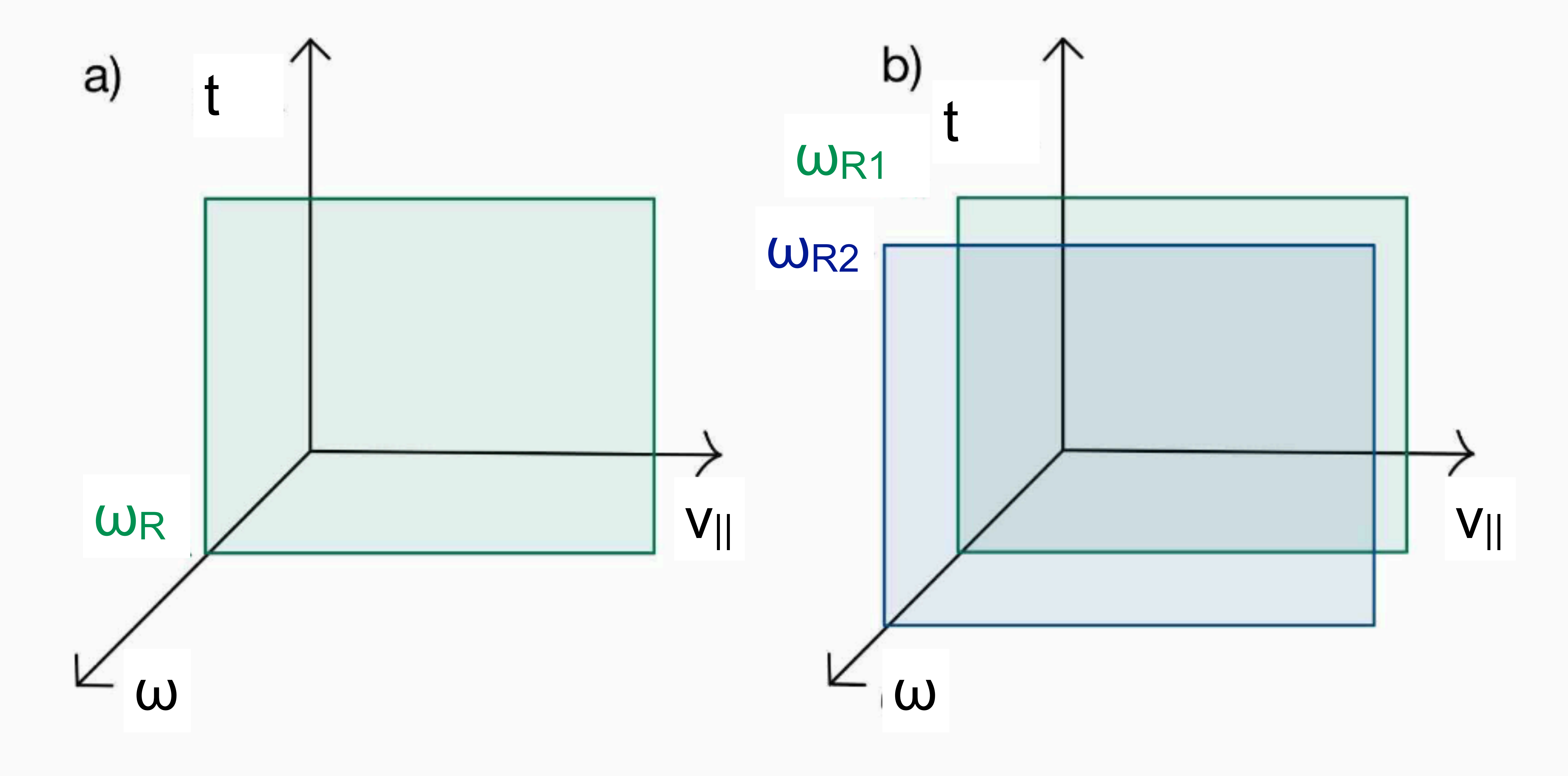

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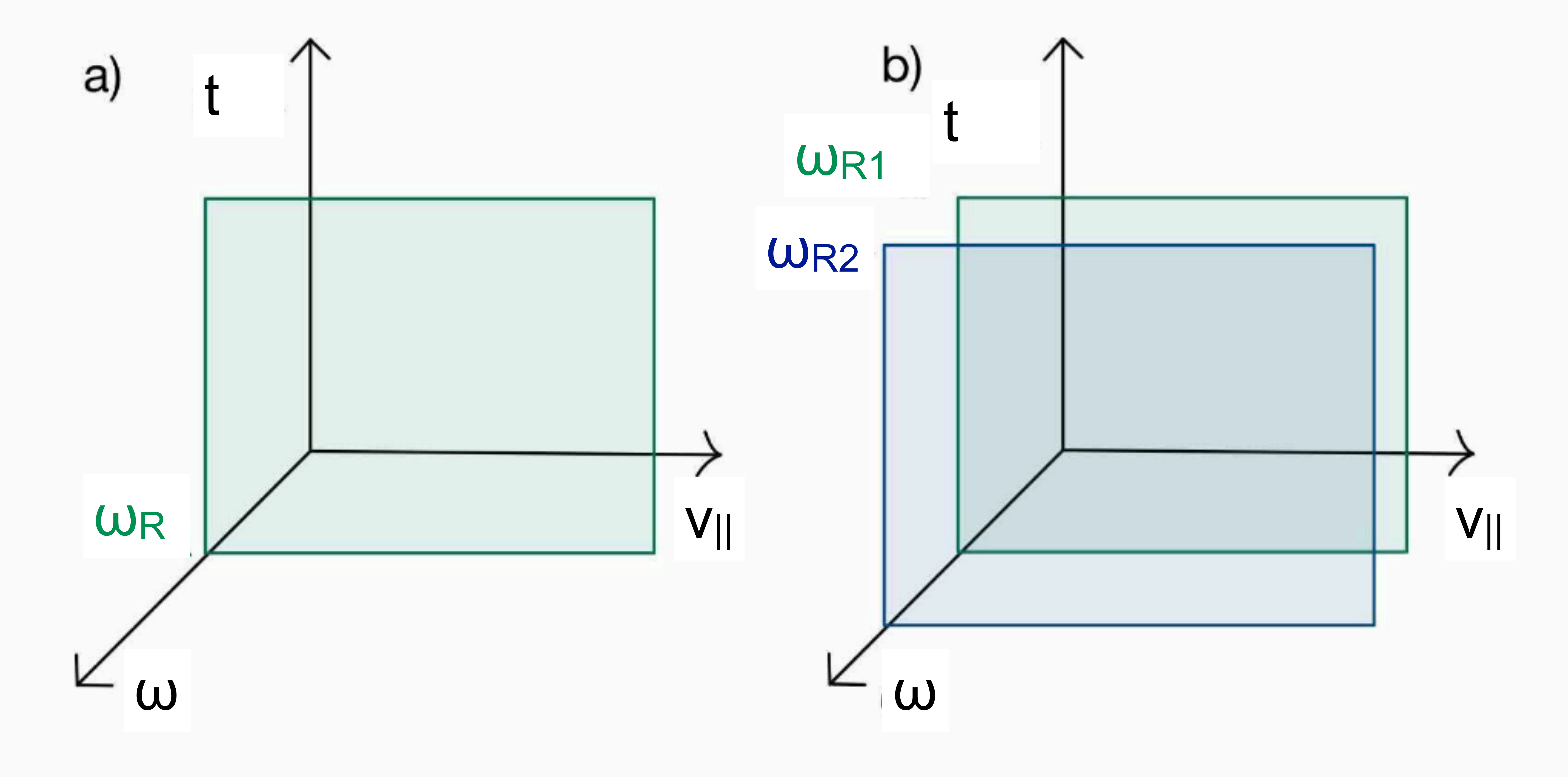

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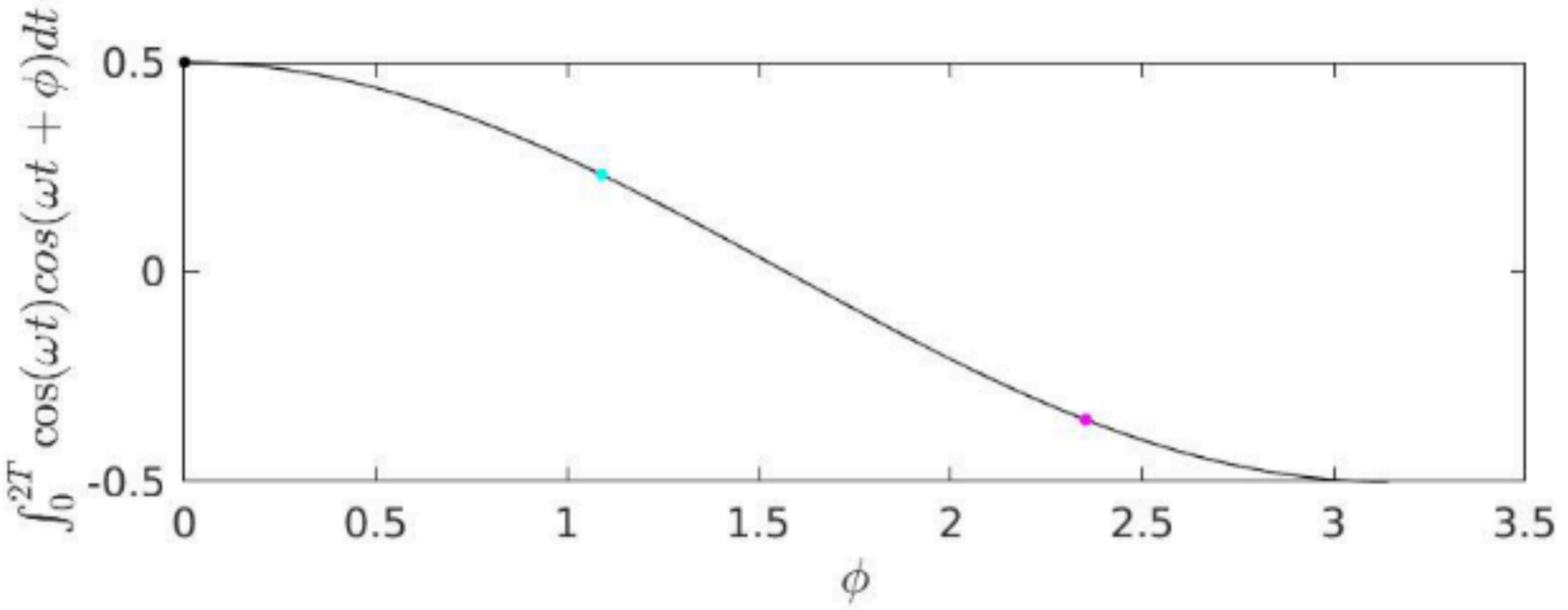
Overview of the differences between the frequency-resolved FPC and traditional FPC

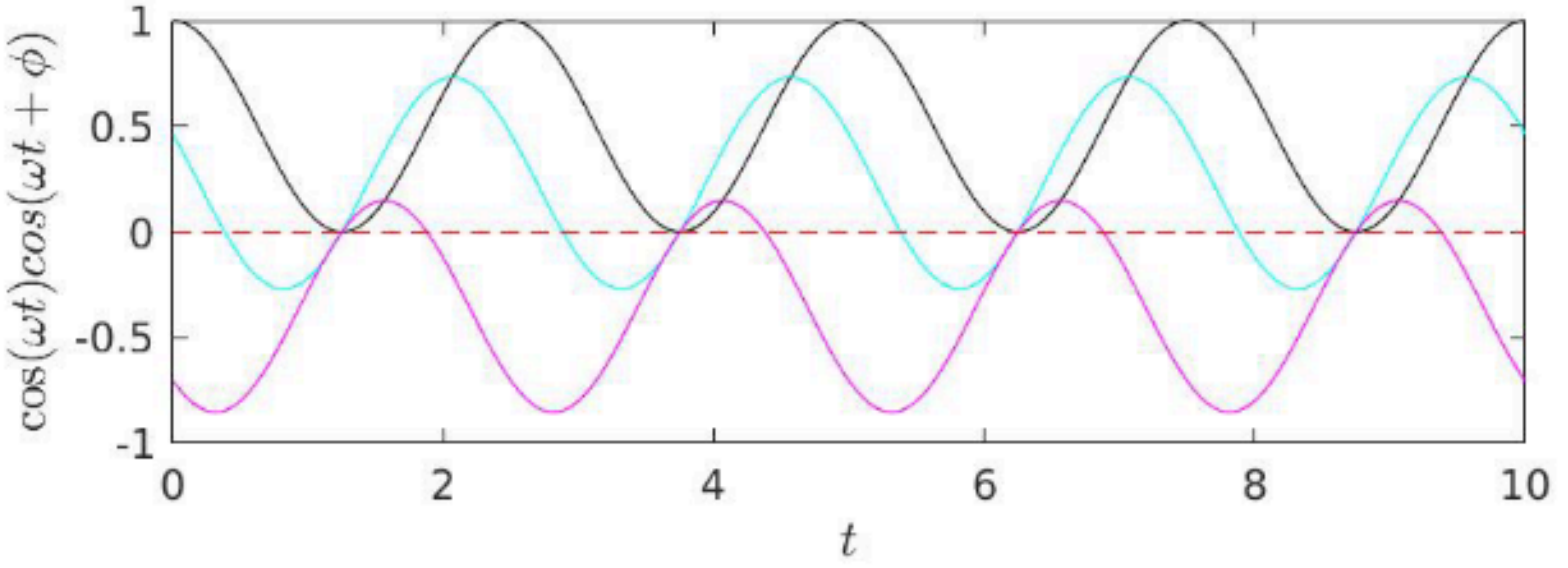
1. Ability to better represent short-lived signals instantaneously
2. Cleaner signals in an environment with multiple dissipation processes

Main issue is that our models for the electric field and distribution function have both a real and imaginary component, but we can only measure the real part of each of these signatures

$$C_{E_\parallel} = \sum_n \int_{-\infty}^{\infty} x_n(t) w(t - t_0) \left[\cos(\omega_{R,n}) + i \sin(\omega_{R,n})\right] dt$$

Equivalent to only looking at the real part of a Fourier transform. Both the frequency and phase information is included, but convoluted together.





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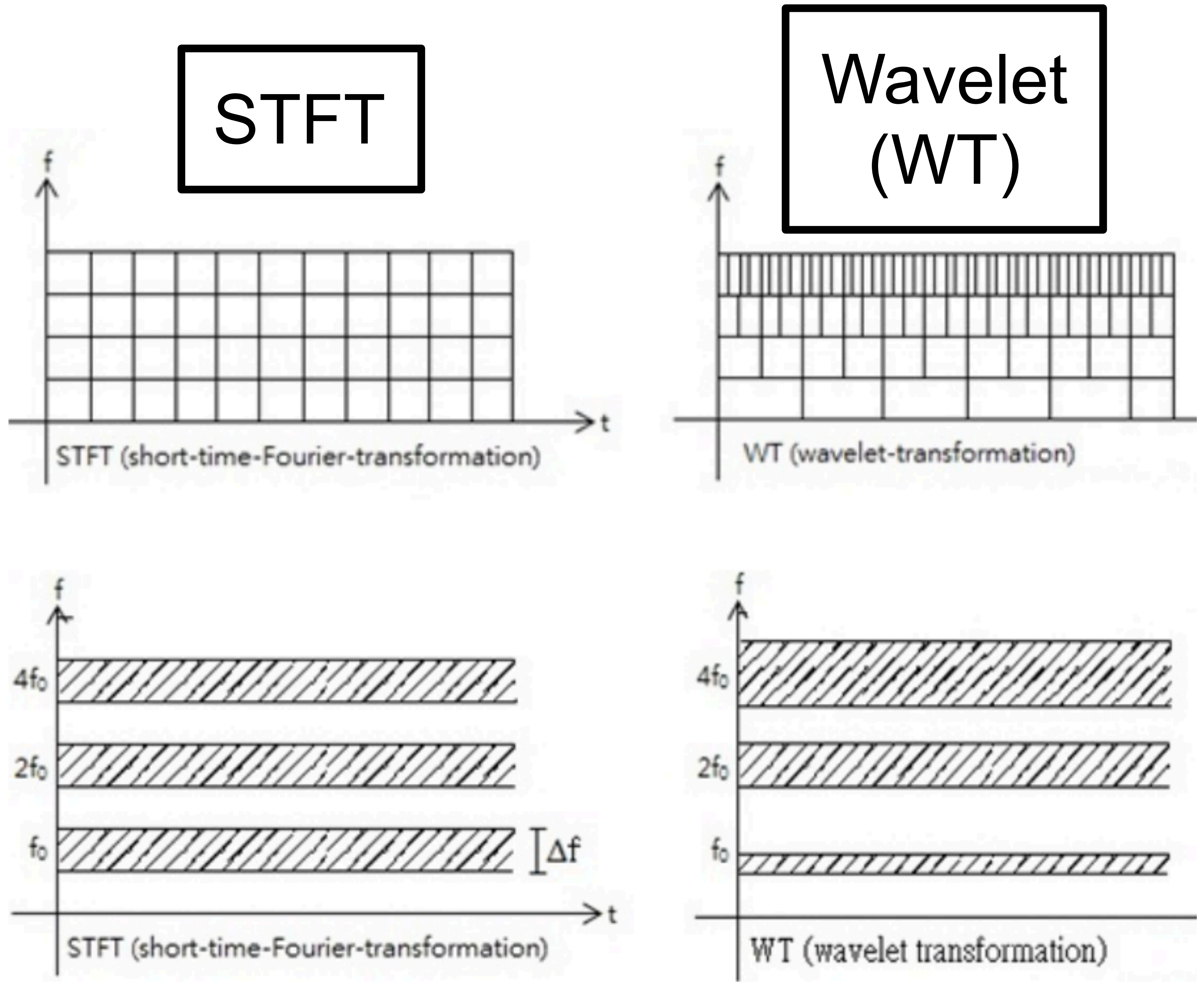
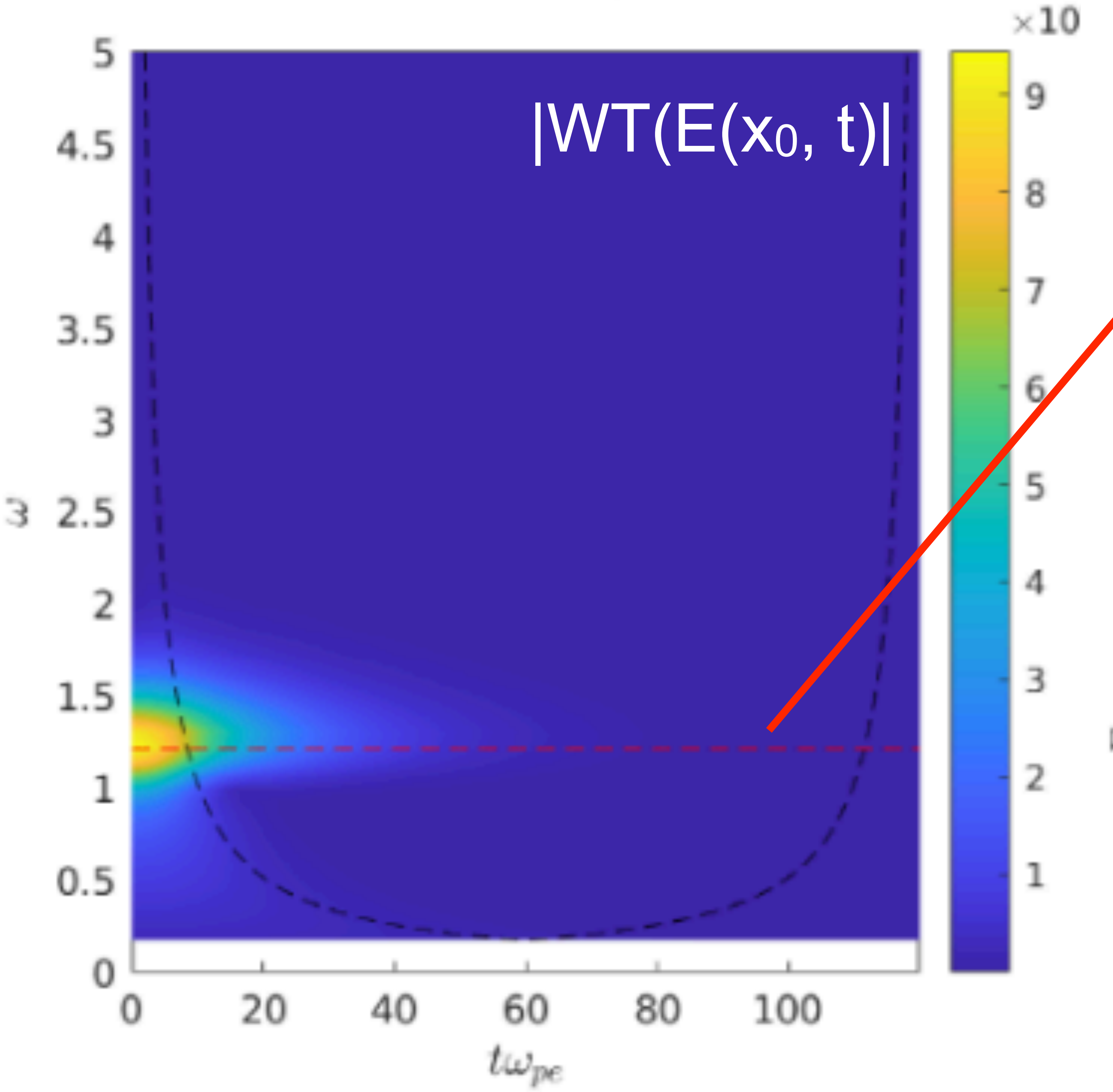
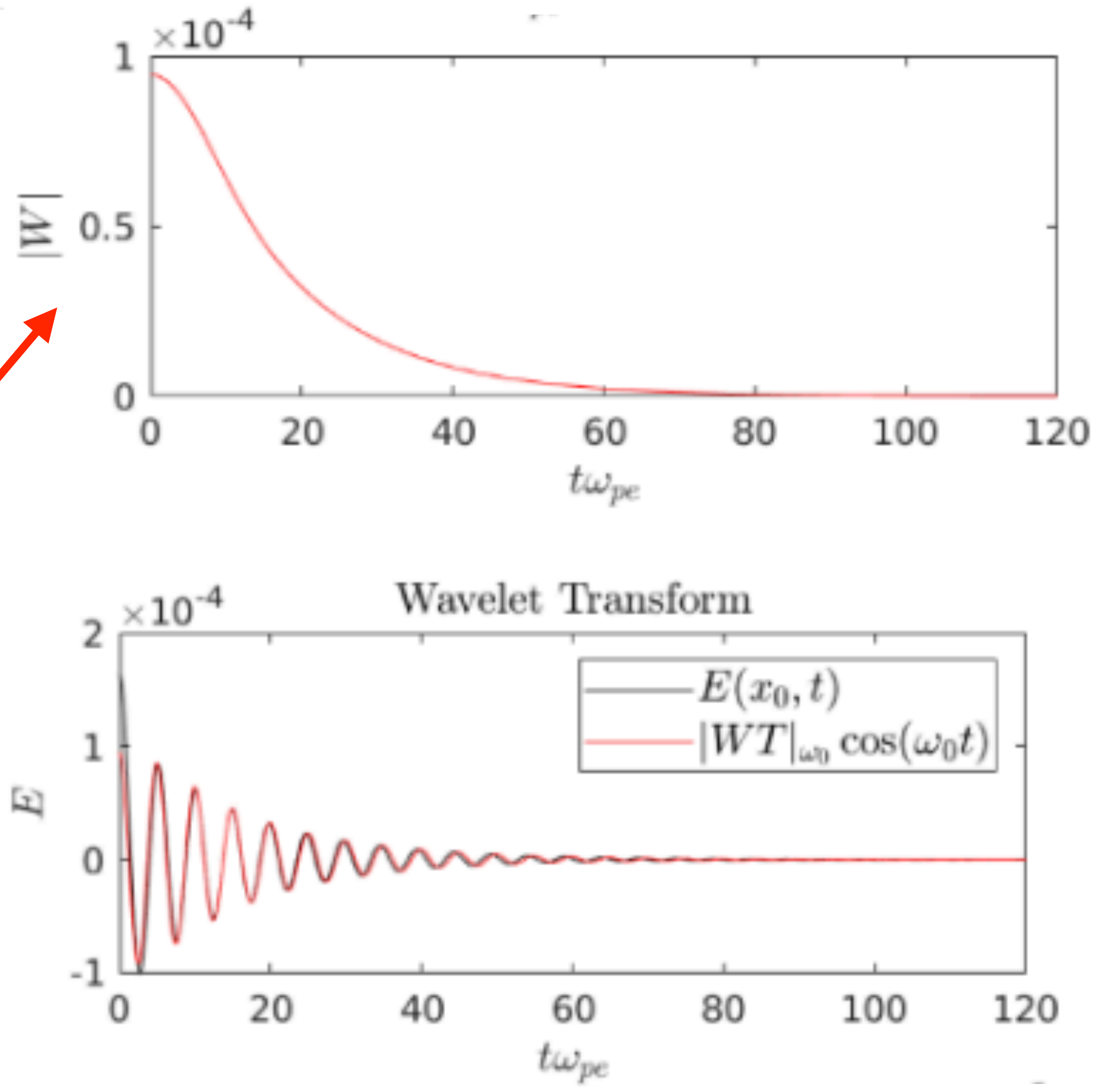
Applying the wavelet transform to electric field data from Gkeyll simulation of electron landau damping



$$\Delta f \Delta t \geq 1$$





Applying the frequency-resolved FPC to a Gkeyll simulation of electron landau damping