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When do waves drive plasma flows?

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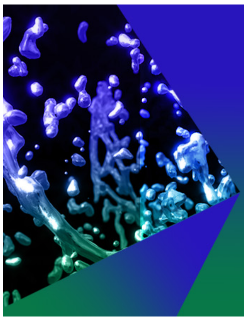

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ABSTRACT

Flows and rotation, particularly $\mathbf{E} \times \mathbf{B}$ rotation, are critical to improving plasma performance, and waves are a primary tool of plasma control. Thus, it is paramount to understand under what conditions waves can drive $\mathbf{E} \times \mathbf{B}$ flows in plasmas. In this didactic review, an invited paper accompanying the 2023 Marshall N. Rosenbluth Doctoral Thesis Award, this question is answered in the context of momentum-conserving quasilinear theory. There are two primary frameworks for momentum-conserving quasilinear theories that can handle both resonant and nonresonant particles: Eulerian averaging theories and oscillation-center Hamiltonian theories. There are also two different paradigmatic wave problems: plane-wave initial value problems, and steady-state boundary value problems. Here, it is shown that each of these frameworks "naturally" works better with a different problem type. By using these theories, one finds a great difference in the behavior of time- vs space-dependent waves. A time-evolving plane wave can only drive flow if the electromagnetic momentum of the wave, given by the Poynting flux, changes. This result precludes flow drive by any planar electrostatic wave. In contrast, a steady-state spatially evolving wave can drive flow whenever there is divergence in the flux of Minkowski momentum, a completely different physical quantity. This review aims to provide a high-level, intuitive understanding of the very different behaviors observed for these two types of problem.

I. INTRODUCTION AND GOAL OF THIS REVIEW

The present review is an invited paper corresponding to the Marshall N. Rosenbluth Doctoral Thesis Prize lecture at the 2023 American Physical Society Division of Plasma Physics meeting. The dissertation in question[1] covered many topics, including wave coupling[2,3] and poloidal rotation[4] in toroidal confinement systems; mass separation in linear plasma devices;[5] particle transport,[6–8] current channel evolution,[9,10] and rotation[11] in Z pinches; and magnetic field generation mechanisms in astrophysics.[12,13] However, the unifying topic underlying these seemingly disparate areas was the deep connection in magnetized plasma physics between charge and momentum—namely, that charge cannot be pushed across magnetic field lines without momentum input. This connection, which I first was introduced to through my study of collisional classical transport,[7,8,14–16] gnawed at me, since it seemed to present a conundrum for wave-driven transport processes driven by electrostatic waves. After all, purely electrostatic waves, which contain no electromagnetic momentum, would seem to be precluded from extracting charge from the plasma and driving $\mathbf{E} \times \mathbf{B}$ flow, in direct contrast with established claims in the literature.[17–19] In addition to its academic interest, $\mathbf{E} \times \mathbf{B}$ flow drive underlies shear stabilization,[20–27] turbulence suppression,[28–32] centrifugal confinement,[33–35] and plasma mass separation schemes.[6,36–44] Thus, creating a self-consistent quasilinear theory to elucidate the conditions under which of wave-driven flow drive is possible became the core motivation of the dissertation.

The requisite self-consistent quasilinear theory was developed in several steps of increasing generality. The first effort[45] focused solely on electrostatic plane waves that grow or damp in time, building off the textbook[46–50] Eulerian quasilinear theory of the bump-on-tail instability. In this theory, electromagnetic forces are straightforwardly averaged over plasma volumes, and momentum conservation can be rigorously established. Already, this theory showed that something was critically missing from the earlier treatments that focused solely on the resonant particles, as it was shown that the growing wave drove charge transport in the nonresonant particles that completely canceled any charge transport from the resonant particles. This result seemingly contradicted earlier claims of charge transport and $\mathbf{E} \times \mathbf{B}$ flow in the literature.[17–19] Thus, flow drive by an electrostatic plane wave was forbidden, consistent with the intuition that the electrostatic wave had no momentum to give to the plasma.

This analysis was subsequently extended to waves with spatial structure.[51] Here, as in the theories of electrostatic and ion-cyclotron-frequency turbulence[52–60] and nonresonant current drive,[61,62] the Reynolds stress was shown to play a critical role. Indeed, for lower hybrid waves, for a steady-state wave driven from the boundary, the Reynolds stress was shown to serendipitously cancel the force on nonresonant particles, leaving only the force on resonant particles, and thus allowing the waves to extract charge and drive $\mathbf{E}\times\mathbf{B}$ rotation. Because of the importance of momentum conservation, great care was taken to show how both local and global momentum conservation worked out for this case.[51]

Finally, the theory was extended to electromagnetic waves.[63] This allowed the application to a much broader variety of plasma waves, including waves with electromagnetic momentum, as well as more clearly revealing the connection to earlier ponderomotive theories[64] and the Minkowski momentum of geometrical optics.[65]

A fairly comprehensive and didactic review of how flows can be driven by electrostatic waves has been given in Ref. 66. That review basically followed the historical development of the dissertation, with a focus on the Eulerian quasilinear theories. In these theories, the vanishing of the force on nonresonant particles appeared as a rather fortuitous cancelation between the Reynolds stress and the Lorentz force. Only at the end was the deeper reason for this cancelation—arising from oscillation-center Hamiltonian theory—briefly explored.

In the present review, I attempt a different approach, treating the oscillation-center theories and Eulerian theories on a more equal footing. My goal in doing so is to demonstrate the strength and weakness of each type of theory in addressing the different paradigmatic type of problems that appear in quasilinear theory: namely, steady-state boundary value problems, and plane-wave initial value problems. In this way, I hope to establish an intuition for what is happening in each type of problem, especially with regard to demystifying the flow of momentum between wave and plasma. In contrast to the other papers I have written on this topic, this one will be somewhat simpler and less formal, and will hopefully allow a reader somewhat new to quasilinear theory to be introduced to the fundamental theoretical tools and conservation principles, with plenty of signposts as to where they can learn more.

II. A SIMPLE QUESTION: CAN A WAVE MOVE CHARGE ACROSS MAGNETIC FIELD LINES?

Consider a uniform slab of plasma, infinite along z and periodic along y, with a magnetic field $\mathbf{B} \parallel \hat{z}$. Charged particles in this system will follow helical paths, streaming along $\hat{z}$ and Larmor-rotating in the x–y plane. Now let there be a wave with a wavevector $\mathbf{k} \parallel \hat{y}$, with constant amplitude in y and z, which damps in the plasma (Fig. 1). The fundamental question to be answered is whether net charge can be moved along $\hat{x}$ during this interaction. If the wave can move net charge, then it can generate an electric field $\mathbf{E} \parallel \hat{x}$, producing $\mathbf{E}\times\mathbf{B}$ flow in the $\hat{y}$ direction (Fig. 2). Conversely, if the wave cannot move net charge, then no flow can be driven.

If one looks only at particles which interact resonantly with the wave, there is reason to expect charge transport. For instance, an electrostatic wave (with oscillating wave field $\tilde{\mathbf{E}} \parallel \mathbf{k}$) is Landau resonant with a particle twice during its orbit if the particle perpendicular velocity $v_\perp$ exceeds the wave phase velocity v_p. During the resonant interaction, energy and momentum are exchanged with the wave.[67,68] When the particle absorbs momentum, its gyrocenter shifts in the $\mathbf{k}\times\mathbf{B}$ direction, whereas when it loses momentum, it shifts in the opposite direction (Fig. 3). For a strong enough wave[67] or in the presence of sufficient collisional scattering,[69] the interaction phase each time the resonance is encountered is random, and repeated interactions with the wave lead to a coupled diffusion in gyrocenter and energy space. If there is a source of particles at the plasma center and a sink at the plasma edge, as for fusion-born alpha particles, then this diffusion draws resonant particles from the source to the sink, moving the resonant particles across field lines—the basis for a wave amplification process known as alpha channeling.[70–76] If one assumes that these resonant particles carry out their associated charge, then this forms the basis of a highly efficient rotation drive scheme that can transfer fusion ash energy directly into rotation energy.[17–19]

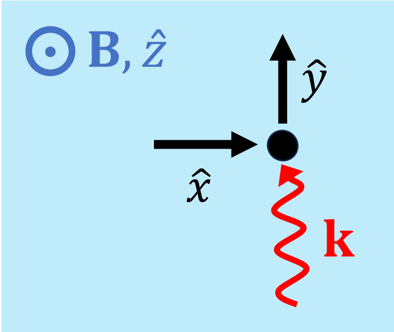

FIG. 1. The core problem considered in the paper: can a wave with $\mathbf{k} \parallel \hat{y}$ damping in a plasma with magnetic field $\mathbf{B} \parallel \hat{z}$ drive net charge along $\hat{x}$.

However, this rotation drive hypothesis relies on an important assumption: that one can ignore the response of the nonresonant (bulk) plasma; and it is by no means clear that this assumption is justified. While the nonresonant response is generally weaker than the resonant response for any given particle, it is also true that there are far more resonant than nonresonant particles, so the weak response can have a large effect on the total charge transport. Furthermore, two more facts provide cause for concern. First, it is known from textbook quasilinear problems such as the bump-on-tail instability that the nonresonant distribution is important in enforcing momentum and energy conservation. Second, it is known from textbook transport theory that

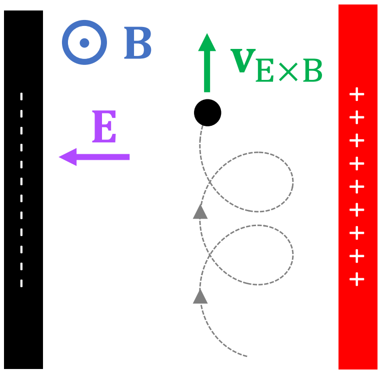

FIG. 2. Charge transport in $\hat{x}$ can produce an electric field $\mathbf{E} \parallel \hat{x}$, producing $\mathbf{E}\times\mathbf{B}$ transport along $\hat{y}$.

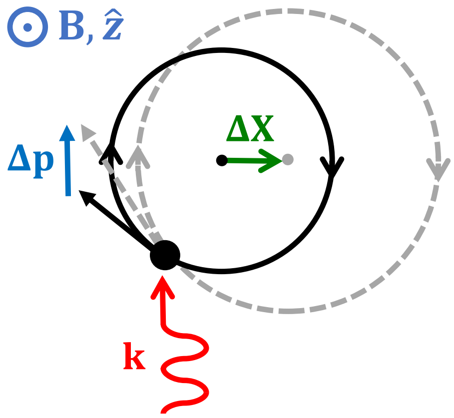

FIG. 3. A particle undergoing Larmor rotation is Landau-resonant with a perpendicularly-propagating electrostatic wave twice per orbit if the particle perpendicular velocity exceeds the wave phase velocity. This resonance allows the particle to exchange energy and momentum with the wave, leading to wave damping or amplification, and to particle diffusion in both gyrocenter and energy.

there is a deep link between momentum conservation and charge transport. These facts imply that self-consistent treatment of the nonresonant particles is particularly important for the problem at hand.

In order to clearly pose and solve the problem, we will begin in Sec. III with a review of the relationship between charge transport and momentum conservation in collisional transport theory, which exposes an important area of concern for rotation drive via wave damping. This example leads us in Sec. IV to think more clearly about what is meant by the wave momentum, as two competing wave momenta, the Minkowski momentum and electromagnetic momentum, make frequent appearances in the literature. Then, in Sec. V, we clarify what we mean by wave damping and amplification, distinguishing between the initial value problems in which waves evolve in time versus the boundary value problems in which waves evolve in space. Having properly posed the problem, in Sec. VI, we review the theoretical frameworks that allow us to tackle both the resonant and nonresonant particles self-consistently: the Eulerian averaging theory, and the oscillation-center Hamiltonian theory. In Sec. VII, we apply these theories to the problem of flow drive, showing that each type of theory has a natural problem type to which it is more suited. In this way, we show that boundary value and initial value problems behave very differently, with rotation drive determined by the divergence of the wave Minkowski momentum flux in the former, and the by the time change of the electromagnetic field momentum in the latter.

III. CHARGE TRANSPORT AND MOMENTUM CONSERVATION

A similar diffusion process to that in Fig. 3 occurs in the classical transport of particles due to collisions across a magnetic field. In this process, in the place of the resonant wave interaction, particles undergo collisions with other particles, which cause a displacement in the gyrocenter. This leads to a diffusion process with a characteristic timescale of the collision frequency, and a characteristic length scale of the Larmor radius.

Now, looking at any individual particle, one might assume that the charge in the system also diffused on the same time and length scales. However, this turns out to be definitively wrong. In a homogeneous plasma with a constant electric field, it turns out that every time a particle collides, leading its gyrocenter to move, the particle with which it collided experiences a corresponding and canceling change in gyrocenter that ensures that there is no net displacement of the center of charge (Fig. 4). This fundamental cancelation follows immediately if one considers the canonical momentum conservation of the plasma particles in the $\mathbf{E} \times \mathbf{B}$-drifting frame. For a particle s, the canonical momentum in a uniform magnetic field $\mathbf{B} \parallel \hat{z}$ is given by

$$p_{s,y} = mv_{s,y} + \frac{q}{c}A_y(x) = mv_y + \frac{q}{c}Bx_s. \tag{1}$$

The gyrocenter position X_s comes from averaging the above equation so that v_y drops out

$$p_{s,y} = \frac{q}{c}B \underbrace{\langle x_s \rangle}_{X_s}. \tag{2}$$

During a collision, momentum is conserved, so we have immediately

$$\sum_s q_s \Delta X_s = \frac{c}{B}\sum_s \Delta p_{s,y} = 0. \tag{3}$$

The quantity on the left-hand side of the above equation is nothing more than the charge transport, so we see that no net charge moves. Thus, classical transport is often said to be "ambipolar."

There are many interesting effects, which come from the plasma's attempt to increase entropy[77] while respecting the ambipolarity of classical transport, including nonintuitive separation[7,8,14–16] and cooling[78] effects. It can also be interesting to see the conditions under which the ambipolarity constraint is broken; for instance, in a plasma with shear in the electric field, allowing for viscosity.[1,15,66]

However, the main point of this result for the present purpose is to see what light it sheds on the problem of wave-driven rotation. In simple terms, Eq. (3) says that in order to transport charge and drive $\mathbf{E} \times \mathbf{B}$ rotation, one must provide momentum that is not already in the plasma particles themselves. Or, in even simpler terms, one must push on the plasma to make it move.

This result presents a fundamental problem for the rotation drive process from Sec. II. Recall that the wave in Fig. 3 was taken to be a plane electrostatic wave, and thus has no Poynting flux, and no electromagnetic momentum. Thus, Eq. (3) would seem to imply that no net charge transport can be driven by the wave.

IV. WAVE MOMENTA: MINKOWSKI MOMENTUM vs FIELD MOMENTUM

Since wave momentum has proven to be so important to the problem of charge transport, it is good at this point to discuss the

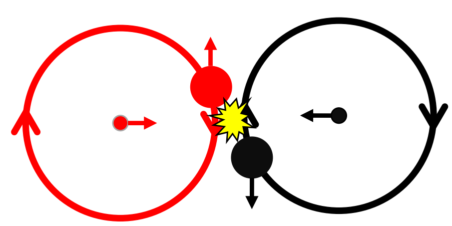

FIG. 4. When two particles collide in a plasma with uniform $\mathbf{E} \times \mathbf{B}$ flow, their Larmor radii shift in such a way that there is no net motion of the center of charge.

multiple definitions of wave momentum that exist in the literature. It turns out that there are two primary types of momentum that are useful to consider when thinking about the wave.

First, and most familiar, is the electromagnetic momentum, which is given by the time-averaged Poynting flux:

$$\mathbf{p}_{\mathrm{EM}} \equiv \frac{\langle \mathbf{E} \times \mathbf{B} \rangle}{4\pi c}. \quad (4)$$

For a wave which varies as $\mathbf{E} = \mathrm{Re}(\tilde{\mathbf{E}} e^{i\mathbf{k}\cdot x - i\omega t})$ and $\mathbf{B} = \mathrm{Re}(\tilde{\mathbf{B}} e^{i\mathbf{k}\cdot x - i\omega t})$, with ω and $\mathbf{k}$ real, this becomes

$$\mathbf{p}_{\mathrm{EM}} = \frac{\mathrm{Re}(\tilde{\mathbf{E}} \times \tilde{\mathbf{B}}^*)}{8\pi c}. \quad (5)$$

Thus, without an oscillating magnetic field component, the wave will have no electromagnetic momentum.

It is worth briefly noting that electrostatic modes with spatial structure generally have a small magnetic field component[79] and are thus often referred to as "quasi-electrostatic." While this small magnetic component is important for the energy flow, it contributes negligibly to the plasma momentum as long as the phase velocity is well below the speed of light,[1,51] and thus can be ignored in the momentum balance for such modes.

The second type of momentum is variously called the "Minkowski momentum," "plasmon momentum," or "geometrical optics" momentum. This momentum can be defined for waves in a slowly varying medium, i.e., where the length and time scales of the variation of the medium are much longer than the wavelength and wave period. It also generally requires that the waves be in the weak-turbulence limit, where wave amplitudes are small enough that quasilinear effects and wave–wave interactions are significantly ordered down from the linear interactions. For a plasma with dielectric tensor $\boldsymbol{\epsilon}$, the Minkowski momentum is given by[65,79]

$$\mathbf{p}_{\mathrm{M}} \equiv \frac{\mathbf{k}}{16\pi\omega}\left[\tilde{\mathbf{E}}^* \cdot \frac{\partial(\omega\boldsymbol{\epsilon})}{\partial\omega} \cdot \tilde{\mathbf{E}} + \tilde{\mathbf{B}}^* \cdot \tilde{\mathbf{B}}\right]. \quad (6)$$

This is the momentum most closely related to the concept of photon momentum in quantum mechanics: for example, it is related to the Minkowski energy W_{M} by $\mathbf{p}_{\mathrm{M}}/W_{\mathrm{M}} = \mathbf{k}/\omega$.

A. Closed systems

To make use of the two momenta above, it is important to understand their roles in the plasma. Consider an infinite homogeneous plasma, with an electromagnetic wave present. The momentum in this system can be partitioned in various ways. In particular, we can identify three important "boxes" of momentum, which together form the whole (Fig. 5). First, there is the momentum $\mathbf{p}_{\mathrm{EM}}$ in the electromagnetic field, given (on average) by Eq. (5). Second, there is the momentum $\mathbf{p}_{\mathrm{NP}}$ of the bulk plasma particles, nonresonant with the wave. Third, there is the momentum $\mathbf{p}_{\mathrm{RP}}$ of the particles that are resonant with the wave. The latter two can be thought of either as separate populations, or as a single total particle population with momentum $\mathbf{p}_{\mathrm{P}}$.

Clearly, in an infinite, homogeneous plasma, the momentum of the electromagnetic field and the particles forms a closed system, in the sense that

$$\frac{d}{dt}(\mathbf{p}_{\mathrm{EM}} + \mathbf{p}_{\mathrm{P}}) = 0. \quad (7)$$

However, it turns out that there is also a second closed system, which is formed by the Minkowski momentum and the resonant particles:

$$\frac{d}{dt}(\mathbf{p}_{\mathrm{M}} + \mathbf{p}_{\mathrm{RP}}) = 0. \quad (8)$$

The existence of both of these closed systems shows that the Minkowski momentum contains information both about the momentum in the electromagnetic fields, and about the nonresonant particles.

B. Example: Bump-on-tail instability

To get a better sense of the role of these momenta, it is helpful to see them in action in a concrete problem. Thus, consider the bump-on-tail instability, the classic paradigm problem of quasilinear analysis.[46–50] This instability can occur for high-frequency electrostatic waves in an homogeneous plasma. In such a plasma, the ions can be treated as a neutralizing background, and only the electron dynamics need be considered.

Electrons near the wave phase velocity interact resonantly with the wave, receiving a random kick that leads to diffusion in velocity space, flattening out the distribution near resonance (Fig. 6, top). The instability occurs when, near the phase velocity of the wave, there is a population inversion $df_e/d|v| > 0$ in the electron distribution function (Fig. 6, bottom). In this case, electrons lose energy due to the diffusion, and thus the wave gains energy; this is the bump-on-tail instability.

How do the various momenta change during this process of diffusive flattening? From the diffusion, the resonant particles experience a change in momentum $\Delta\mathbf{p}_{\mathrm{RP}} < 0$. Because the resonant particle momentum and Minkowski momentum form a closed system [Eq. (8)], the wave thus gains Minkowski momentum $\Delta\mathbf{p}_{\mathrm{M}} = -\Delta\mathbf{p}_{\mathrm{RP}} > 0$.

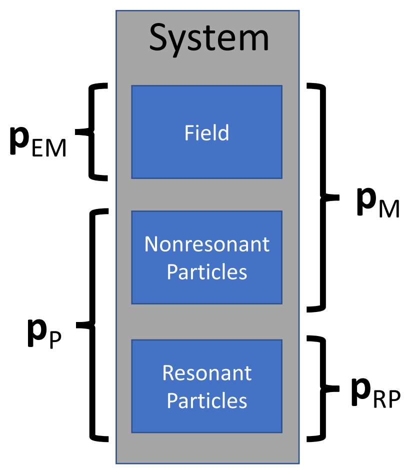

FIG. 5. The total momentum in an infinite plasma-wave system can be partitioned in several ways. Here, separate partitions are drawn for the electromagnetic momentum $\mathbf{p}_{\mathrm{EM}}$, nonresonant particle momentum $\mathbf{p}_{\mathrm{NP}}$, and resonant particle momentum $\mathbf{p}_{\mathrm{RP}}$. The momenta in Sec. IV form parts of two different closed systems; one given by the electromagnetic momentum $\mathbf{p}_{\mathrm{EM}}$ and the total particle momentum $\mathbf{p}_{\mathrm{P}} \equiv \mathbf{p}_{\mathrm{NP}} + \mathbf{p}_{\mathrm{RP}}$, and the second given by the Minkowski momentum $\mathbf{p}_{\mathrm{M}}$ and the resonant particle momentum $\mathbf{p}_{\mathrm{RP}}$.

However, the wave is electrostatic, which means that $\mathbf{p}_{\mathrm{EM}} = 0$ regardless of the wave amplitude, so $\Delta\mathbf{p}_{\mathrm{EM}} = 0$. Because the electromagnetic momentum and total particle momentum form a closed system [Eq. (7)], $\Delta\mathbf{p}_{\mathrm{P}} = -\Delta\mathbf{p}_{\mathrm{EM}} = 0$. Since the total particle momentum consists of the nonresonant and resonant particle momentum (Fig. 5), it is clear that $\Delta\mathbf{p}_{\mathrm{NP}} = -\Delta\mathbf{p}_{\mathrm{RP}}$, i.e., the momentum of the nonresonant distribution shifts equal and opposite to the shift in the resonant momentum.[48,49] This shift in the distribution, which can be thought of as a momentum-conserving "ponderomotive recoil,"[63] shows that the nonresonant particles perform a critical role in enforcing momentum conservation during the wave–plasma interaction. Thus, they cannot be ignored in evaluating wave-based $\mathbf{E} \times \mathbf{B}$ flow drive mechanisms.

It is worth briefly noting that some subtleties have been glossed over in the above discussion of the bump-on-tail instability. Namely, in contrast to the diffusion perpendicular to the magnetic field from Fig. 3, for diffusion in the absence of a magnetic field the formal ordering implies that the diffusion occurs in an infinitely narrow region around the Landau resonance. For the diffusion region to have a finite width, the wave must have finite amplitude, at which point the diffusion region width is set by the electric potential change between wave peaks. In the case of multiple waves, broader diffusion across multiple resonances occurs when the finite diffusion width causes the resonances to overlap, a condition known as the Chirikov criterion.[80] In such a case, the total Minkowski momentum summed over all waves forms a conserved system with the resonant particles. In weakly collisional plasmas, the collisions can further broaden the resonance.[92,93]

Single-wave Landau-resonant diffusion perpendicular to the magnetic field, as in Fig. 3, actually has a much broader diffusion region than that parallel to the magnetic field, since particles with a broad spectrum of $v_\perp > \omega/k$ can interact with the wave. However, it is necessary for the particle to dephase from the gyroresonance through either collisions[69] or nonlinearity induced by finite amplitude.[67]

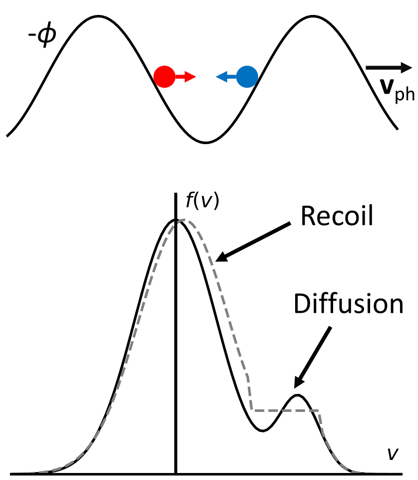

FIG. 6. Schematic description of the bump-on-tail instability. Top: an electrostatic wave with potential ϕ interacts strongly with electrons traveling near the phase velocity $\mathbf{v}_{\mathrm{ph}}$, giving them a random kick, which heats some particles and cools others. This leads to a quasilinear diffusion in velocity space. Bottom: if there are more particles at high velocity than low velocity, the wave will grow, as momentum flows from the resonant particle into the wave Minkowski momentum $\mathbf{p}_{\mathrm{M}}$ according to Eq. (8). However, because the wave is electrostatic, it has no electromagnetic momentum $\mathbf{p}_{\mathrm{EM}}$, and so, the total particle population $\mathbf{p}_{\mathrm{P}}$ loses no momentum according to Eq. (7). This momentum conservation is enforced by a shift in the nonresonant particles, called the ponderomotive recoil.

C. Momentum conservation in systems with spatial structure

Up until now, everything has been done in infinite, homogeneous plasmas. However, the typical plasmas of interest in fusion are not infinite, but contained in a laboratory, and thus, they contain spatial gradients. These gradients can alter the form of the momentum balance, as they allow fluxes of momentum from outside of the plasma into it.

The general form of momentum conservation for a closed system, allowing for spatial fluxes of momentum, is

$$\sum_{\text{subsystems}} \left[\frac{\partial \mathbf{p}}{\partial t} + \nabla \cdot \mathbf{\Pi}\right] = 0. \tag{9}$$

Here, $\mathbf{\Pi}$ is known as the stress, with Π^{ij} referring to the flux of j-directed momentum in the i direction.

In the presence of spatial gradients, the electromagnetic and particle subsystems still combine to form a closed system. For the electromagnetic field, the relevant momentum flux is given by the (negative) Maxwell stress tensor

$$\Pi^{ij}_{\mathrm{EM}} = \frac{1}{4\pi}\left\langle \frac{\delta^{ij}}{2}E^2 - E^iE^j + \frac{\delta^{ij}}{2}B^2 - B^iB^j \right\rangle, \tag{10}$$

while for the particles, the stress is given (nonrelativistically) by

$$\Pi^{ij}_{\mathrm{P}} = n_{s0}\int m_s f_s \mathbf{v}\mathbf{v}d\mathbf{v}, \tag{11}$$

where f_s is the particle distribution function in velocity space, normalized to 1, and n_{s0} is the particle density.

Things are a bit more complicated for the Minkowski and resonant particle subsystems. In fact, these no longer form a fully closed subsystem. However, for the components of the momentum oriented along a symmetry direction (such as y or z in the fundamental problem of this paper), the "momentum along the symmetry direction" is still closed between the resonant particle momentum and the Minkowski momentum; i.e., Eq. (9) holds for the y- and z-directed momenta when summed over the subsystems RP and M.[51] The off-diagonal elements of the Minkowski stress, which enter this equation, are given by

$$\Pi^{ij}_{\mathrm{M}} = v^i_g p^j, \tag{12}$$

where $v^i_g = \partial\omega/\partial\mathbf{k}$ is the group velocity of the wave.

V. PROBLEM TYPES: EVOLUTION IN SPACE vs TIME

It is clear that the temporal and spatial structure of the wave make a difference in the momentum conservation relations. Thus, it is important to clarify the time and space structure of waves.

Generally, the geometric optics limit is assumed, where the wave amplitude and dispersion properties vary on length and times scales much longer than the wavelength and wave period. For now, take the dispersion properties to be constant in space. Let ρ_s and $\mathbf{j}_s$ be the charge and current densities of species s. Then, for $A \in \{\rho, \mathbf{j}, \mathbf{E}, \mathbf{B}\}$, the wave takes the form:

$$A = \mathrm{Re}\left[\tilde{A}_0 e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}\right], \tag{13}$$

with (in general,) complex values of $\mathbf{k}$ and ω.

We can separate out the complex behavior by writing:

$$\omega = \omega_r + i\omega_i; \quad \mathbf{k} = \mathbf{k}_r + i\boldsymbol{\kappa}. \tag{14}$$

The local wave amplitude is then given by

$$\tilde{A} = \tilde{A}_0 e^{-\boldsymbol{\kappa}\cdot\mathbf{x}+\omega_i t}. \tag{15}$$

Here, $\boldsymbol{\kappa}$, the imaginary part of $\mathbf{k}$, encodes the spatial variation of the wave amplitude, and ω_i, the imaginary part of ω, encodes the temporal variation. For the wave to be "quasi-monochromatic" and the geometric approximation to hold, one must have $|\omega_r| \ll |\omega_r|$ and $|\boldsymbol{\kappa}| \ll |\mathbf{k}_r|$.

In general, it is possible (and common!) for both ω_i and $\boldsymbol{\kappa}$ to be nonzero, so that the wave amplitude evolves in both space and time. However, consideration of two limiting cases can afford insight into a wide variety of problems.

First, in the plane-wave initial value problem (IVP), $\boldsymbol{\kappa}$ is assumed to be 0, so that $\tilde{A}$ is independent of $\mathbf{x}$ [Fig. 7(a)]. This is common in the study of phase-space instabilities in both astrophysical and fusion plasmas, such as the bump-on-tail instability in Sec. IV B, where the local instability growth is more important than the propagation of waves outward from the instability.

Second, in the steady-state boundary value problem (BVP), ω_i is taken to be 0, so that $\tilde{A}$ is independent of t [Fig. 7(b)]. This is common in radio-frequency (RF) plasma control problems, such as for current drive and heating, where a wave is driven from the plasma edge, and the steady-state damping profile is of more interest than the transient behavior as the wave antenna starts up.

It is worth noting how these momentum considerations relate to current drive. Like rotation drive, current drive relies on setting up flows, now parallel to the magnetic field, in the plasma. However, because an electron has comparable charge but negligible mass compared to an ion, the net current is not constrained in the same way as $\mathbf{E}\times\mathbf{B}$ drive, since any flow driven in the ions can easily offset the change in electron momentum without substantially affecting the current. Thus, current can usually be driven by directly driving flow in the resonant electrons.[81] However, in the leading and most efficient current drive techniques, which rely on wave damping on superthermal electrons, such as lower hybrid current drive[82] or electron cyclotron current drive,[83] it turns out that the momentum absorption is not responsible for the majority of the driven current. Instead, the current drive arises mainly from the asymmetric perpendicular heating, which renders resonant electrons less collisional than nonresonant electrons. Thus, for these current drive techniques, any momentum-conserving role played by thermal, nonresonant electrons, would not substantially affect the net current—and indeed, both forms of current drive have enjoyed extensive experimental confirmation.[84,85]

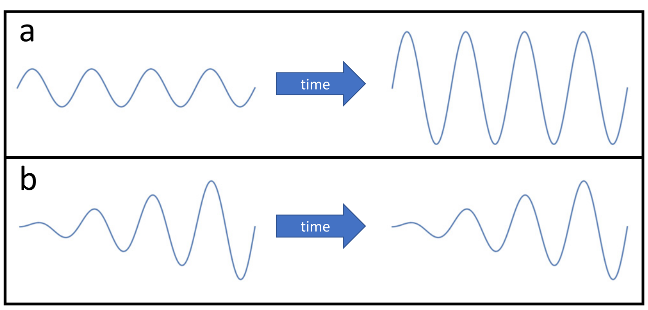

FIG. 7. Although in general wave structure can evolve in both space and time, it is useful to consider two limiting cases. (a) In a plane-wave initial value problem, a plane wave grows in time in an infinite, homogeneous plasma. (b) In a steady-state boundary value problem, a wave comes in from the plasma boundary, damping its energy and momentum in space, with no evolution of the wave envelope in time (though the wave phase oscillates).

Returning to the $\mathbf{E}\times\mathbf{B}$ drive problem, and following this discussion of wave structure, it is clear that in Fig. 1, it was incomplete to say, that a wave "damps in the plasma," since this did not fully specify the problem. In general, the behavior can be different depending on whether a local plane-wave damps or amplifies in time, or a wave propagates in from the boundary, with its amplitude damping or amplifying in space. Both types of problems will be treated in Secs. VI and VII.

VI. THEORETICAL FRAMEWORKS

There are two generally self-consistent momentum-conserving theoretical frameworks with which one can tackle the wave–particle interaction problem. These are the local Eulerian averaging approach, and the oscillation-center Hamiltonian approach. It is worthwhile to review each, since they turn out to be well-adapted to different types of problems.

A. Local Eulerian averaging

The local Eulerian averaging theory is very straightforward; one simply takes a box of plasma, and averages the force over a wave period (either in space or time). This type of theory is the basis for textbook[48,49] quasilinear analysis of the bump-on-tail instability and Landau damping.

The Eulerian theory can be sketched out fairly quickly, but for more details refer to Refs. 45, 66 (for the electrostatic theory), and 63 (for the electromagnetic theory). Recalling from Eq. (15) the definition of the local amplitude $\tilde{A}$ for $A \in \{\rho_s, \mathbf{j}_s, \mathbf{E}, \mathbf{B}\}$, averaging the Lorentz force over a wave cycle yields

$$\langle \mathbf{F}_s \rangle = \left\langle \rho_s \mathbf{E} + \frac{1}{c}\mathbf{j}_s \times \mathbf{B} \right\rangle \tag{16}$$

$$= \frac{1}{2}\mathrm{Re}\left[\tilde{\rho}_s \tilde{\mathbf{E}}^* + \frac{1}{c}\tilde{\mathbf{j}}_s \times \tilde{\mathbf{B}}^*\right], \tag{17}$$

where the * operator denotes the complex conjugate. Both ρ_s and $\tilde{\mathbf{j}}_s$ can be expressed in terms of the susceptibility tensor $\boldsymbol{\chi}_s$ for species s (i.e., the species contribution to the dielectric tensor $\boldsymbol{\epsilon}$):

$$\tilde{\mathbf{j}}_s = -\frac{i}{4\pi}\omega \boldsymbol{\chi}_s \cdot \tilde{\mathbf{E}}, \tag{18}$$

$$\tilde{\rho}_s = \frac{\mathbf{k}\cdot\tilde{\mathbf{j}}_s}{\omega} = -\frac{i}{4\pi}\mathbf{k}\cdot\boldsymbol{\chi}_s\cdot\tilde{\mathbf{E}}. \tag{19}$$

Now, to get the quasilinear force, one Taylor expands in small $|\boldsymbol{\kappa}|/|\mathbf{k}_r|$, ω_i/ω_r, and $|\boldsymbol{\chi}_s^A|/|\boldsymbol{\chi}_s^H|$, where the superscripts A and H refer to the anti-Hermitian and Hermitian parts of the susceptibility, respectively. After some algebra, one arrives at the form of the force[63]

$$\langle \mathbf{F}_s \rangle = -\frac{\partial}{\partial \mathbf{x}}\cdot \boldsymbol{\Pi}_{Ns} + \frac{\partial}{\partial t}\mathbf{p}_{Ns} + \mathbf{F}_{Rs}, \tag{20}$$

where

$$\mathbf{\Pi}_{Ns} = \frac{1}{8\pi}\mathrm{Re}\left[\left(\boldsymbol{\chi}_s^H\cdot\mathbf{E}\right)\mathbf{E}^* - \frac{1}{2}\mathbf{I}\left(\mathbf{E}^*\cdot\boldsymbol{\chi}_s^H\cdot\mathbf{E}\right) + \frac{1}{2}\left(\frac{\partial}{\partial\mathbf{k}_r}\left(\mathbf{E}^*\cdot\boldsymbol{\chi}_s^H\cdot\mathbf{E}\right)\right)\mathbf{k}_r\right], \tag{21}$$

$$\mathbf{p}_{Ns} = \frac{1}{8\pi}\mathrm{Re}\left[\left(\boldsymbol{\chi}_s^H\cdot\mathbf{E}\right)\times\left(\frac{\mathbf{k}_r}{\omega_r}\times\mathbf{E}^*\right) + \frac{\mathbf{k}_r}{2}\frac{\partial}{\partial\omega_r}\left(\mathbf{E}^*\cdot\boldsymbol{\chi}_s^H\cdot\mathbf{E}\right)\right], \tag{22}$$

$$\mathbf{F}_{Rs} = \frac{1}{8\pi}\mathbf{k}_r\left(\mathbf{E}^*\cdot\boldsymbol{\chi}_s^A\cdot\mathbf{E}\right). \tag{23}$$

Here, the tildes on wave quantities have been dropped for clarity. The three contributions to the force can be interpreted, respectively, as (a) a stress from spatial gradients of the wave amplitude, (b) a momentum associated with putting a plane wave in the plasma, and (c) a force on the resonant particles. These forces can be shown to respect momentum conservation with the electromagnetic field, in the sense that Eq. (9) is satisfied when summed over all species as well as the electromagnetic momentum and momentum flux.[63] It should be noted that Eq. (20) is the electromagnetic generalization of Eq. (122) in Ref. 51.

B. Oscillation centers

The oscillation-center (OC) theories, originally conceived by Dewar,[86] start quite a bit differently and fundamentally require a kinetic treatment. One starts by considering the single-particle Hamiltonian of a particle in a wave field

$$H = \frac{1}{2m_s}\left(\mathbf{p} - \frac{q_s}{c}\mathbf{A}(\mathbf{x},t)\right)^2 + q_s\phi(\mathbf{x},t). \tag{24}$$

The evolution of a particle distribution under this Hamiltonian is determined by the Liouville equation

$$\frac{\partial f}{\partial t} + \frac{\partial H}{\partial\mathbf{p}}\cdot\frac{\partial f}{\partial\mathbf{x}} - \frac{\partial H}{\partial\mathbf{x}}\cdot\frac{\partial f}{\partial\mathbf{p}} = 0. \tag{25}$$

Now, in deriving a quasilinear theory from a Hamiltonian, one generally starts by assuming that the perturbing oscillating wave field is small compared to the zeroth-order Hamiltonian. One then splits up the distribution function into $f = f_0 + f_1$, with $\langle f_1\rangle = 0$. Fourier analyzing then yields $\tilde{f}_1$ as a function of the oscillating field quantities. Once f_1 is known, one can average Eq. (25) and substitute in for $\tilde{f}_1$ to obtain the quasilinear diffusion equation.

Now, in the original coordinates $(\mathbf{x},\ \mathbf{p})$, the quasilinear diffusion operator thus obtained contains forces on both the resonant and nonresonant particles. Indeed, integrating this diffusion equation over velocity recovers the Eulerian volume force from Sec. VI A. The idea behind oscillation-center theories is to find a near-identity transform to a different set of canonical momenta $\boldsymbol{\mathcal{X}}$ and $\boldsymbol{\mathcal{P}}$ where the quasilinear diffusion operator only impacts the resonant particles. This transformation is accomplished via a generating function $F_2(\mathbf{x},\boldsymbol{\mathcal{P}},t) = \mathbf{x}\cdot\boldsymbol{\mathcal{P}} + S(\mathbf{x},\boldsymbol{\mathcal{P}},t)$, which changes the coordinates and Hamiltonian according to

$$\boldsymbol{\mathcal{X}} = \mathbf{x} + \frac{\partial S}{\partial\boldsymbol{\mathcal{P}}}, \tag{26}$$

$$\boldsymbol{\mathcal{P}} = \mathbf{p} + \frac{\partial S}{\partial\mathbf{x}}, \tag{27}$$

$$\mathcal{H} = H + \frac{\partial S}{\partial t}. \tag{28}$$

These equations are solved in orders by (a) demanding that $\langle dS/dt\rangle = 0$ along a particle trajectory at each order [by combining Eqs. (26)–(28) with Eq. (25)], (b) appropriately averaging over the nonresonant particles while "pulling out" the resonances, and (c) combining with Maxwell's equations to form a closed system. The details of this procedure are laid out in Ref. 86. (Note that an alternate approach exists that uses a second canonical transformation to more systematically go to higher orders.[87]) The end result is a new set of coordinates given by

$$\boldsymbol{\mathcal{X}} = \mathbf{x} + \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial\mathbf{p}} - \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial\mathbf{x}}\cdot\frac{\partial^2 S(\mathbf{x},\mathbf{p},t)}{\partial\mathbf{p}\partial\mathbf{p}}, \tag{29}$$

$$\boldsymbol{\mathcal{P}} = \mathbf{p} - \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial\mathbf{x}} + \frac{\partial S(\mathbf{x},\mathbf{p},t)}{\partial\mathbf{x}}\cdot\frac{\partial^2 S(\mathbf{x},\mathbf{p},t)}{\partial\mathbf{p}\partial\mathbf{x}}. \tag{30}$$

Crucially, the function S turns out to be a function of the wave amplitude. This makes the coordinate transformations between the original and OC coordinates both complicated and generally time-dependent.

In the new coordinates, the quasilinear diffusion operator, which in the old coordinates affected both nonresonant and resonant particles, now only affects the resonant particles. The net result is that this diffusion operator is thus much more intuitive, as it does not have any recoil effects subtly embedded inside of it—all recoil effects have been transferred into the coordinate transformation [Eqs. (29) and (30)]. Furthermore, the resonant particle diffusion operator (in the original coordinates) is the same as in the averaged Eulerian theory, so that one does not need to actually calculate it with the complex coordinate transformations of OC theory in order to calculate the damped momentum and energy on resonant particles.[86] In particular, it is still true that the total momentum damped on the resonant particles forms a closed system with the Minkowski momentum. Thus, in the new coordinates, *quasilinear diffusion has the sole effect of transferring Minkowski momentum into the resonant particles.*

It turns out to be very instructive to look at the nonresonant particles in the new coordinates. The resulting Hamiltonian depends on the specific system, but generally takes the form an S-dependent "ponderomotive potential" or "renormalization energy," e.g., for electrostatic waves in an unmagnetized plasma

$$\mathcal{H} = \frac{\boldsymbol{\mathcal{P}}^2}{2m} + \Phi(\boldsymbol{\mathcal{X}},t);\quad \Phi(\boldsymbol{\mathcal{X}},t) \equiv \frac{q_s^2}{4m_s}\frac{|\tilde{\mathbf{E}}|^2}{(\omega - \mathbf{k}\cdot\boldsymbol{\mathcal{P}}/m)^2}. \tag{31}$$

Although these oscillation-center coordinates may currently seem very abstract, they will prove to be of great utility in studying certain types of problems.

VII. NATURAL STRENGTHS OF THE THEORIES

So far, two types of problem (plane wave initial value problems and steady-state boundary value problems) and two types of theoretical frameworks (Eulerian averages and oscillation-center theories) have been discussed. Fundamentally, either framework can be used to solve either type of problem. However, it turns out that each framework is much more naturally suited to a different problem type.

A. Steady-state boundary value problem

Consider first the steady-state boundary value problem and consider solving this problem by using the Eulerian theory. For simplicity, consider a fluid model of the species in the plasma. In Sec. III, it was established that it is the *net force on a fluid element* that drives charge transport in the plasma. Thus, we need to calculate this net force on the fluid volume.

Now, Eq. (20) gives the average local force applied by the *electromagnetic field* on species s. However, for a wave with spatial structure, this does not tell the whole story, because a new term enters the analysis: the Reynolds stress. This stress represents fluxes of momentum introduced to Eq. (9) through the fluid oscillations. In a plasma with no zeroth-order flow, it is given by

$$F_{\mathrm{Rey}} = -\nabla\cdot\mathbf{\Pi}_{\mathrm{Rey}};\quad \mathbf{\Pi}_{\mathrm{Rey}} \equiv \frac{1}{2}m_s n_0 \mathrm{Re}[\tilde{\mathbf{u}}_s\tilde{\mathbf{u}}_s]. \tag{32}$$

In a plasma with a zeroth-order flow, it becomes significantly more complicated, as cross terms between the density and flow fluctuations begin to appear at many orders. Even at this simplest level, the Reynolds stress cannot be generally expressed in terms of the susceptibility tensor $\boldsymbol{\chi}_s$ in a way, which yields *general* cancelations or insight. However, in several specific problems, including for ion-cyclotron-frequency turbulence,[52–60] current drive,[61,62] and lower hybrid waves,[66] cancelations tend to appear that eliminate the orbit-averaged force on nonresonant particles while leaving the force on resonant particles, hinting at a deeper reason for the force on nonresonant particles to cancel (though it should be noted that these theories generally still allow for *local* flow drive,[59–62] associated with wave structure along the direction of flow drive, which is neglected in the core model of this paper). These serendipitous cancelations and cascading number of second-order terms suggest that the Eulerian framework is not the best adapted to steady-state boundary value problems with a core symmetry.

Consider instead, then, the oscillation-center theory for the boundary value problem. First consider the resonant particles. From the discussion of closed systems, recall that Minkowski momentum enters the plasma as a spatial flux [Eq. (12)], which is then damped on the resonant particles through the resonant quasilinear diffusion operator (which is the same in the OC and Eulerian theories). This constitutes the total resonant response.

Now consider the nonresonant particles, governed by the ponderomotive Hamiltonian. From Eq. (30), we see that $\langle d\mathbf{p}/dt\rangle$ is equal to $\langle d\boldsymbol{\mathcal{P}}/dt\rangle$ plus terms that depend on $\langle dS/dt\rangle$, and $\langle dS/dt\rangle = 0$, for a steady-state wave. Thus, in steady state, $\langle \mathbf{p}\rangle$ is a constant function of $\langle\boldsymbol{\mathcal{P}}\rangle$. Now, along the symmetry direction y, the ponderomotive potential does not change, so that $d\mathcal{P}_y/dt = -d\mathcal{H}/d\mathcal{Y} = 0$. Thus, $\langle dp_y/dt\rangle = 0$, i.e., there is no force along y for the nonresonant particles. Thus, while analysis of the steady-state boundary value problem was quite complicated in the Eulerian framework, it is almost trivial in the oscillation-center framework.

Indeed, it should be noted the oscillation-center theory makes an even stronger statement than used here, since even if the wave has structure along y, the net force exerted by the wave along y for a closed periodic orbit will always go away, since the force arises from the gradient of a ponderomotive potential.[66] This is the reason nonresonant forces tend not to survive flux surface averages: a frequent result in (Eulerian) fluid and kinetic studies of low-frequency turbulence.[52–58]

In particular, the oscillation-center theory reveals that a wave need not have electromagnetic momentum nor apply a Maxwell stress on the plasma in order to damp momentum in the plasma. It is interesting to consider, then, how this Minkowski momentum ended up in the plasma in the first place, given a vacuum gap between the plasma and the wave antennae. In fact, it turns out that any Minkowski momentum in the plasma must have originated as electromagnetic momentum traversing the vacuum to enter the plasma. For instance, the lower hybrid wave begins its life as an evanescent vacuum wave, which couples to a slow wave in the plasma. While this evanescent wave cannot be treated by geometrical optics, it can be shown that the electromagnetic momentum flux (Maxwell stress) in the evanescent wave region is the same as the Minkowski momentum flux that eventually damps in the plasma.[51]

It should be noted that the gradient structure of the nonresonant ponderomotive Hamiltonian is particularly useful when the original coordinates (before applying Dewar's theory) are action-angle variables, as used in toroidal systems, since it reveals that the net result of the ponderomotive potential is a slight shift to the orbit. Once this key result of OC theory is known (and if one does not particularly care about the details of this shift), it is not necessary to actually calculate the quasilinear diffusion in the OC coordinates. Instead, one can simply apply the purely *resonant* quasilinear diffusion operator in the *original* coordinates to calculate wave-driven transport, without worrying that one is missing major canceling ponderomotive torques. This much more straightforward calculation is the basis for much of the theory of resonant wave-driven transport,[76,88,89] but it must be remembered that the approximation is only valid for static wave structures, as it throws away all momentum-conserving ponderomotive recoil terms.

B. Time-dependent initial value problem

Now consider the plane-wave initial value problem. Attempting to apply the oscillation-center theory is tricky, because now the wave amplitude is time-dependent; thus, the coordinate transformation in Eqs. (29) and (30) is also time-dependent, meaning that the nonresonant reaction is largely hidden in a very complicated coordinate transformation. Thus, the oscillation-center framework is not particularly well-adapted to this problem.

Consider instead the Eulerian approach. In the plane-wave initial value problem, the subtleties involved with the Reynolds stresses vanish, and local force averages are completely straightforward following from Eq. (20). Since this force respects momentum conservation between the particles and the electromagnetic fields, it is clear that the total momentum gained by the plasma will be whatever momentum is lost from the electromagnetic fields due to the change in the Poynting flux. Thus, for the instability, flow is only driven if the wave gains or loses electromagnetic momentum.

It is rather remarkable how the two theoretical frameworks (Eulerian vs Oscillation Center) are so strongly naturally adapted to the two different types of problems (plane-wave IVP vs steady-state BVP), and how these in turn each reveal a connection of that problem to a different type of fundamental wave momentum (electromagnetic vs Minkowski). These relationships are summarized in Fig. 8.

VIII. CONCLUSION: WHEN DO WAVES DRIVE PLASMA FLOWS?

In this review, consideration of the canonical momentum in collisional classical transport problems revealed a fundamental connection between charge transport and momentum conservation, demonstrating the importance a self-consistent, momentum-conserving theory of wave-driven rotation. Consideration was thus given to the two types of

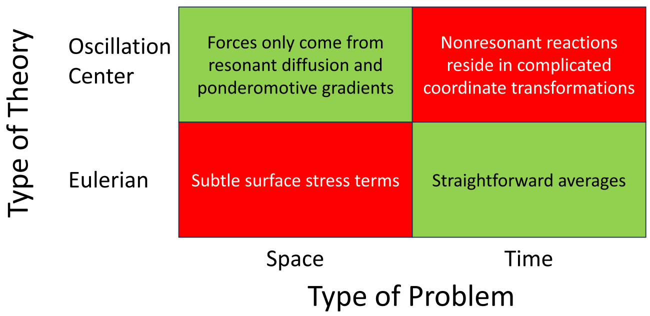

FIG. 8. Each type of theoretic framework in Sec. VI is naturally adapted to a different type of problem in Sec. V. Using the naturally adapted theory to each problem type reveals the role played by the various types of wave momenta in Sec. IV.

momentum contained by a wave. It was shown that two types of momentum-conserving quasilinear theory existed, which were fundamentally well-adapted to two different types of wave problems. This in turn allows one to determine when waves drive plasma flows in a definitive and intuitive manner. For the plane-wave initial value problem, as for instabilities, a wave only drives a plasma flow if, during the growth of the instability, the electromagnetic momentum [Eq. (5)] of the wave changes; this lost momentum then appears as flow momentum in the plasma. For the steady-state boundary value problem, as in current drive and heating, only the resonant particles feel a force along the symmetry direction, and thus flow is driven according to the flux of Minkowski momentum into the plasma [Eq. (12)]. Furthermore, these results hold not just for driving $\mathbf{E}\times\mathbf{B}$ flow drive (Figs. 1–3), but for parallel flow drive as well.

These results imply that it is not sufficient to look only at the resonant particles when asking whether a certain quasilinear diffusion process will drive flow–one must consider the overall wave dynamics as well. For instance, the resonant diffusion process described in Fig. 3 will not drive flow if, as a result of the resonant diffusion, a plane wave grows in time, but it will drive flow if, as a result of the diffusion process, a steady-state wave grows in space.

Of course, in any real system, some combination of these two limiting behaviors will be happening. Further complicating matters, in geometries with broken symmetry, is the fact that an applied torque does not necessarily produce steady-state rotation in the expected direction, as observed for instance in the decay of poloidal rotation into toroidal rotation in tokamak plasmas.[90] Such effects generally result in a complex set of interconnected relations that determine the steady-state flow.[91] Nevertheless, it is hoped that the current discussion provides the tools necessary for approaching wave-driven flows in any given system, in a way which is both intuitive and self-consistent.

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AUTHOR DECLARATIONS

Conflict of Interest

The author has no conflicts to disclose.

Author Contributions

Ian E. Ochs: Conceptualization (equal); Formal analysis (equal); Investigation (equal); Visualization (equal); Writing—original draft (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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