1

Structure + Summary

SAPIENZA
UNIVERSITÀ DI ROMA

- start sph. symm. *-cl., glob., nuclear fluid dynamical model, fully self-c. relaxation (r.h.s.) based on Rosenbl., McD, Judd '57
- get corrections for anisotropy decay and equipartition terms
- better agreement with N-body for core collapse, mass segr., binary heating (not there)
- generalization to axisymm. system poss.
- local ↔ „straight line" ↔ orbit average using E, J or E, J_z for axis. rot. syst.
- basis of Monte-Carlo Hénon style
- relation to ultracold plasma (D. Comparat) Rydberg atoms

$$Kn = \frac{\lambda}{L} \qquad \frac{\Delta v}{v} = \frac{2Gm}{p v^2} \,\mathrm{tg}\, \frac{\theta}{2}$$

1) (A) Now 6-d Vlasov-Poisson, 1p $f(\vec{x},\vec{v},t)$

SAPIENZA
UNIVERSITÀ DI ROMA

sph. symm., „collisional" term

*-cl: $t_{dyn} \sim \frac{r}{\sigma}$; $t_{rx} \sim \frac{N}{\log\Lambda}\, t_{dyn}$; $t_{rx} \gg t_{dyn}$ (in general, not always)

$$\frac{\partial f}{\partial t} + \vec{v}\,\vec{\nabla} f + \underset{\uparrow \atop \text{Poisson}}{\dot{\vec{v}}}\,\vec{\nabla}_v f = \left(\frac{\delta f}{\delta t}\right)_{\substack{rx\\(coll)}} + (\ )_{bin} + (\ )_{diss}$$

Now: $\boxed{\text{(A) l.h.s Vlasov-Poisson}}$ (B) Coll. r.h.s. Rosenbluth, M, J, 57

Moment equations ↔ fluid dyn. model

follow Larson 70, LB + Eggleton 80, Louis + Sp 91

Notation: $\rho = \int_{-\infty}^{+\infty} f(\vec{v})\, d^3\vec{v} = 2\pi \int_0^{\infty}\int_{-1}^{+1} d\mu\, dV\, V^2 f(V,\mu)$

sph. pol. coord. in v-space

$\mu = \cos\theta$, $v_r = V\mu$, $v_t^2 = v_\theta^2 + v_\varphi^2 = V^2(1-\mu^2)$

Moments: $u\rho = \int \vec{v} f(\vec{v})\, d^3v$ bulk rad. motion

$p_r = \rho\sigma_r^2 = \int (v_r-u)^2 f\, d^3\vec{v}$ radial p, σ_r | $p = \frac{1}{3}(p_r + 2p_t)$

$p_t = \rho\sigma_t^2 = \int v_\theta^2 f\, d^3\vec{v} = \int v_\varphi^2 f\, d^3v$ tang. p, σ_t | $e = \frac{3}{2}p$

$F_r = 3p_r v_{er} = \int (v_r-u)^3 f\, d^3\vec{v}$

$F_t = 2p_t v_{et} = \int v_r v_\theta^2 f\, d^3\vec{v} = \int v_r v_\varphi^2 f\, d^3\vec{v}$

radial energy fluxes of rad/tan ~~en~~ thermal en.
net transport velocity of therm energy

$$F = \frac{1}{2}(F_r + F_t) = \frac{1}{2}(3p_r v_{er} + 2p_t v_{et}) \underset{\text{isotr.}}{=} \frac{5}{2} v_e p$$

$v_{er} = v_{et}$

(2)

(A)

SAPIENZA
UNIVERSITÀ DI ROMA

- Eqs. of continuity, lin. mom. cons. (Euler), energy (rad + tan)
(Note: r.h.s. = 0!)

Note Euler: $0 = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \mp \frac{\partial \phi}{\partial r} + \frac{1}{\rho}\frac{\partial p_r}{\partial r} + \frac{2 p_a}{\rho r}$

$\frac{p_a}{\rho} = (\sigma_r^2 - \sigma_t^2)$ anisotropy

- Before going to r.h.s.: heat-flux closure ~~[illegible]~~

Lynden-Bell + Eggleton '80:

$F = -\rho \frac{\lambda^2}{T} \frac{\partial \sigma^2}{\partial r}$; λ: MFP, T: collis. mean time; $\frac{k}{\mu m_H} \cdot T = \sigma^2$

Use: $\lambda^2 = \frac{1}{k_J^2} = \frac{\sigma^2}{4\pi G \rho}$; $T = T_{rx} \underset{here}{=} T_{Larson}$

Isotr.:

$F = \frac{5}{2} \rho \sigma^2 \cdot \underbrace{\frac{C}{4\pi G \rho T} \frac{\partial \sigma^2}{\partial r}}_{v_e} \propto \frac{\rho}{\sigma} \frac{\partial \sigma^2}{\partial r}$

- Generalization to axis. using viscos. $\sim \frac{1}{T}$? (Goodman)

- Kn number related

- external gravity need correction: (Amaro-Seoane, Freitag, Sp 2004)

$\lambda^2 \to \lambda^2 \cdot \frac{M_*(r)}{M_{bh}}$

(only this gets BW cusp $\rho \sim r^{-7/4}$)

③ Ⓑ

SAPIENZA
UNIVERSITÀ DI ROMA

r.h.s.

Rosenbl., McD., Judd '57, Rosenbluth pot.

$$\left(\frac{\delta f}{\delta t}\right)_{rx} = -4\pi G^2 m_b \ln\Lambda \left[\sum_{i=1}^{3} \frac{\partial}{\partial v_i}\left(f(\vec{v})\frac{\partial h}{\partial v_i}\right) + \frac{1}{2}\sum_{i,j=1}^{3} \frac{\partial^2}{\partial v_i \partial v_j} f(\vec{v}) \frac{\partial^2 g}{\partial v_i \partial v_j}\right]$$

$D(\Delta v_i)$/factor $\qquad D(\Delta v_i \Delta v_j)$/factor

$$h(\vec{v}) = (m + m_b) \int \frac{f_b(\vec{v}_b)}{|\vec{v} - \vec{v}_b|} d^3\vec{v}_b$$

$$g(\vec{v}) = m_b \int f_b(\vec{v}_b) |\vec{v} - \vec{v}_b| d^3\vec{v}_b$$

local at $\vec{r}$, „straight lines"

$$\left[\vec{\nabla}_v^2 h = -4\pi\left(1 + \frac{m}{m_b}\right) f_b(\vec{v})\,;\ \vec{\nabla}_v^4 g = -8\pi f_b(\vec{v})\right]$$

NB: RMJ: $d^3v \leftrightarrow 2\pi V dV d\mu$ (sph. coord. in vel. space)

Other possibility: orbit average, use E, J or E, J_z

l.h.s. $= 0$ along orbit;

r.h.s. $= \left(\frac{\delta f}{\delta t}\right)_{rx} = \cancel{\ldots G^2 \ldots \ln\Lambda} \left[A \ldots \langle\Delta E\rangle \langle\Delta J\rangle \langle\Delta J^2\rangle \langle\Delta E^2\rangle \langle\Delta E \Delta J\rangle \right.$

orbit averaged

$$d^3\vec{v} \leftrightarrow \pi \frac{dE}{v_r} J dJ\,;\ v_r = \sqrt{2\left(E - \phi + \frac{J^2}{\ldots}\right)}$$

$\mu = \cos\theta\,;\ v_r = V\mu\,;\ v_t^2 = v_\theta^2 + v_\varphi^2 = V^2(1-\mu^2)$

4. B

SAPIENZA
UNIVERSITÀ DI ROMA

- Ansatz following RMB:

$$f(v,\mu) = \sum_{\ell=0}^{\infty} A_\ell(v)\, P_\ell(\mu)$$ Legendre Polyn.

$$f_b(v,\mu) = \dots$$ $A_0(v) = MB$

$A_\ell(v)$ defines ℓ-th moments, eg

$$A_2(v) = \frac{\sigma_r^2-\sigma_t^2}{3\sigma^2}\,\frac{v^2}{\sigma^2}\,A_0(v)$$ $\sigma^2 = \frac{1}{3}(\sigma_r^2+2\sigma_t^2)$

Larson 1970: $f_b = A_0$, f_a up to $\ell=4$

Louis + Sp 91: $f_b = f$ up to $\ell=2$

Schneider + Sp 2011: $f_b = f$ up to arbitrary ℓ (for r.h.s.), $\ell=5$ l.h.s.

- 1st non-tr. result: (anisotropy w. core collapse)

$$\left(\frac{\delta p_a}{\delta t}\right)_{trx} = -\frac{9}{10}\,\frac{p_a}{T}$$ (Larson: $-\frac{6}{5}\frac{p_a}{T}$ not self-cons. $p_a/\rho = \sigma_r^2-\sigma_t^2$)

$$T = \frac{9}{16\sqrt{\pi}}\,\frac{\sigma^3}{G^2 m \rho \log\Lambda}$$ („Larson trx")

5 15

SAPIENZA
UNIVERSITÀ DI ROMA

- 2nd non-trivial result: (equipartition) w. mass segr. (Sp + Takahashi 95)

two comp. $m, f\,;\; m', f'$

$$\left(\frac{\partial p_r}{\partial t}\right)_{rx} = \frac{C_1 + C_2 + C_3 + C_4}{T}$$

$$C_1 = -\frac{3}{4}\rho' \left(\sigma^2 - \frac{m'}{m}\sigma'^2\right)$$

Spitzer Eq.

$$C_2 = \frac{1}{5}\rho' \sigma_a^2 \left(1 + \frac{3}{2}\frac{m'}{m}\right) + \frac{1}{5}\rho' \sigma_a'^2 \left(\frac{3}{2} + \frac{m'}{m}\right)$$

$$C_3 = -\frac{3}{10}\rho' \left(\sigma_a^2 + \sigma_a'^2\right) \frac{\left(1 + \frac{m'}{m}\right)}{\left(1 + \sigma^2/\sigma'^2\right)}$$

$$C_4 = -\frac{27}{140}\rho' \sigma_a^2 \sigma_a'^2 \left(\sigma^2 + \sigma'^2\right)^{-2} \times \left[\sigma^2\left(1 + \frac{16}{27}\frac{m'}{m}\right) - \frac{28}{27}\sigma'^2\left(1 + \frac{39}{28}\frac{m'}{m}\right)\right]$$

Interesting for SIDM?